t-tests (comparing means)

Ling250/450: Data Science for Linguistics
C.M. Downey
Spring 2025

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  - $p < 0.05$ means "less than 5% chance the Null would give us the data we observe"

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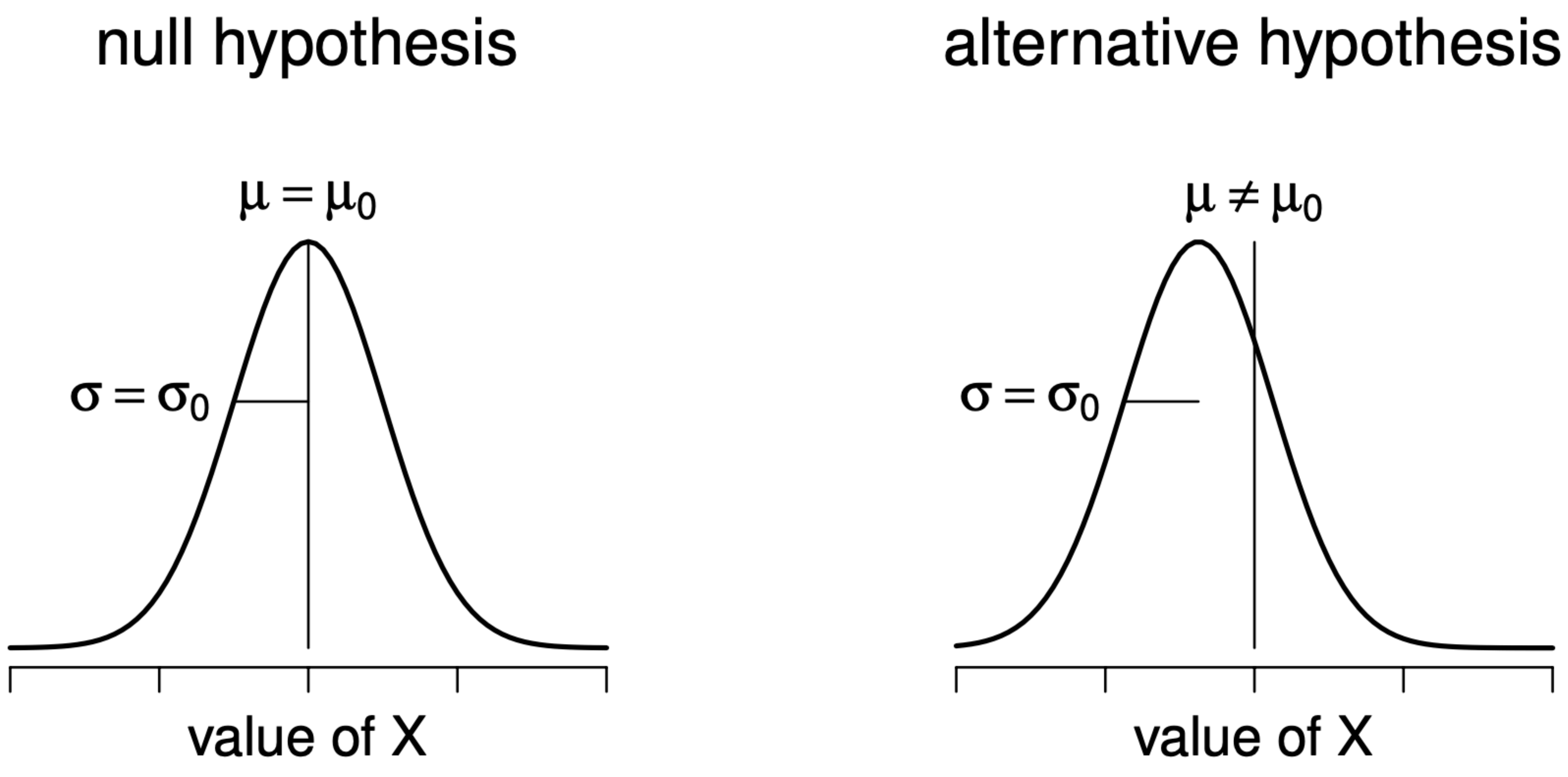
- We'll only look at the math of the one-sample test in detail

One-sample t-test

First, the z-test



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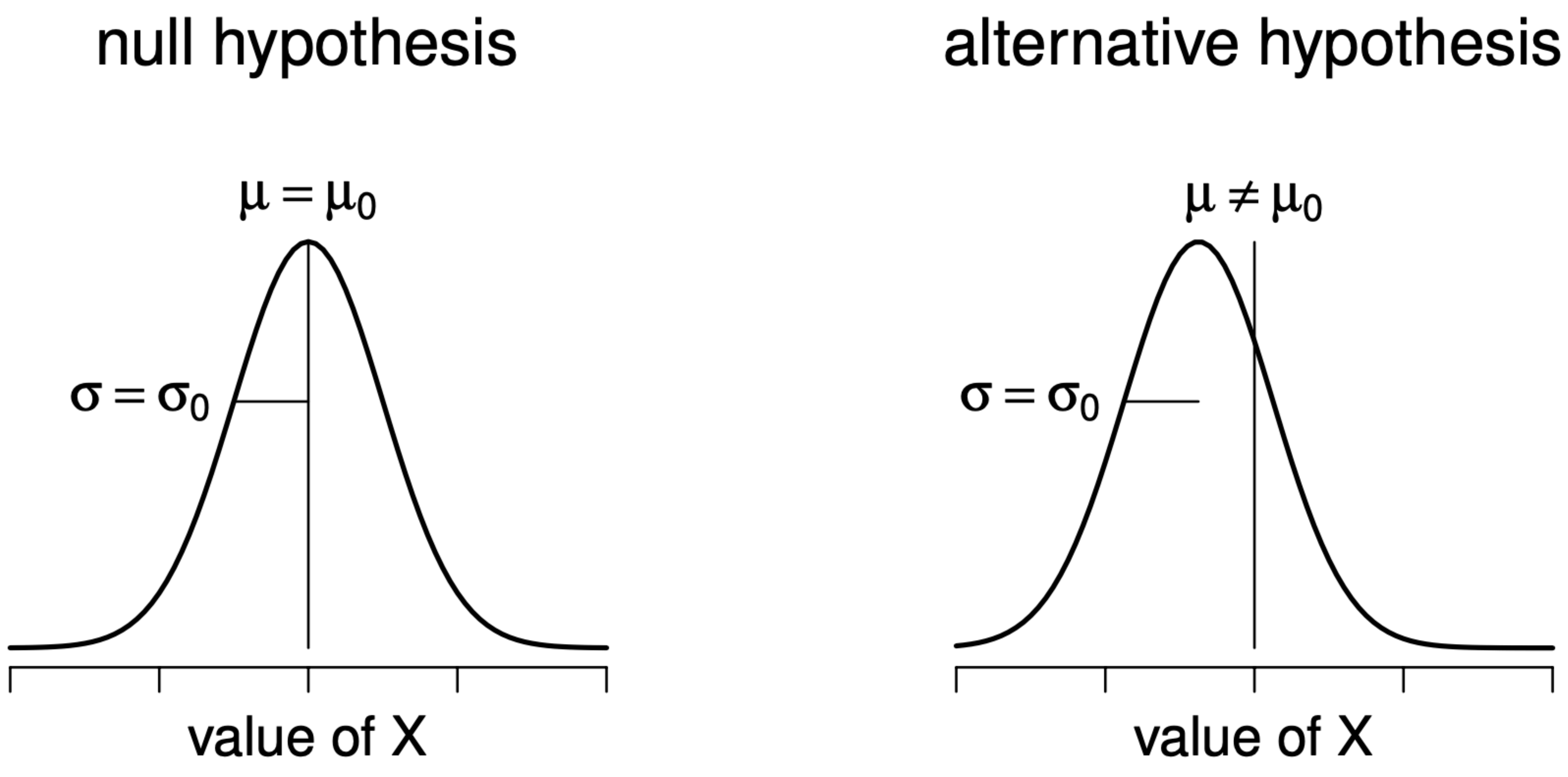

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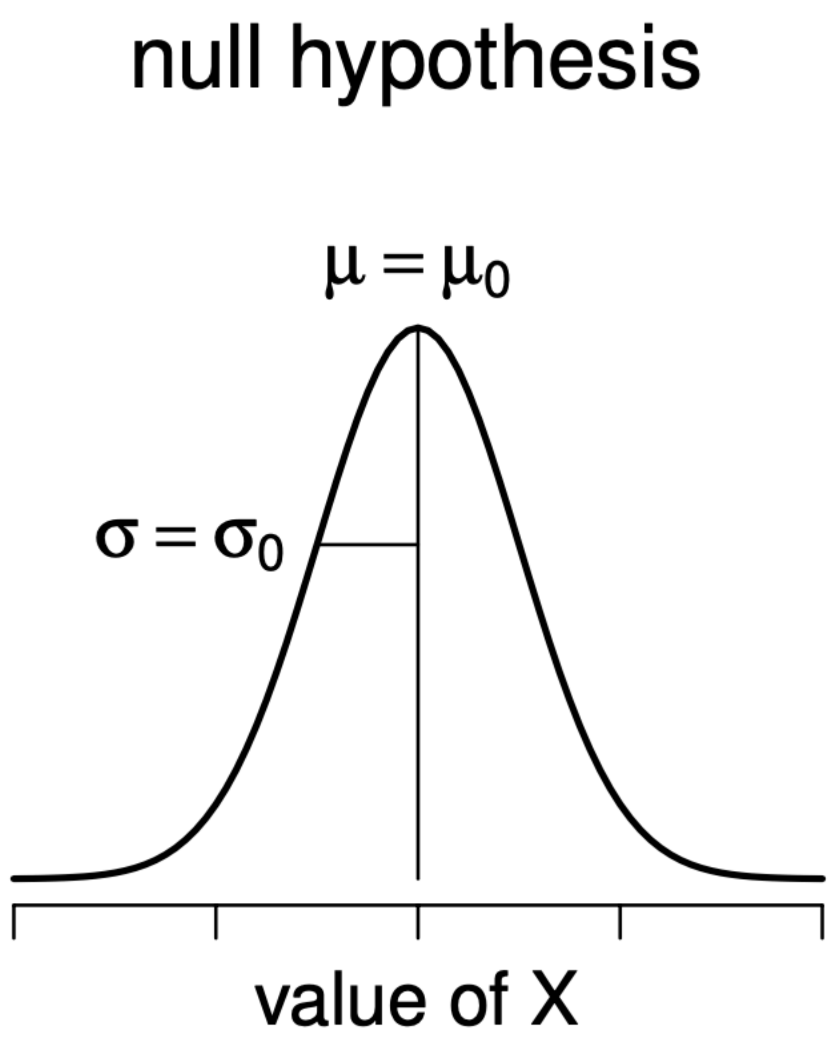

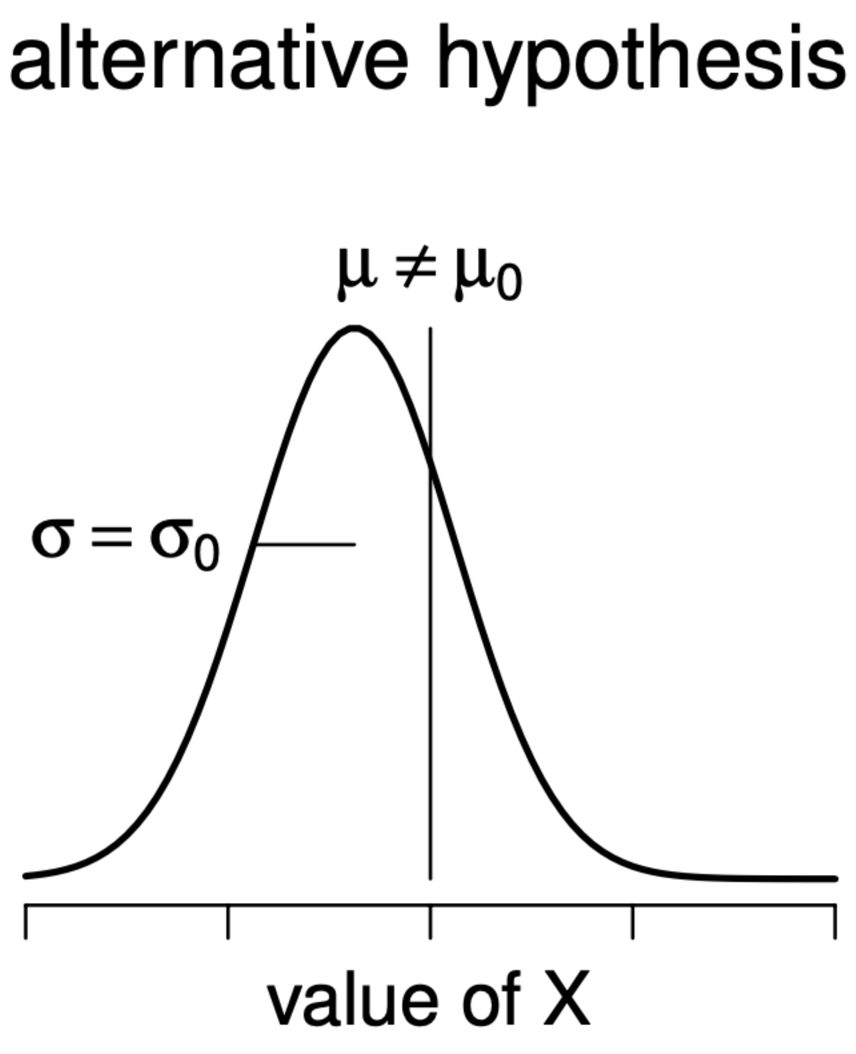

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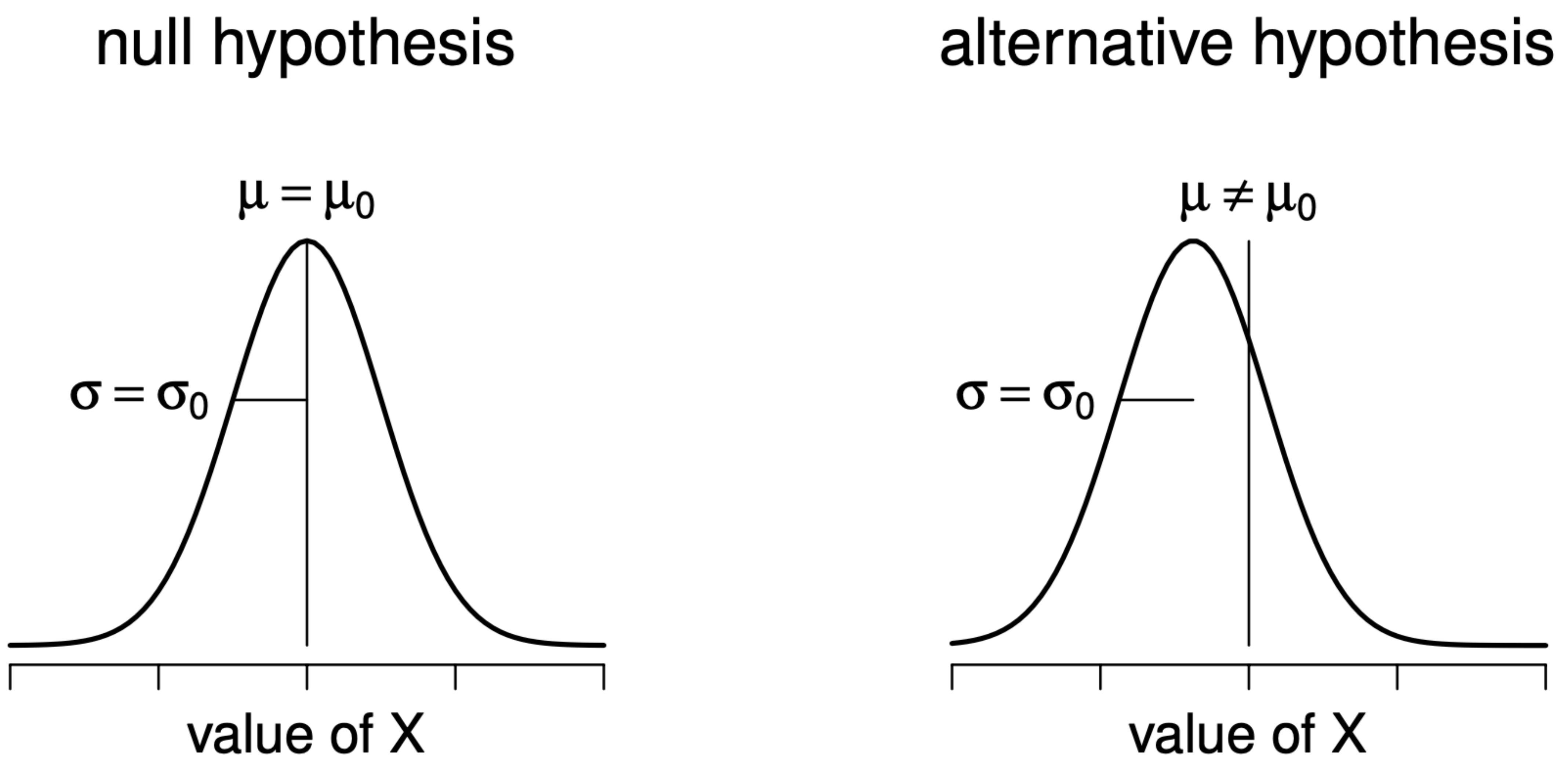

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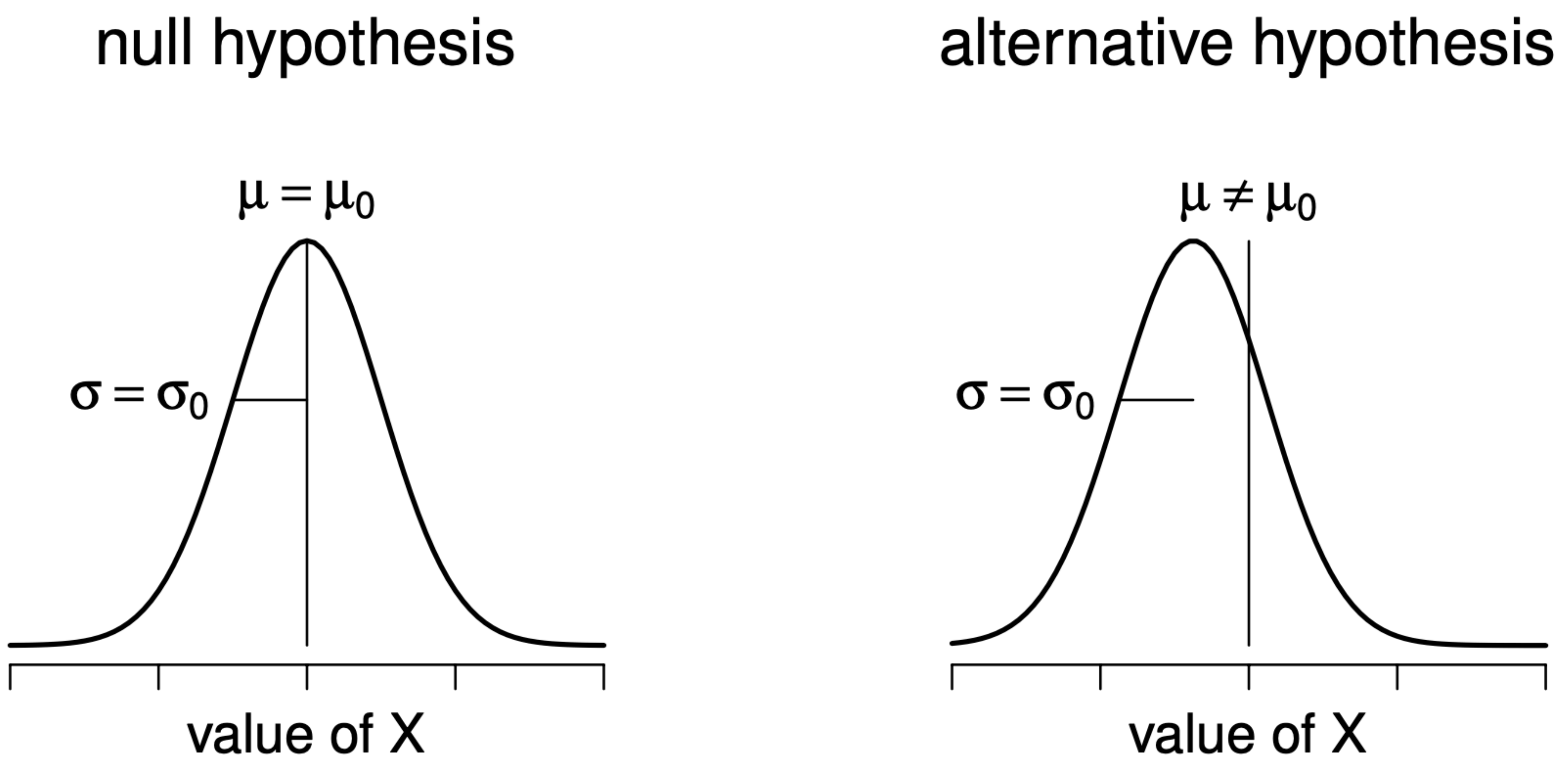

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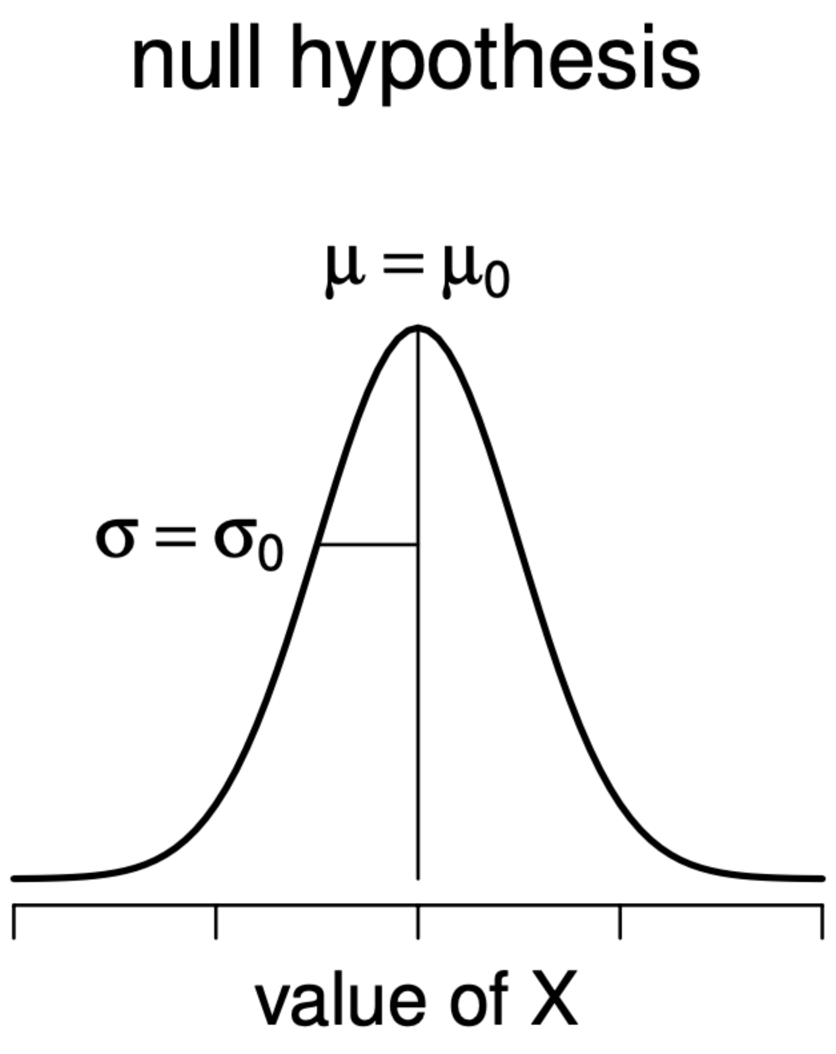
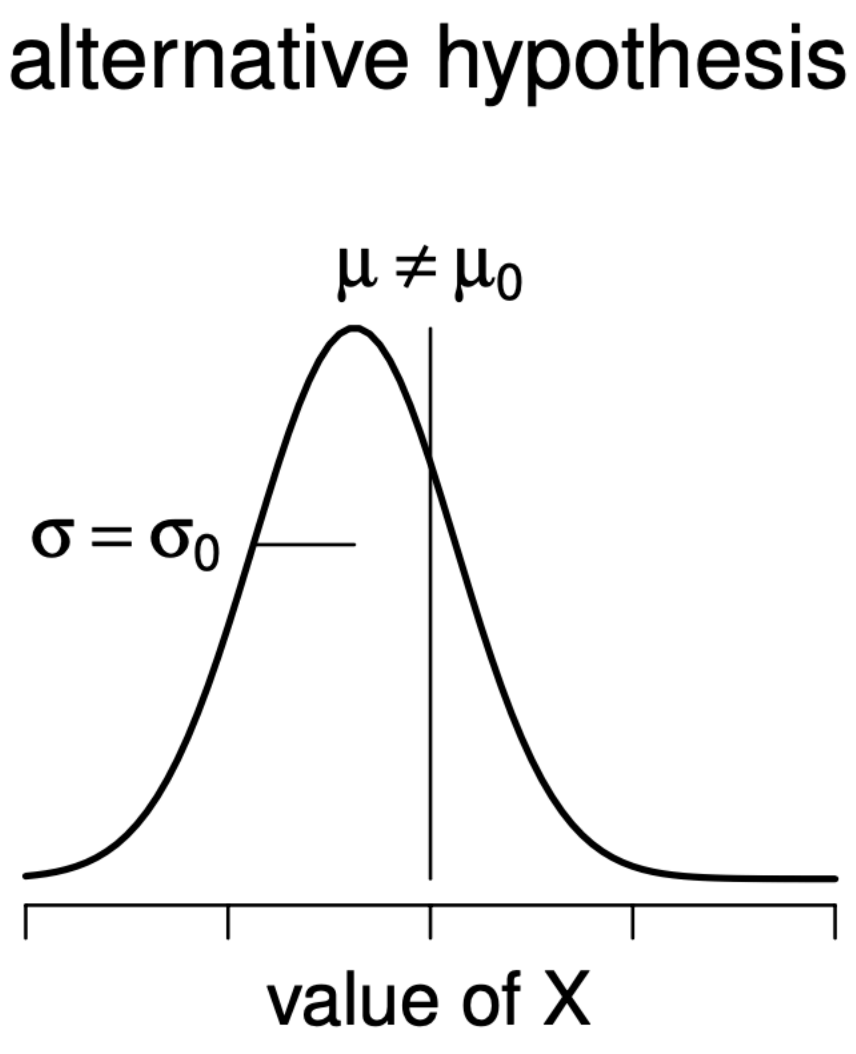
  - Pretend example: do linguistics students have the **same average grade** as the entire class taking a foreign language?





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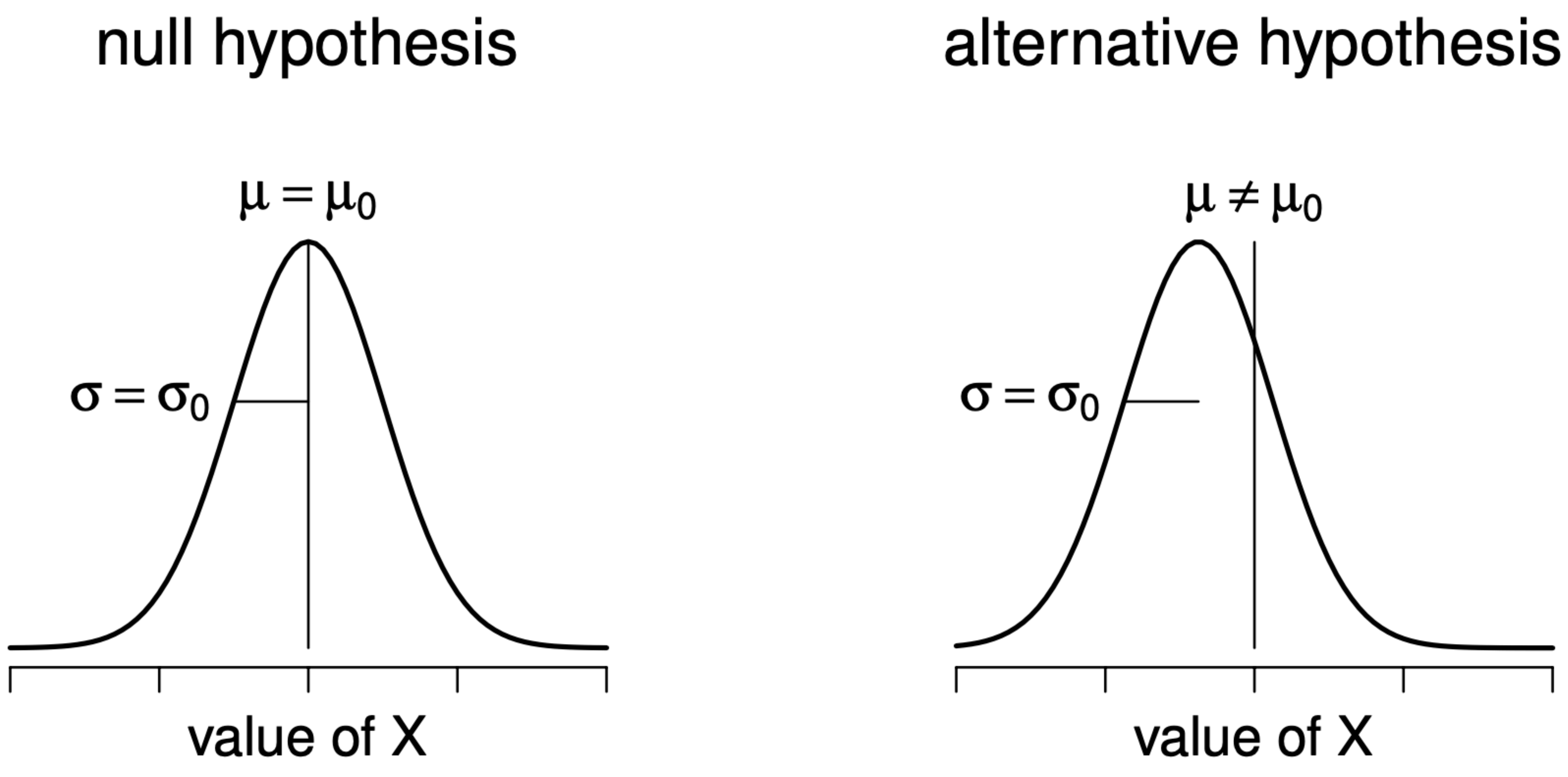

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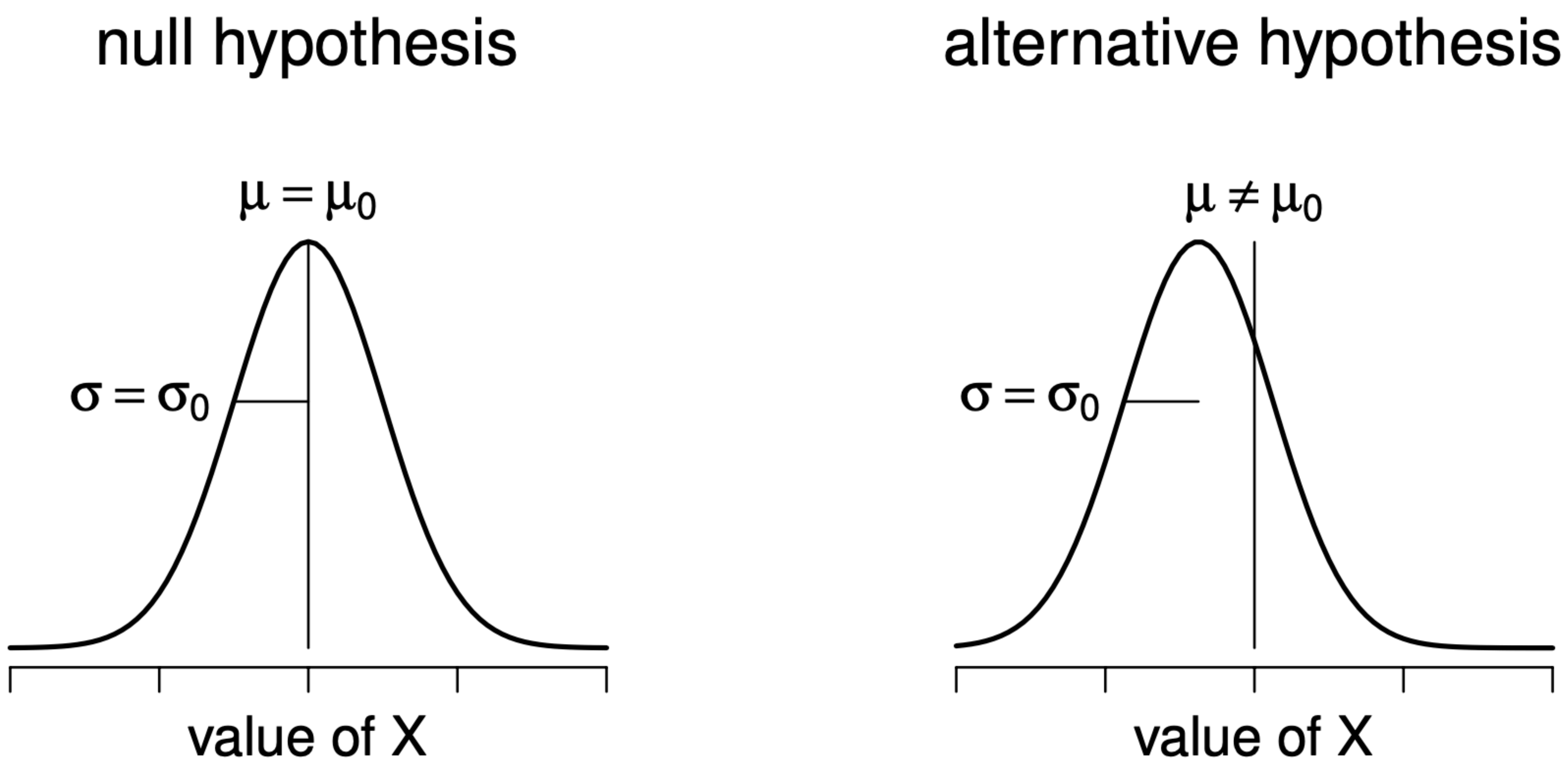
- Our **Null hypothesis** is that the Ling students have the **same mean grade** as the course overall
  - μ_0 is the mean course grade (67.5)
  - $H_0 : \mu = \mu_0 = 67.5$



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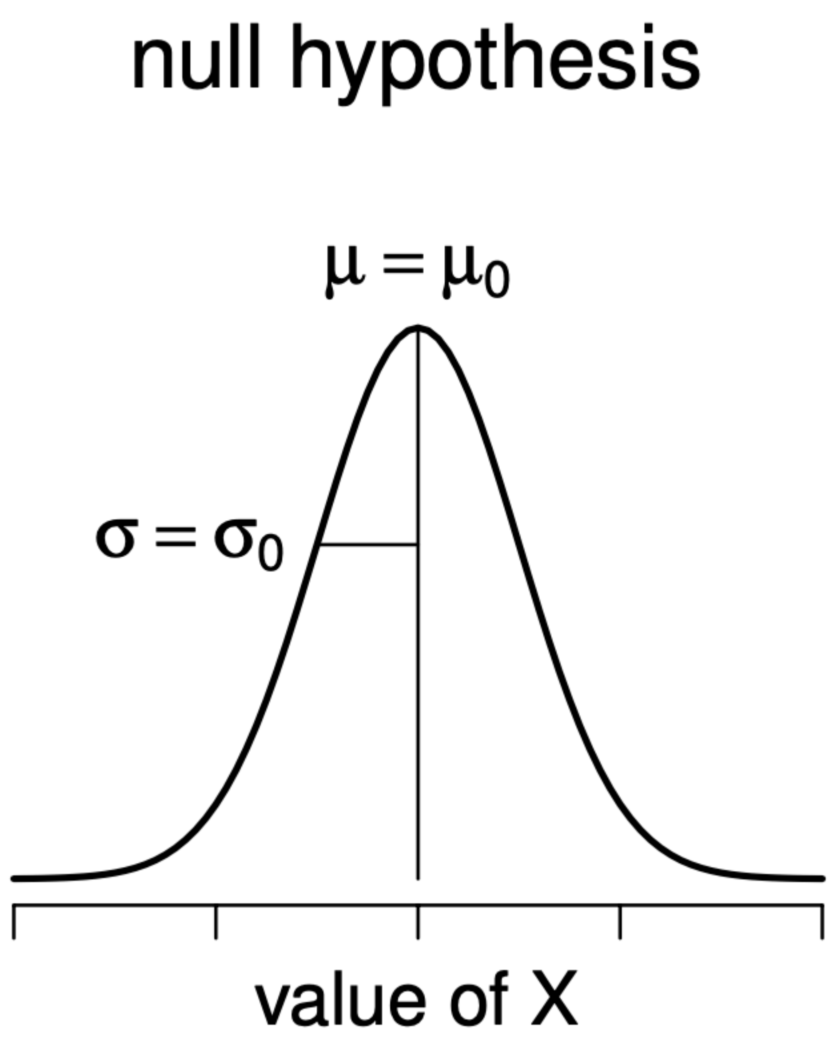
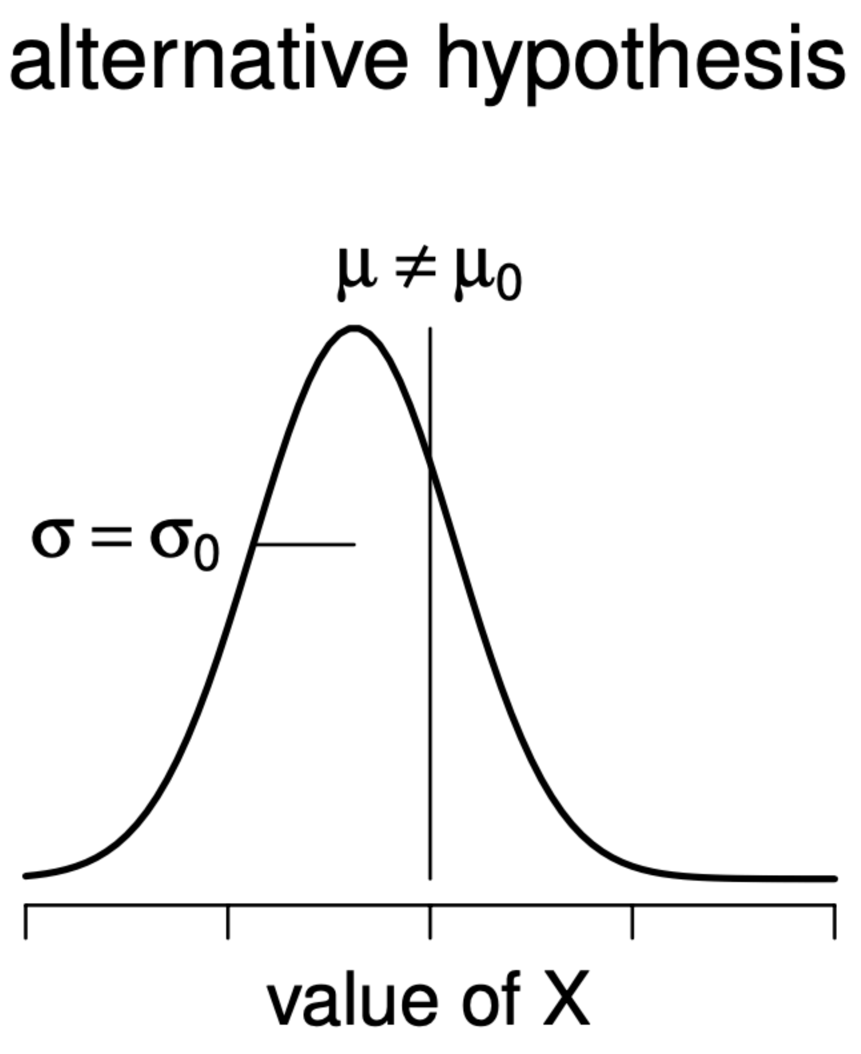
- The **alternative hypothesis** is the Ling students have a **different mean grade**
  - $H_1 : \mu \neq \mu_0 = 67.5$





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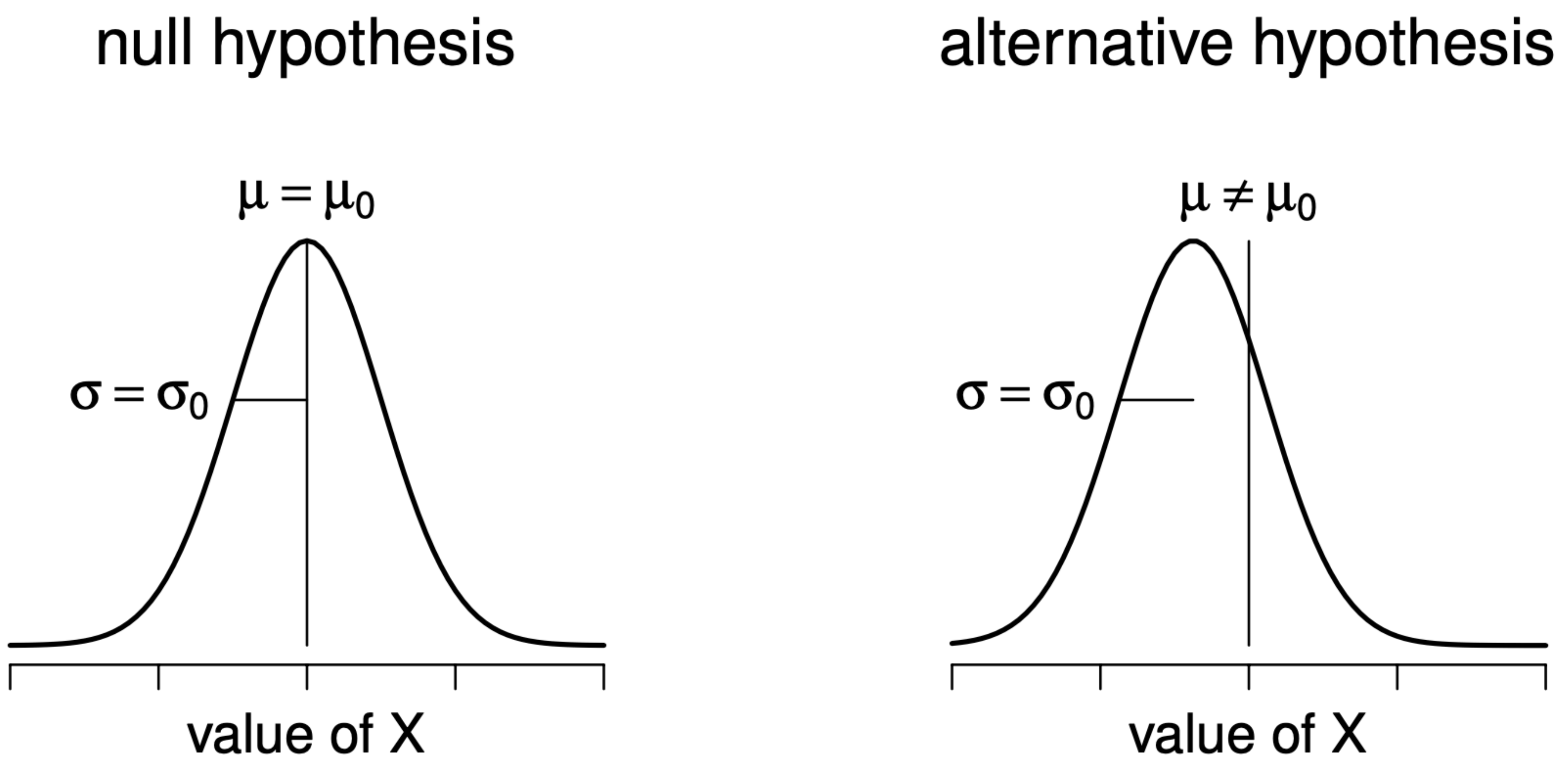

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- **Big assumption**: we assume we know the **standard deviation** of the **population we compare against** (e.g. the whole class; σ_0)
  - Also that our **sample population** (Ling students) has the same stdev
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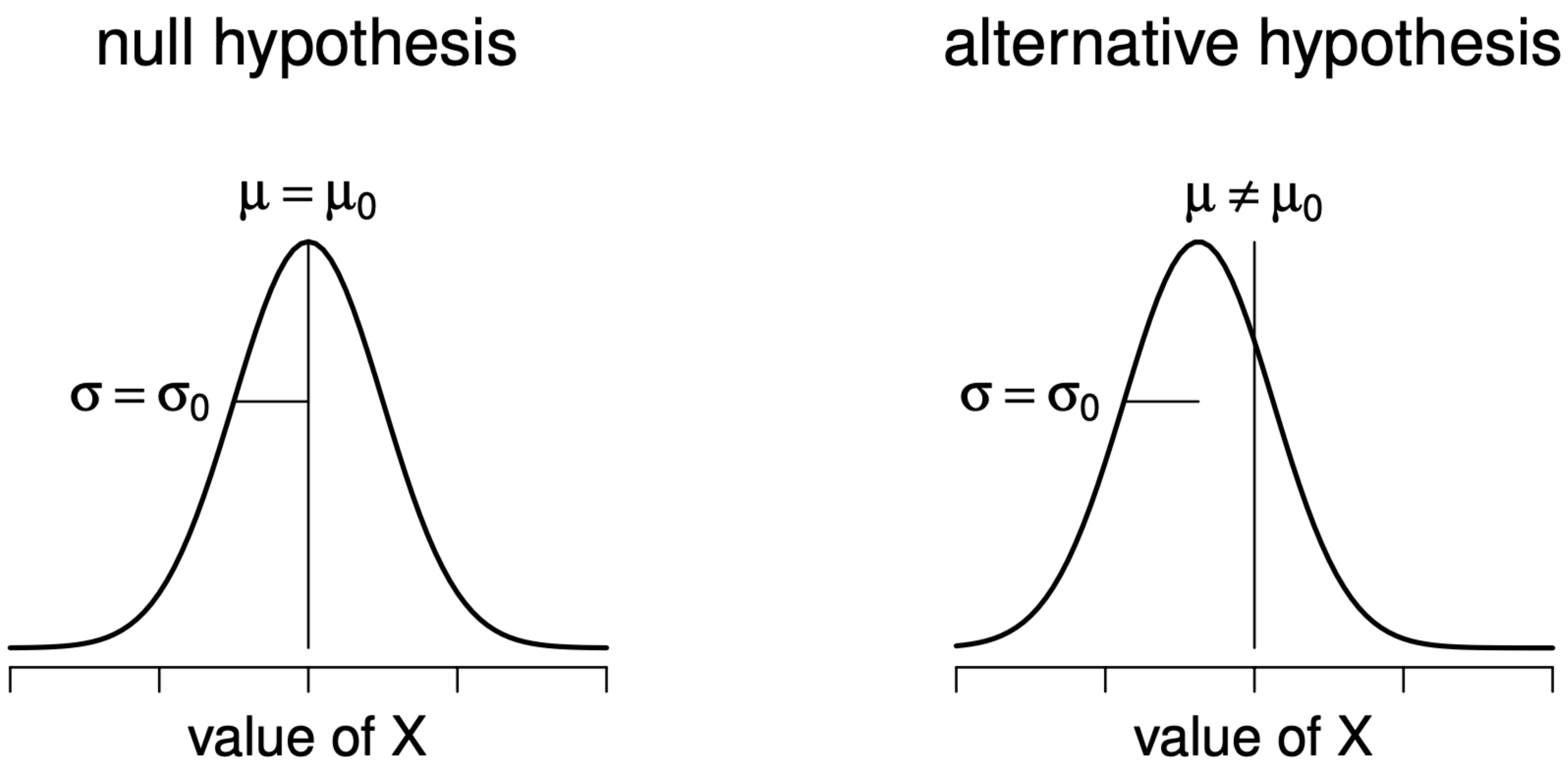

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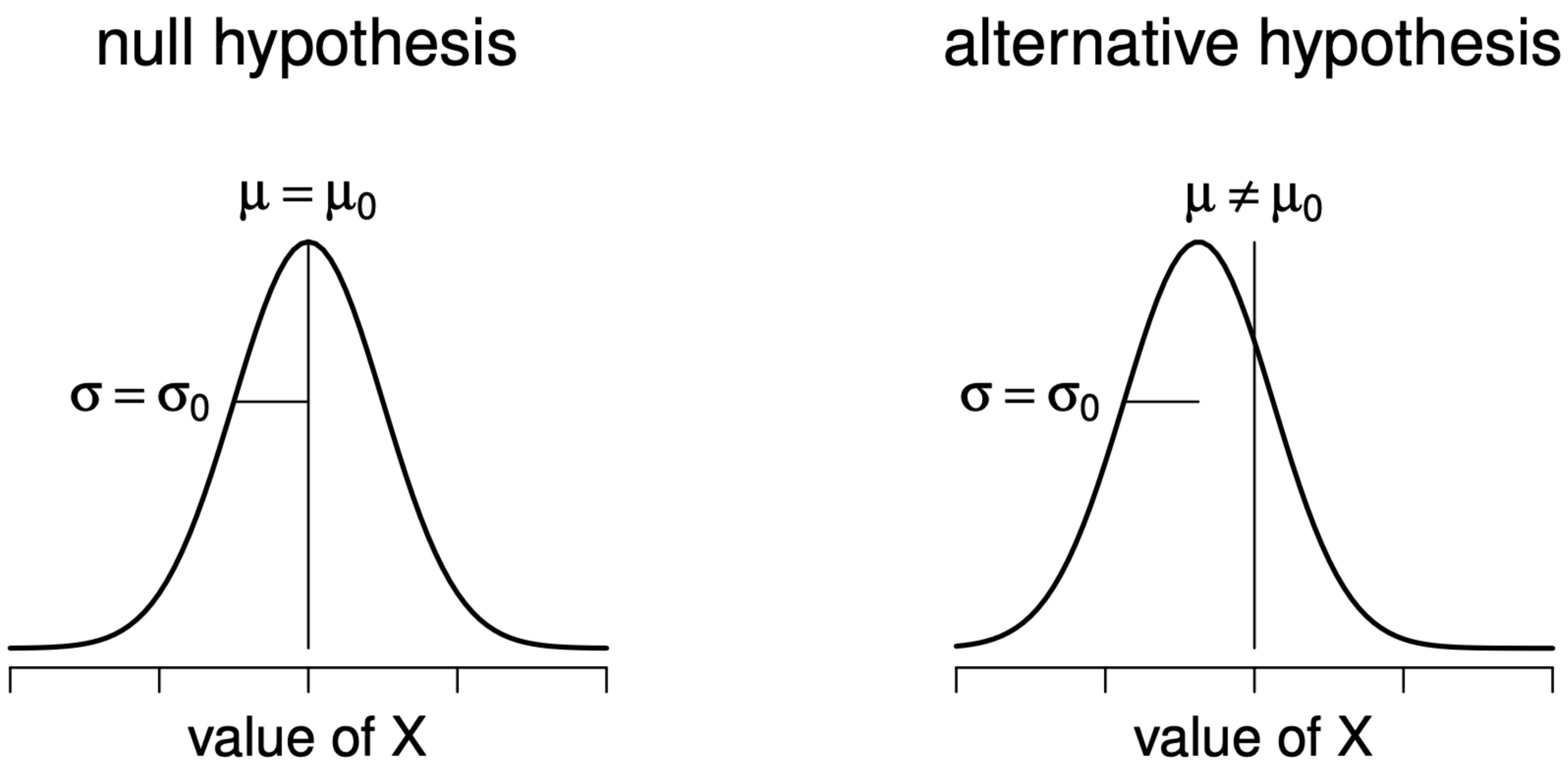
  - These are **bad assumptions**, which will lead us to the t-test
- We also assume the **sampling distribution** of the mean is **Normal**
  - This is usually a **safe-ish** assumption (see lecture on probability distributions)



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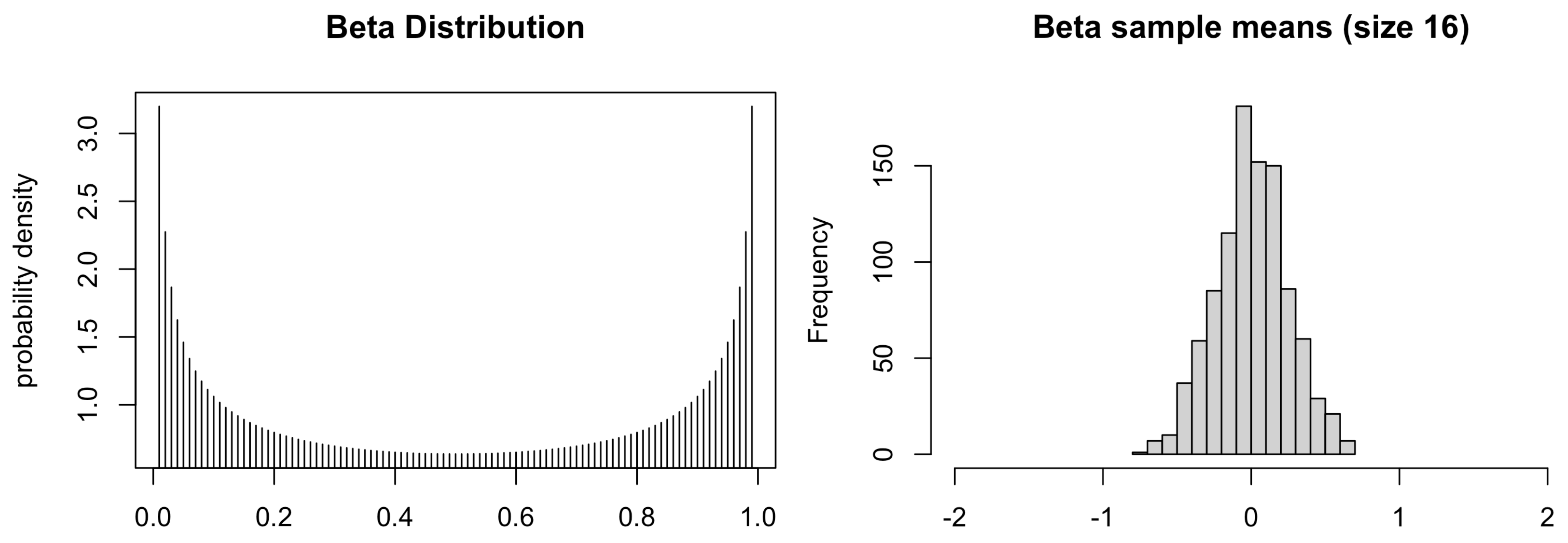
Recall: Sampling distributions



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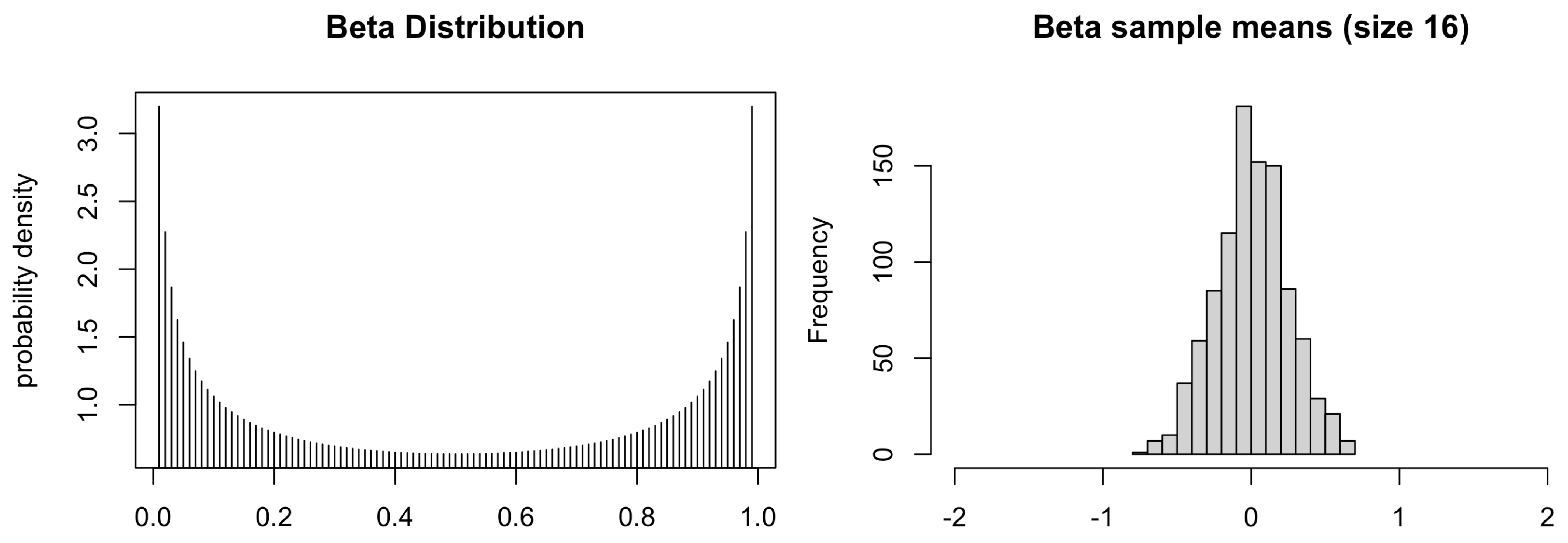

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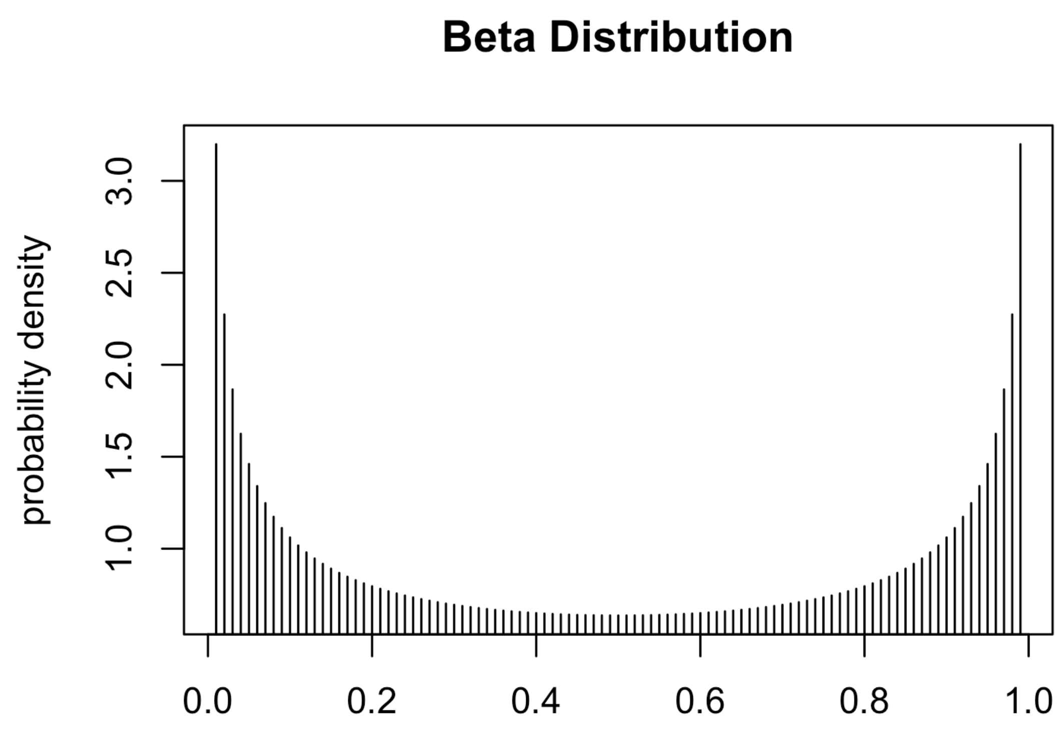

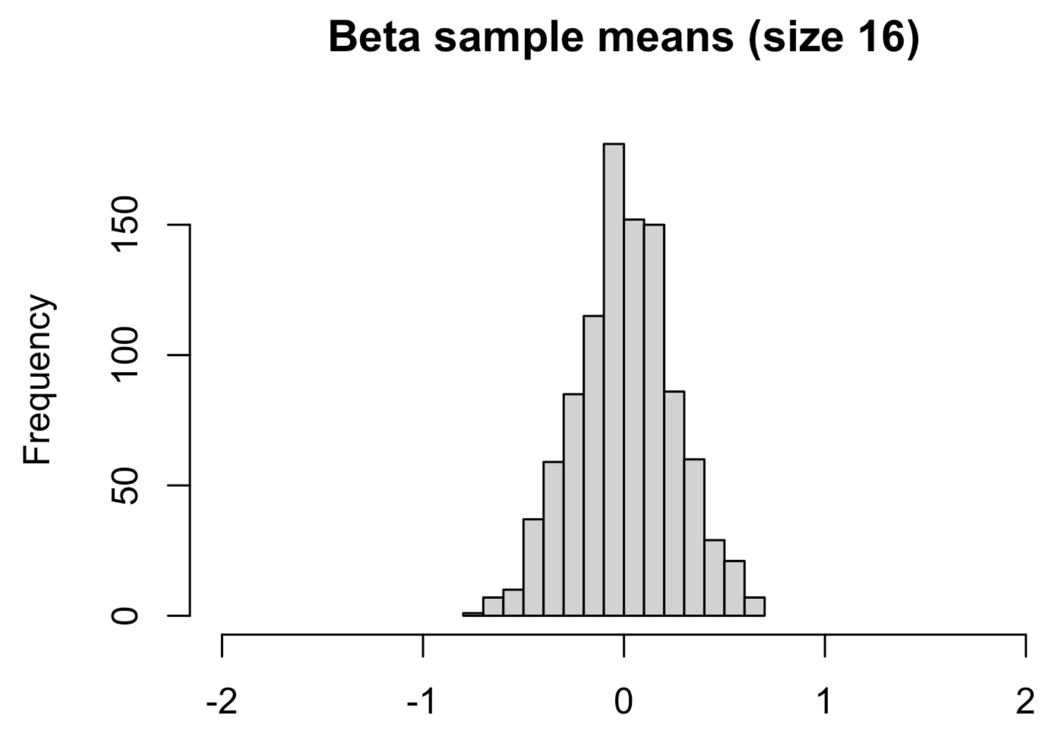

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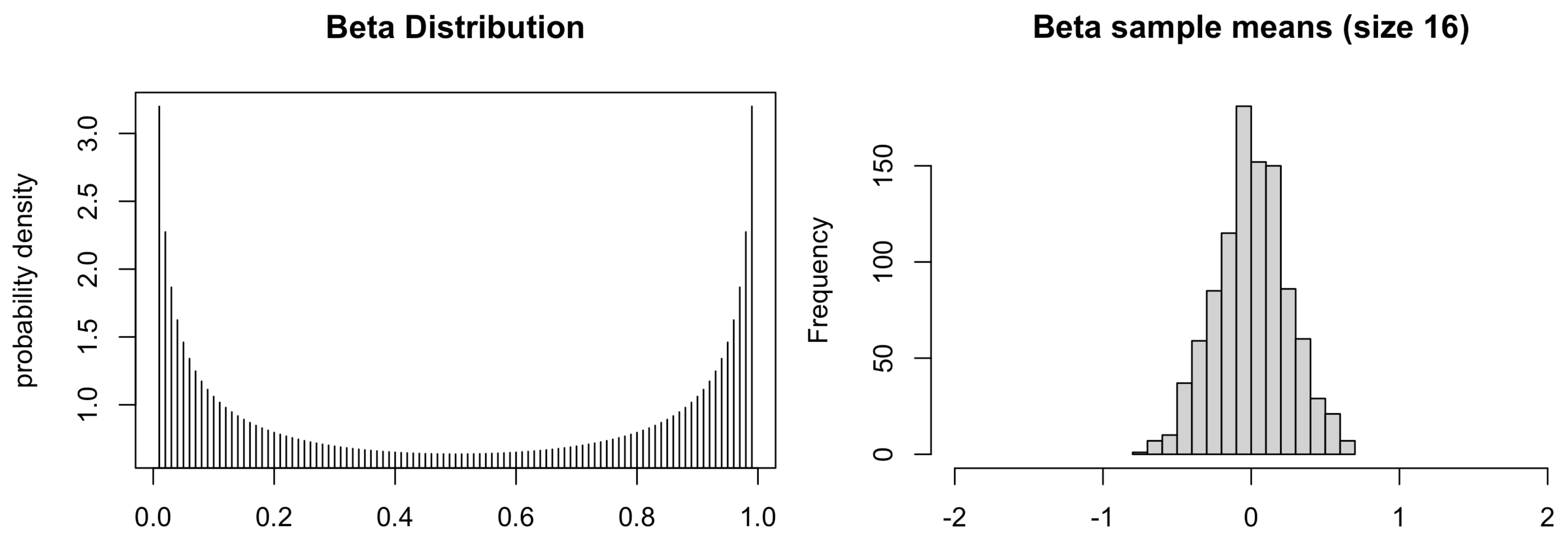

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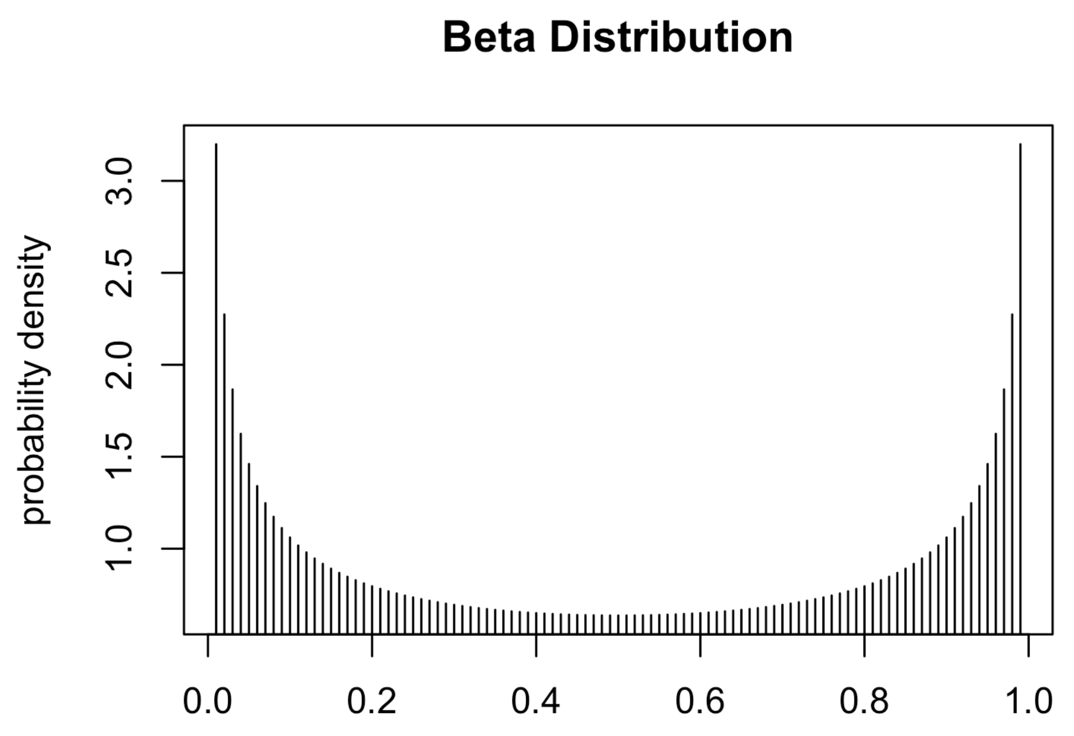

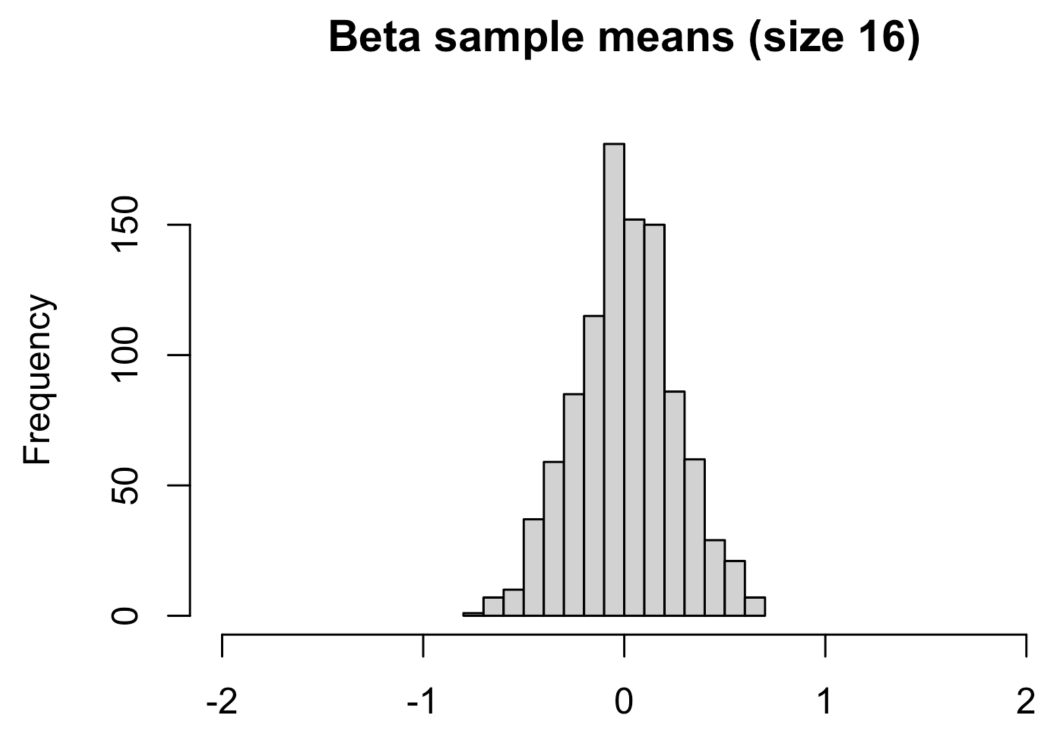

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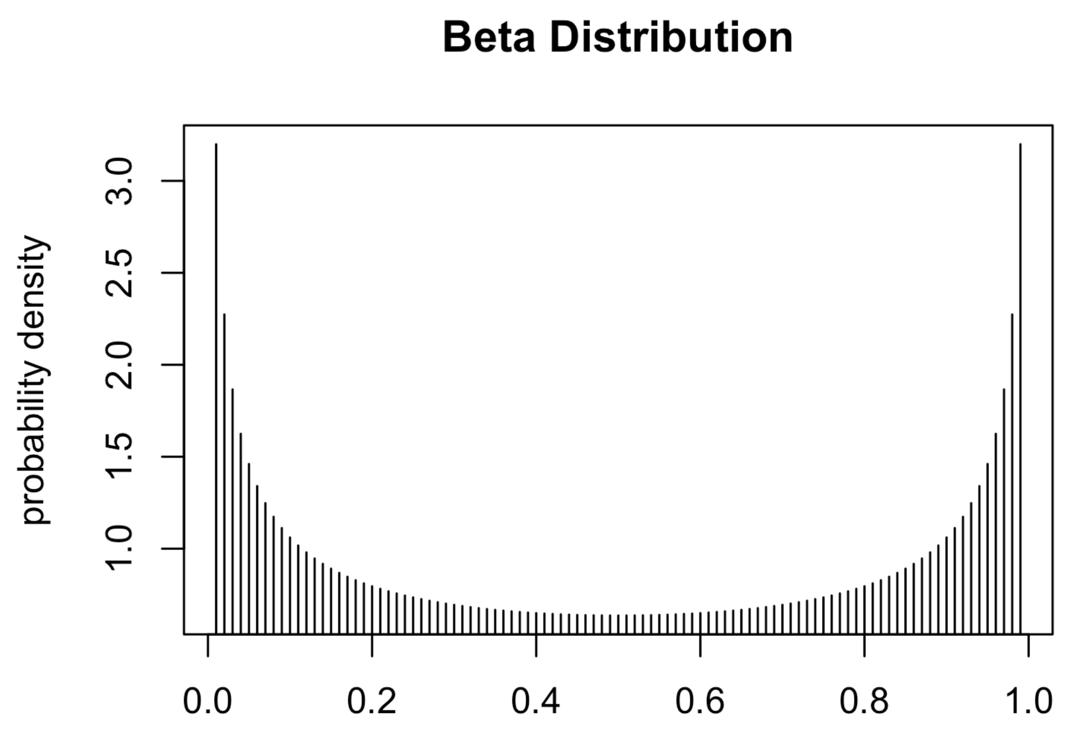

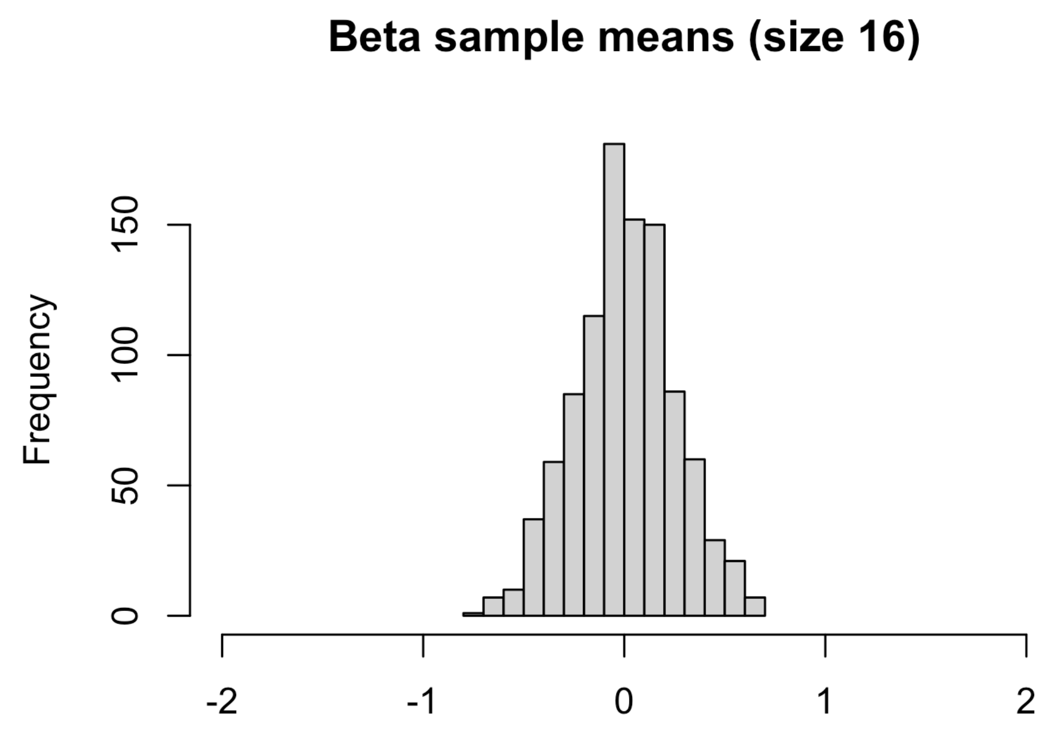

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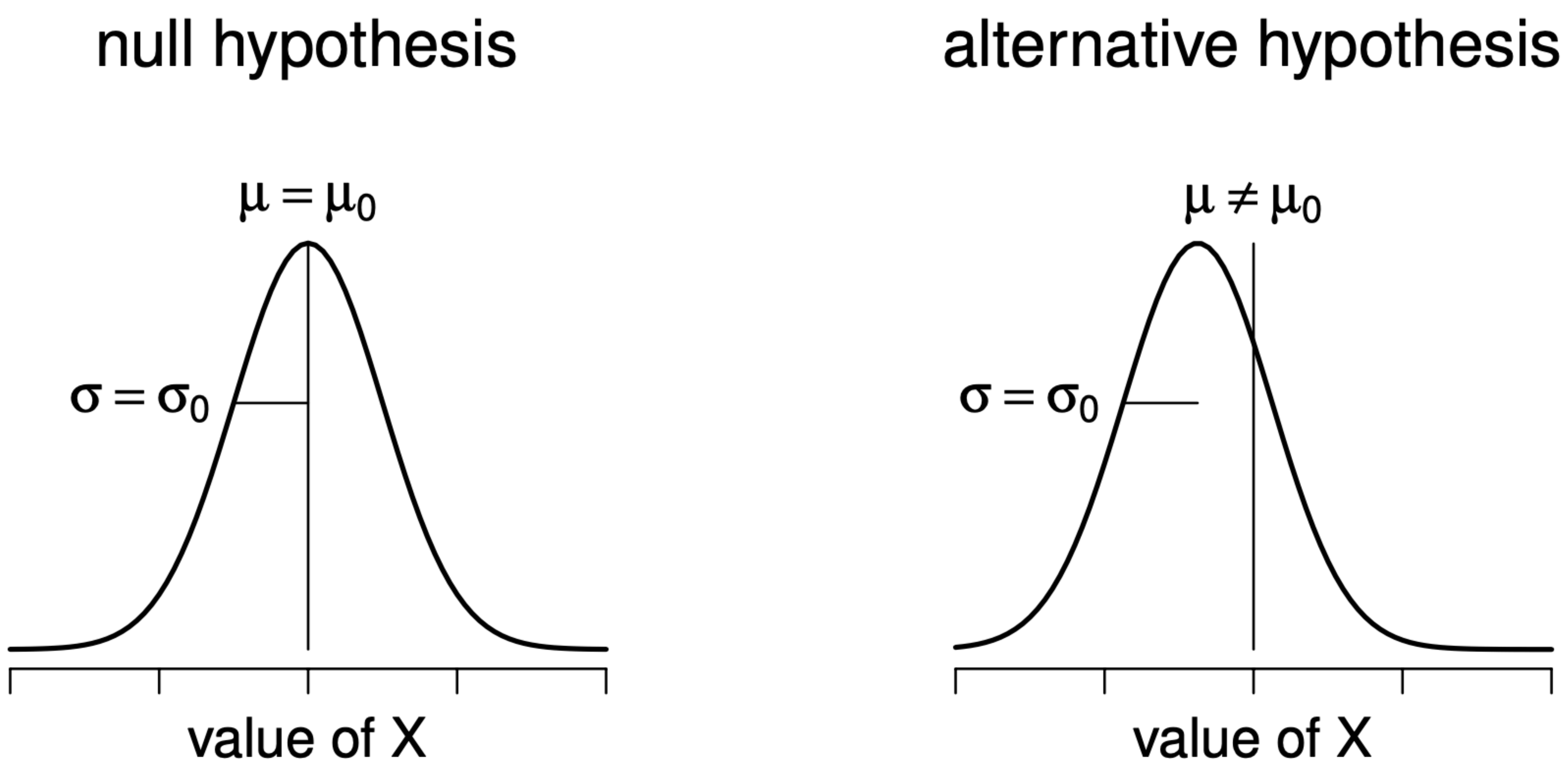

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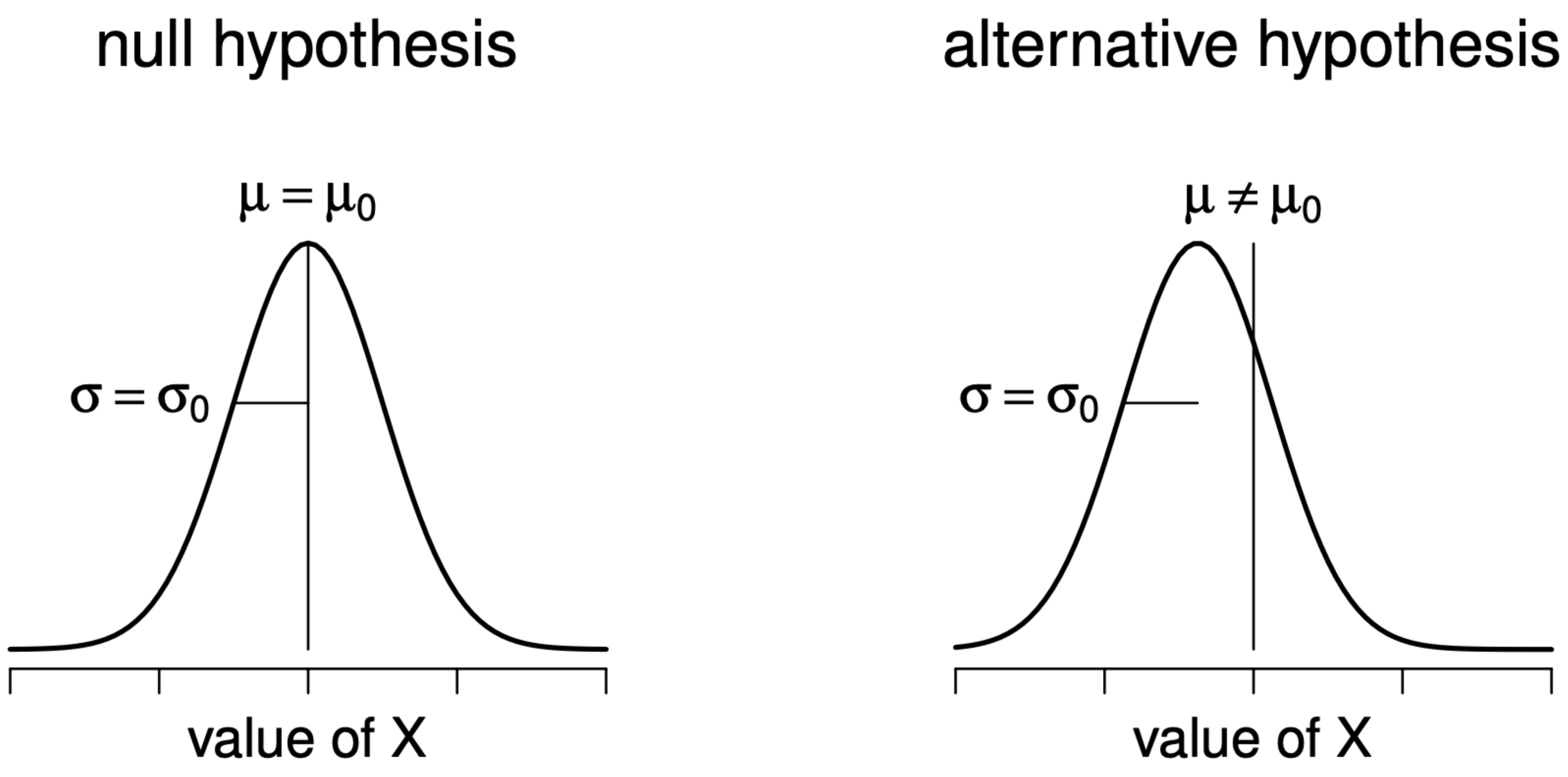

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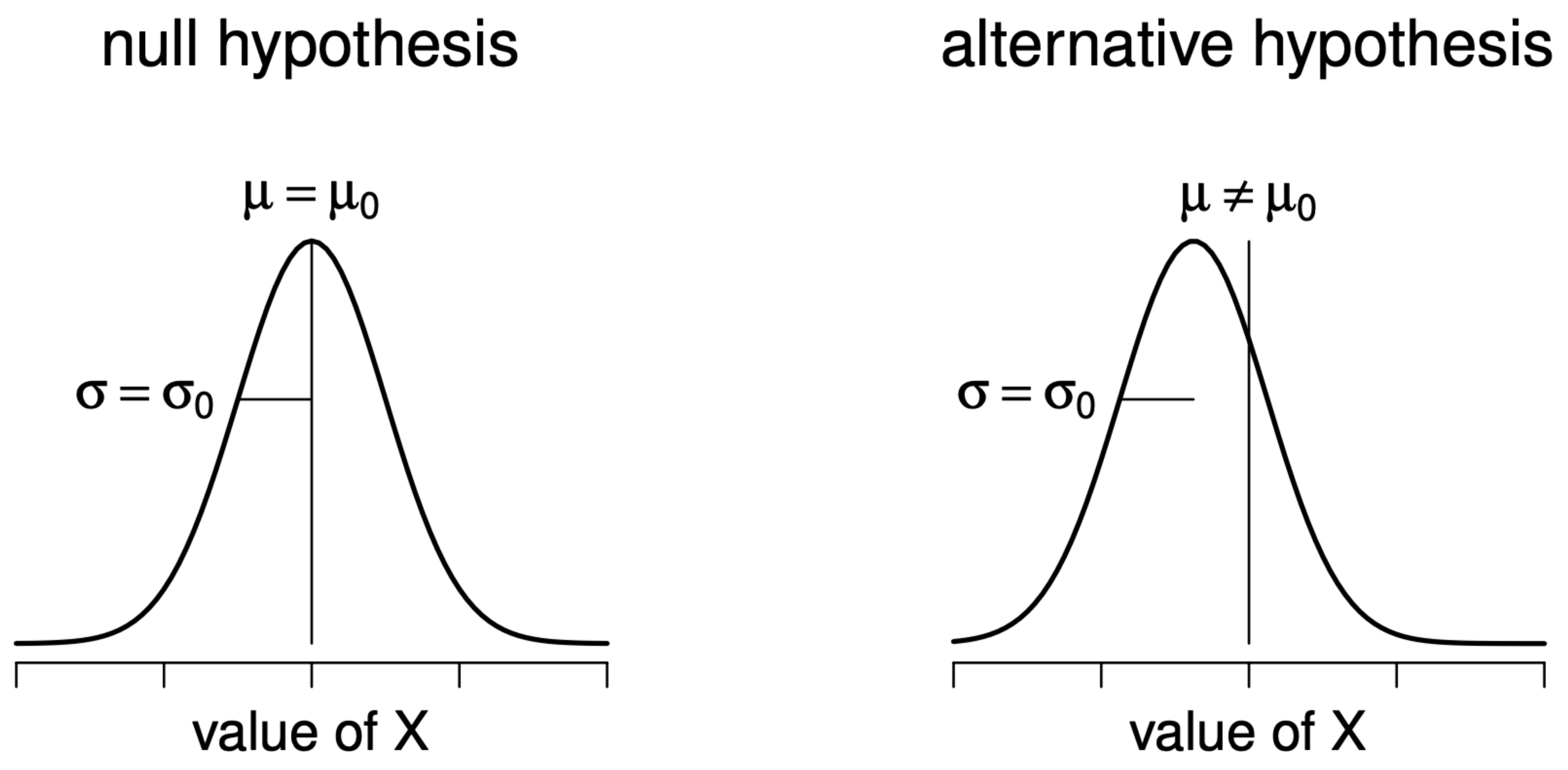

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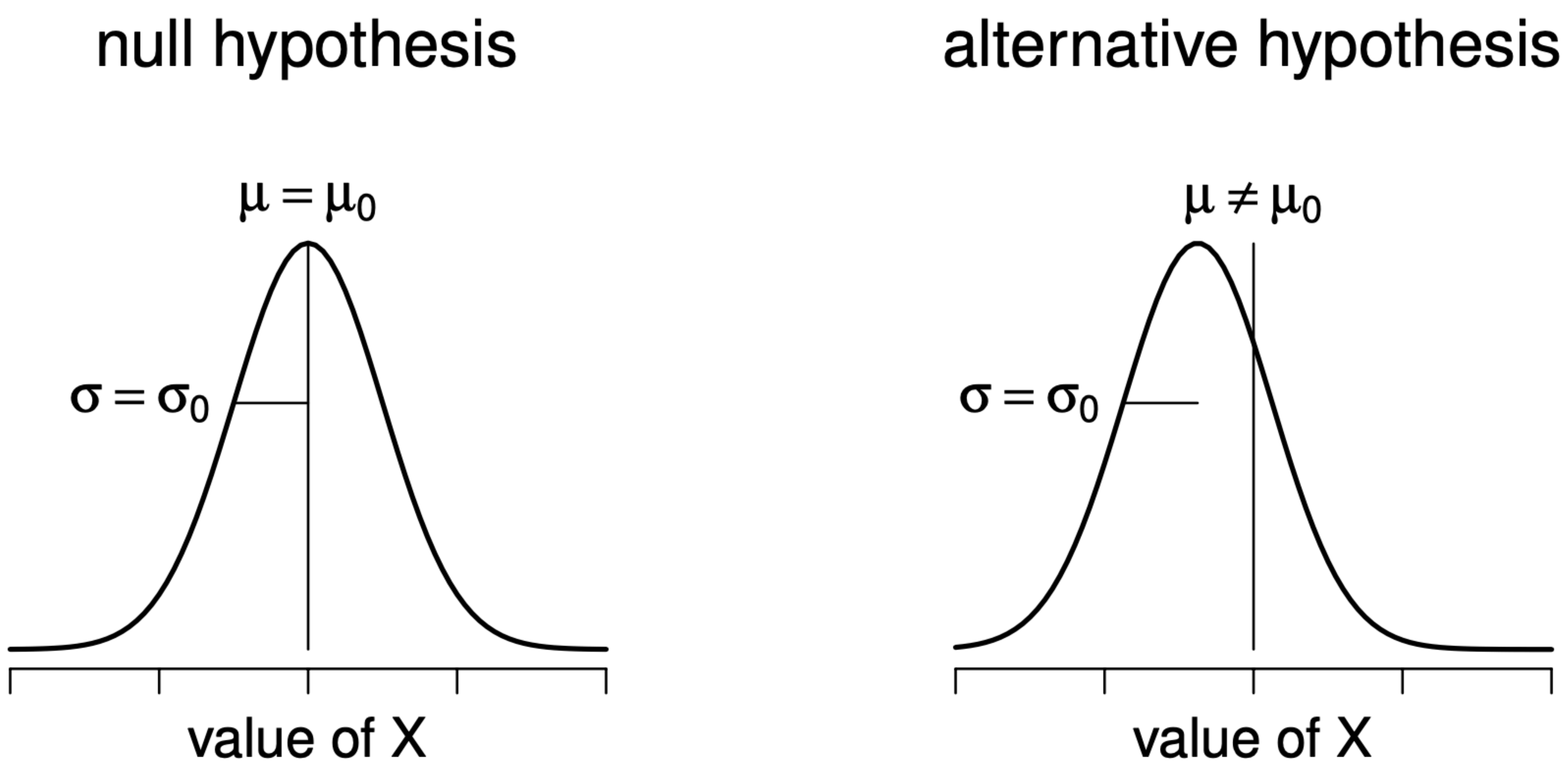

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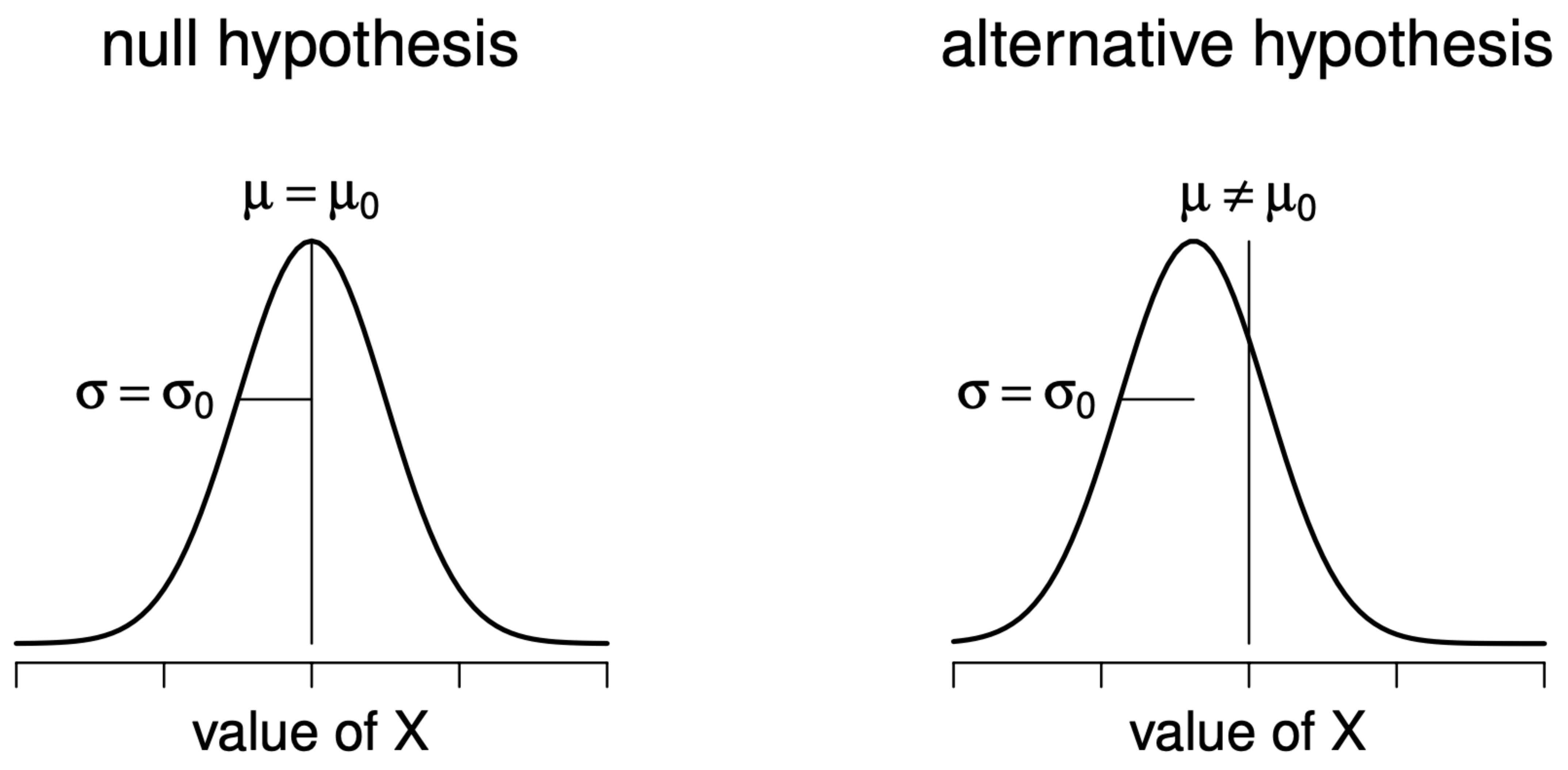

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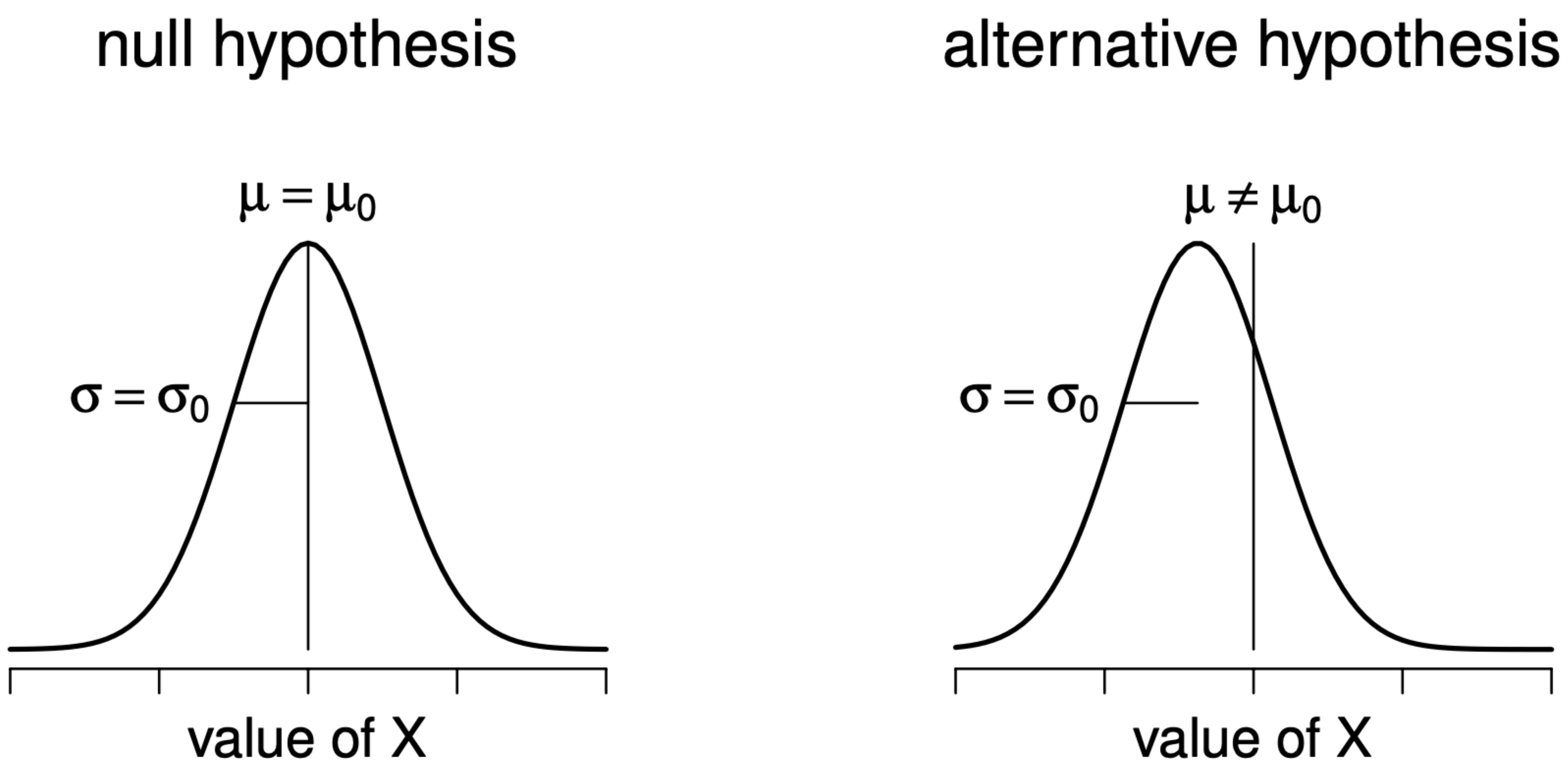

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- This tells us **which values of $\overline{X}$ are more probable**, assuming the Null!

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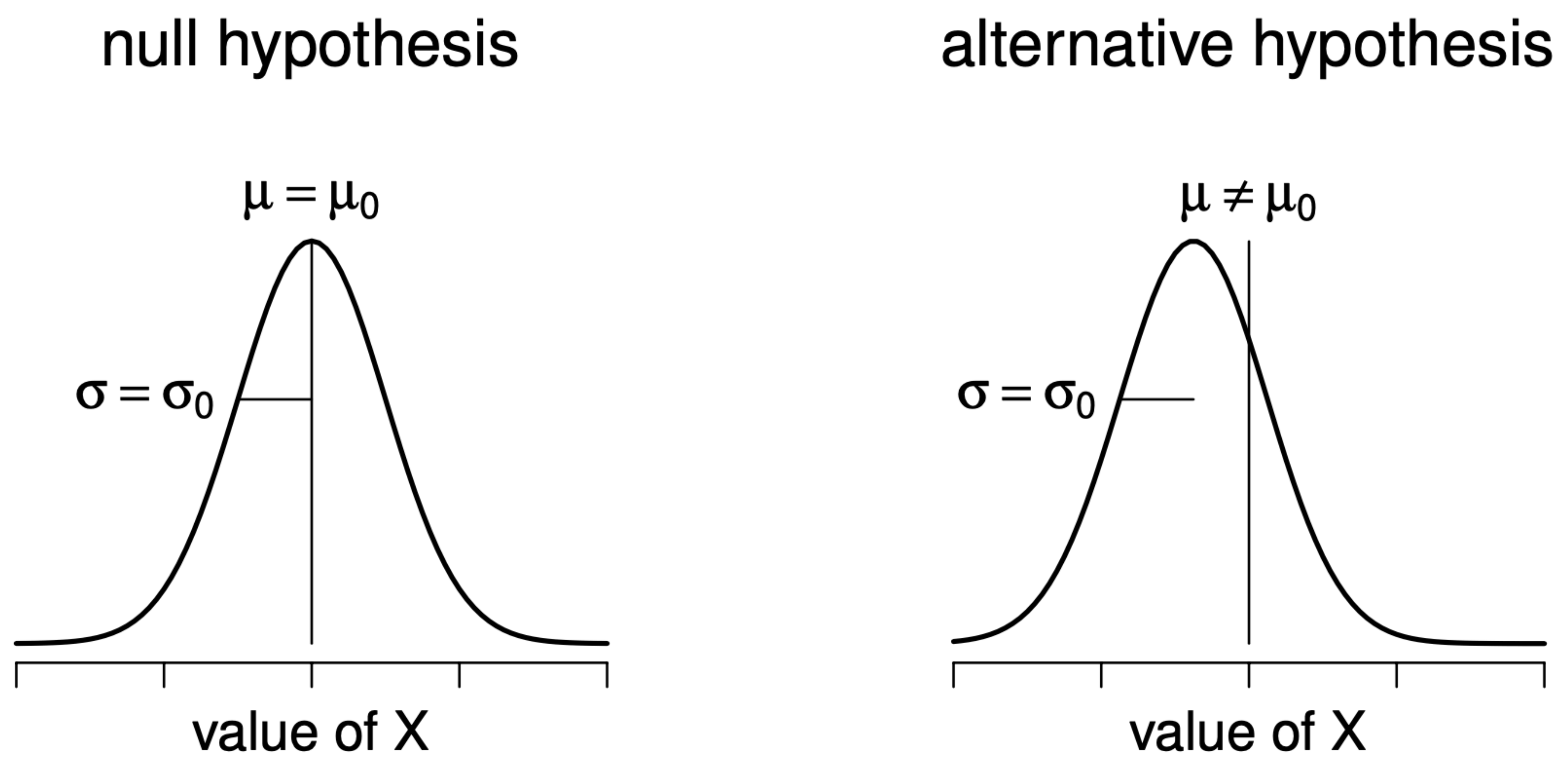

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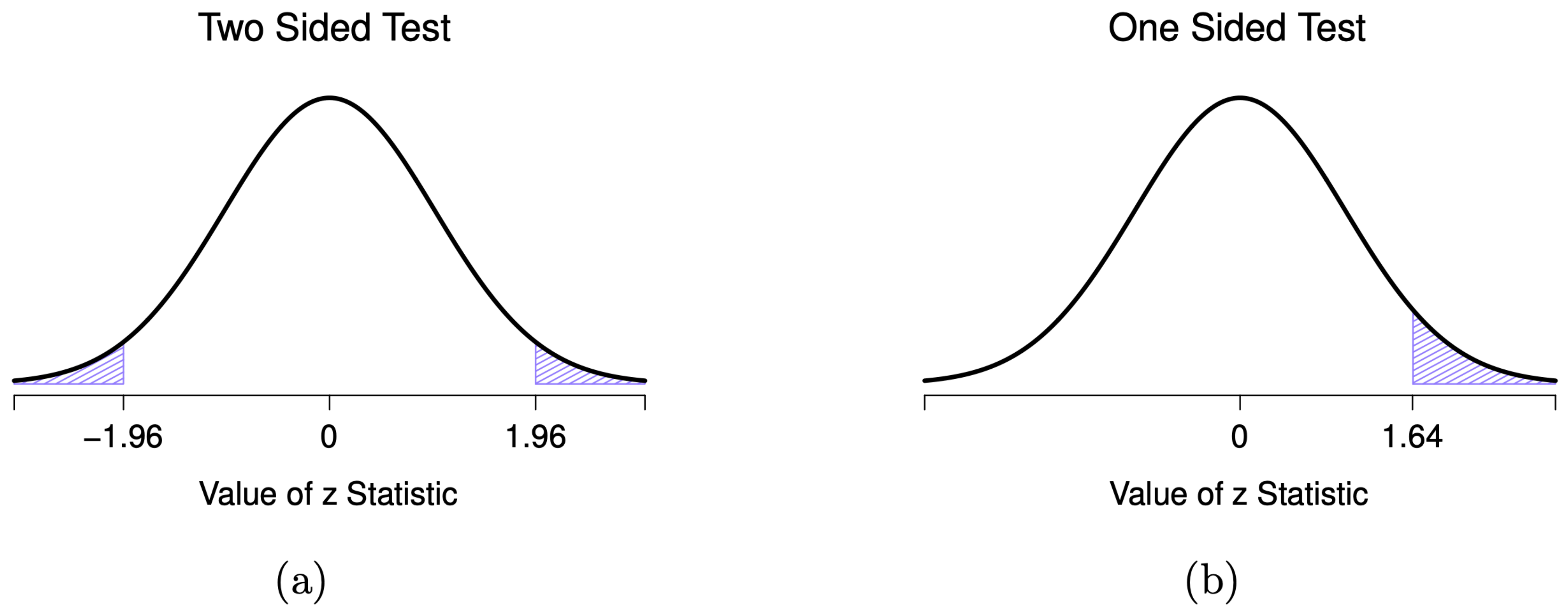

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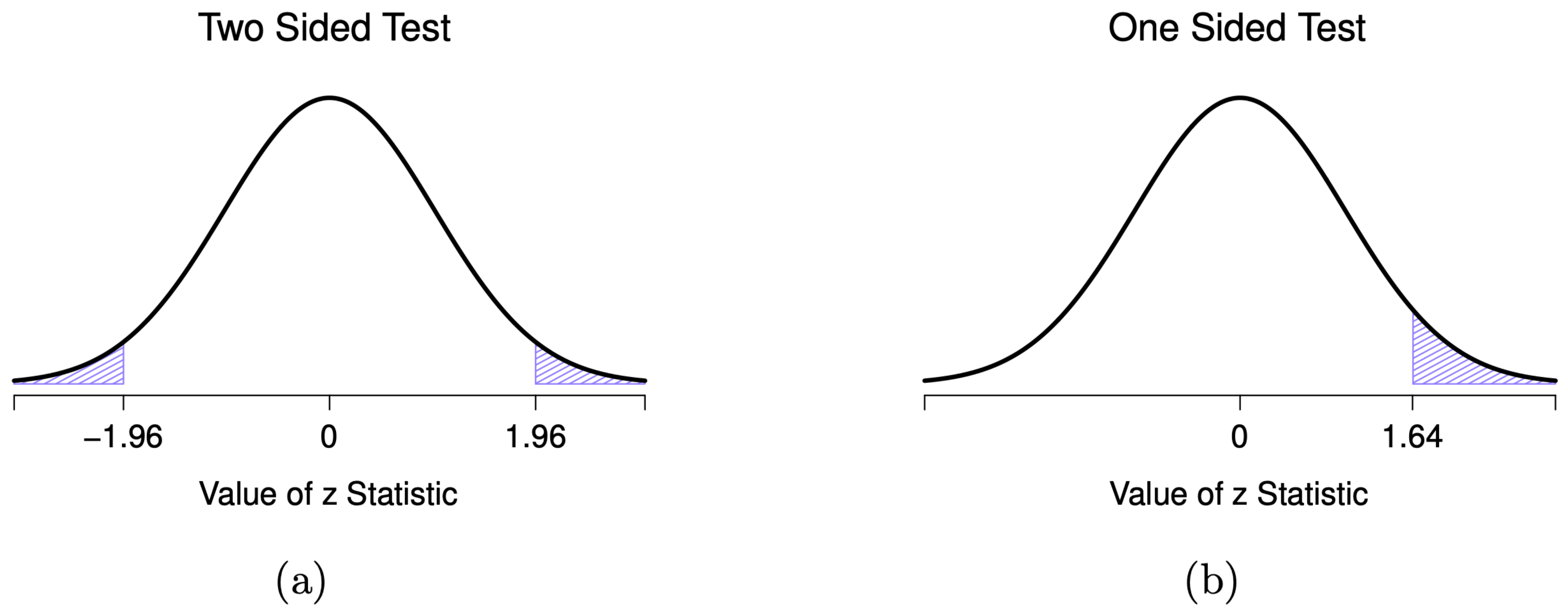

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  - The **area under the curve** is easily calculable from z-scores!

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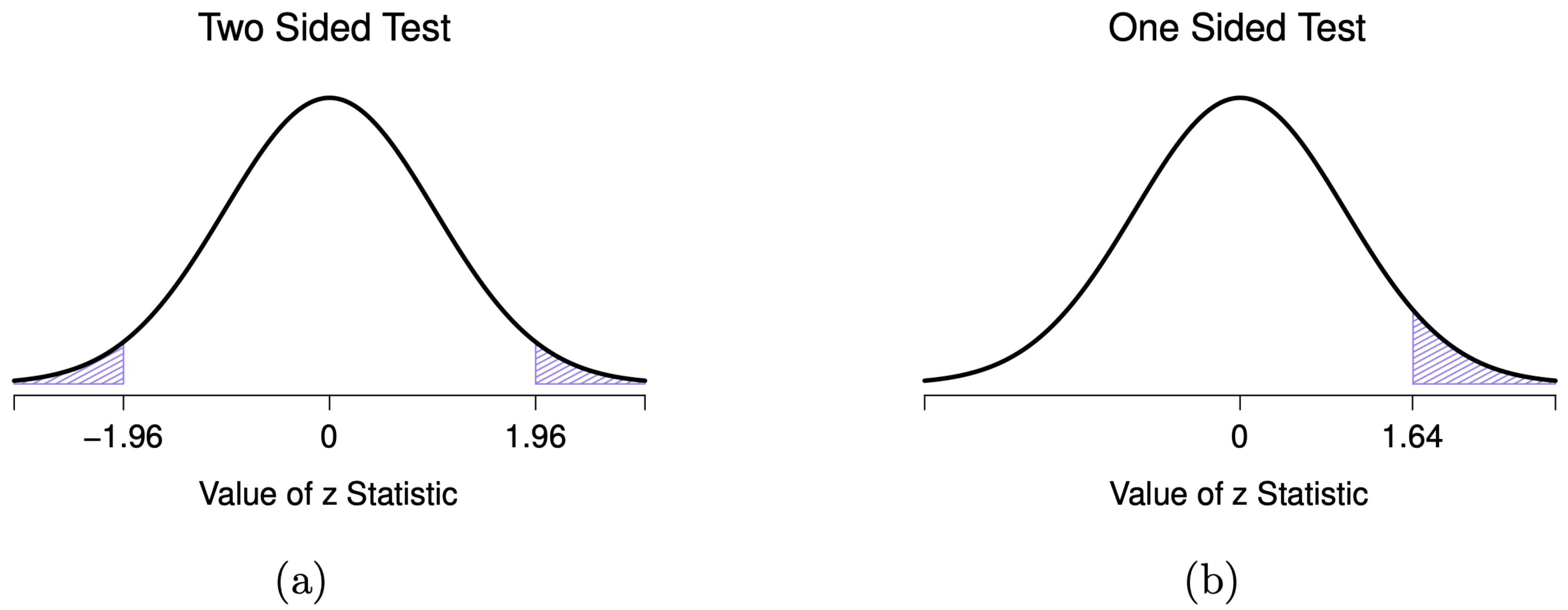

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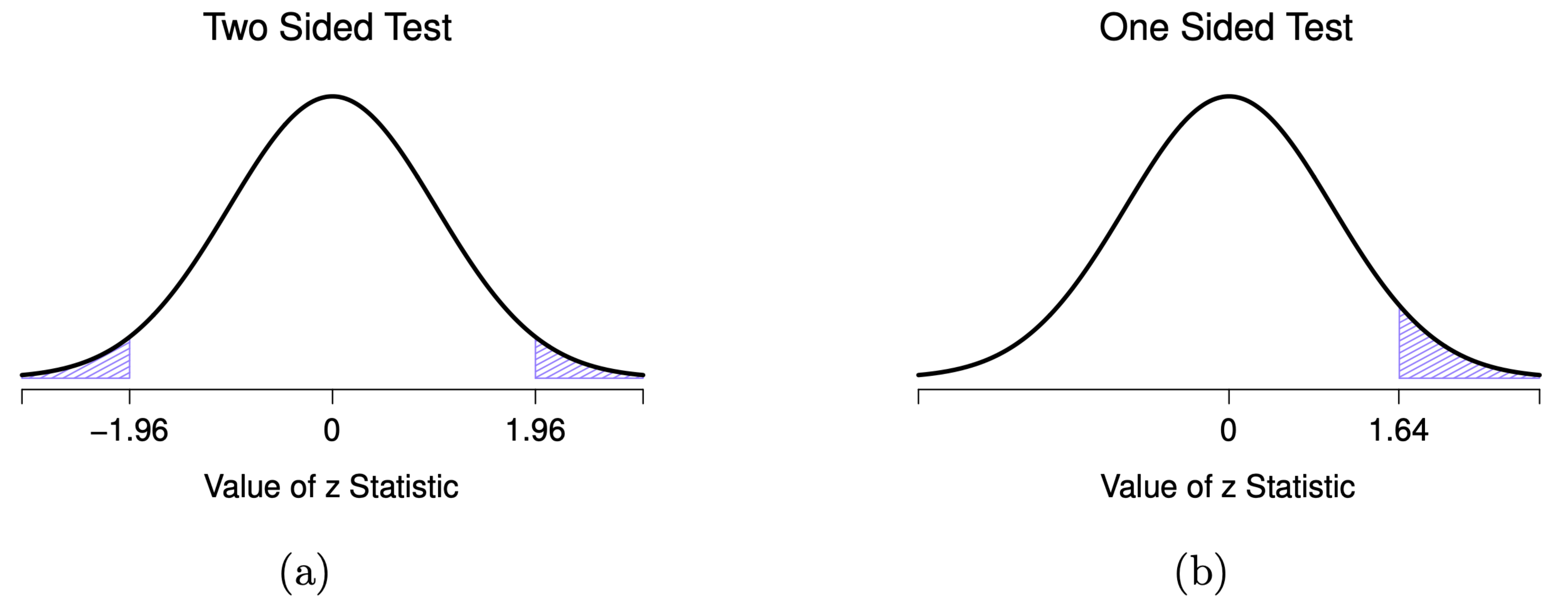
- The z-score is **how many standard deviations away** a certain value is from the mean, in a **Normal distribution**
  - The **area under the curve** is easily calculable from z-scores!
- This lets us define a **critical region** for a significant result!

$$z_{\overline{X}} = \frac{\overline{X} - \mu_0}{SEM(\overline{X})}$$



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- This lets us define a **critical region** for a significant result!
  - i.e. if $\overline{X}$ is a **certain z-score away** from μ_0, we can say $\mu \neq \mu_0$ with **statistical significance!**

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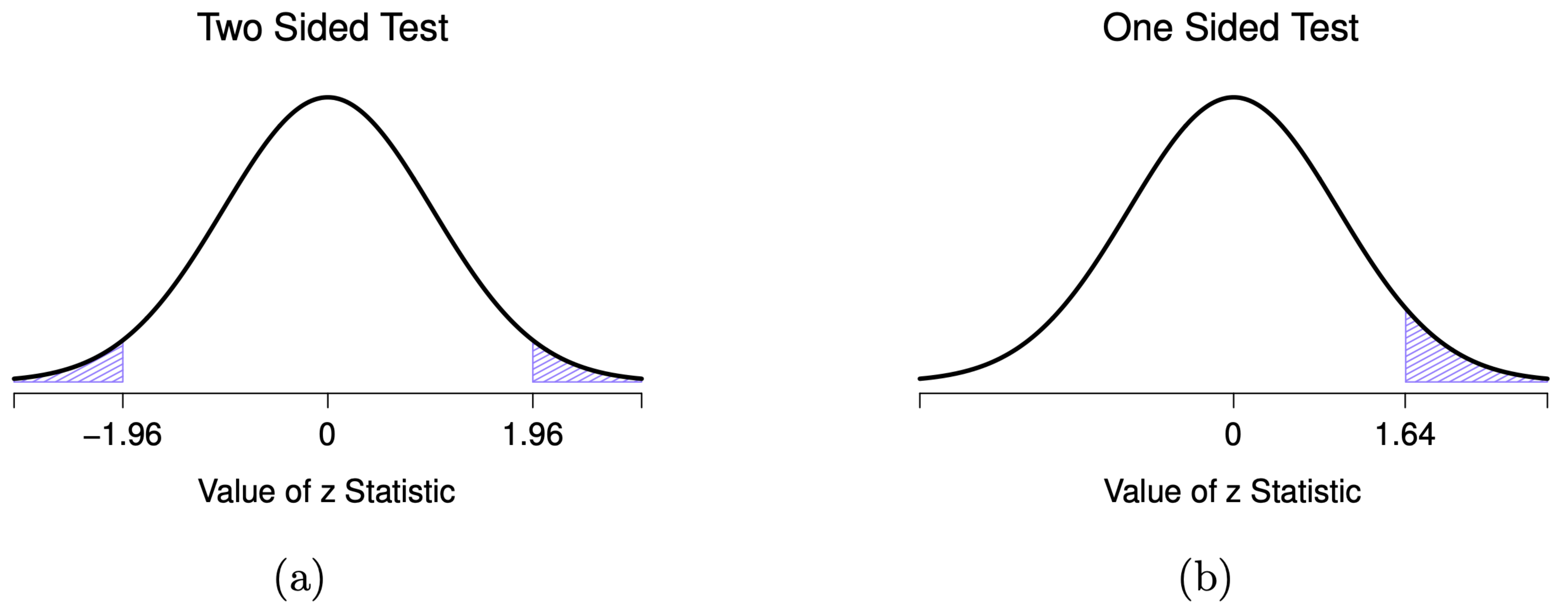

Figure 13.3: Rejection regions for the two-sided z-test (panel a) and the one-sided z-test (panel b).

Calculating the p-value

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> null_stdev = 9.5
> N = 20
> SEM = null_stdev / sqrt(N)
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> ling_mean = 72.3
> z_score = (ling_mean - null_mean) / SEM
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> upper_area = pnorm(z_score, lower.tail=FALSE)
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- Let's plug in some values
  - **Sample size** (number of Ling students): $N = 20$
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- Let's plug in some values
  - **Sample size** (number of Ling students): $N = 20$
  - **Sample mean** (their average grade): $\overline{X} = 72.3$
- Calculation on the right →
- p-value is **0.012**
  - We'd have a **1.2% chance** of seeing a sample mean as high as that if the Null were true
  - (i.e. a fairly significant result)

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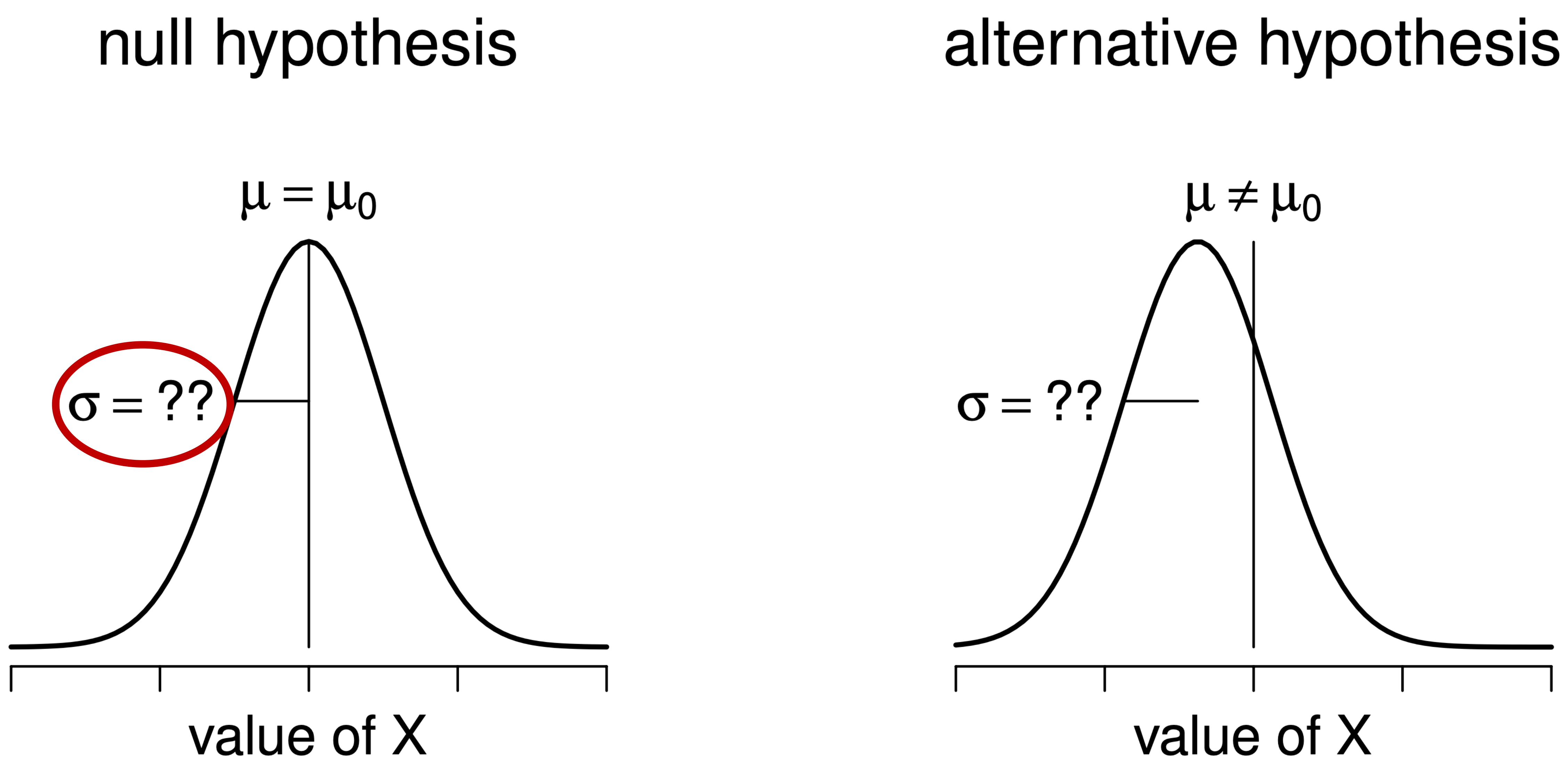
Why we need a t-test



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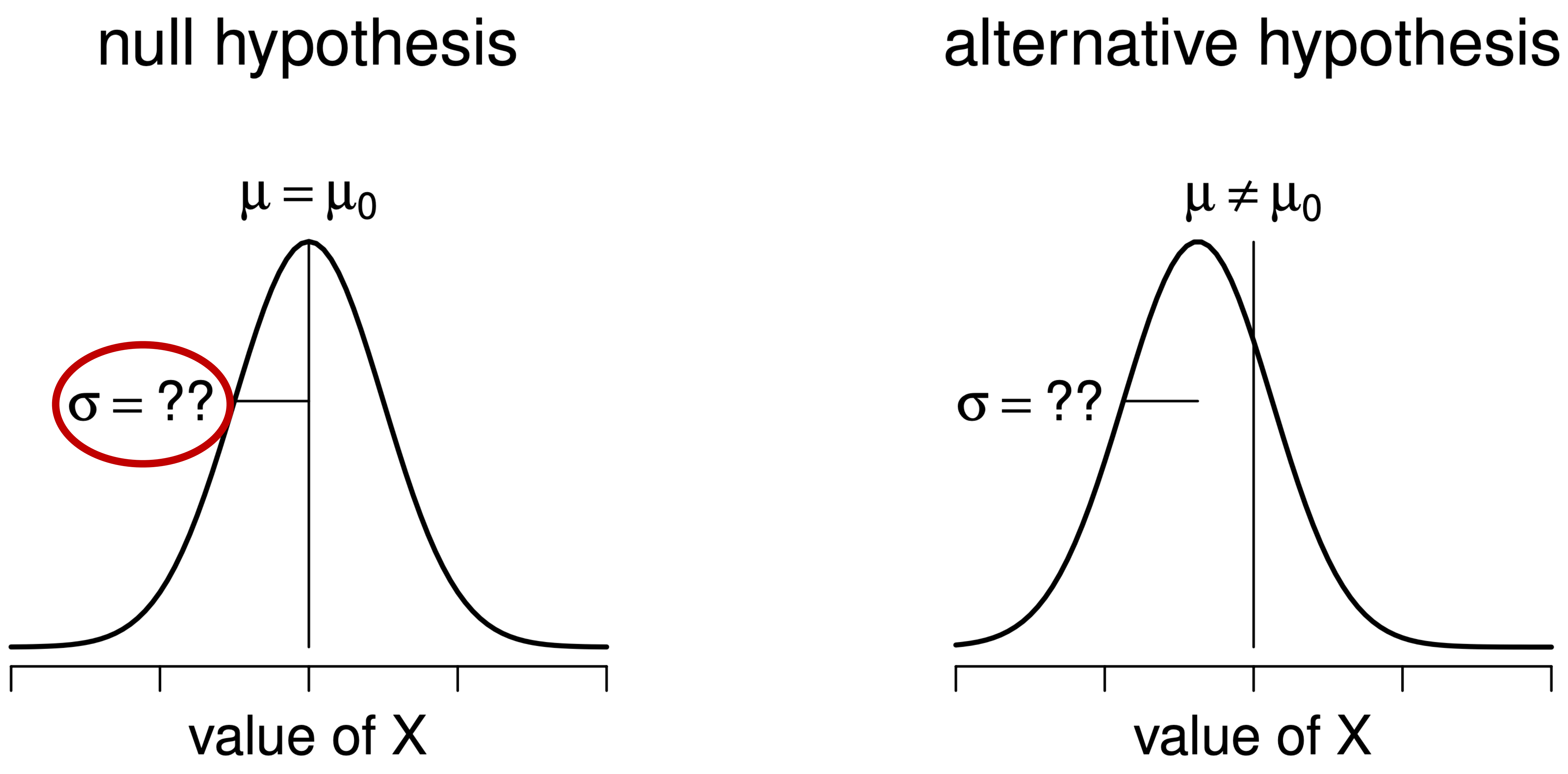

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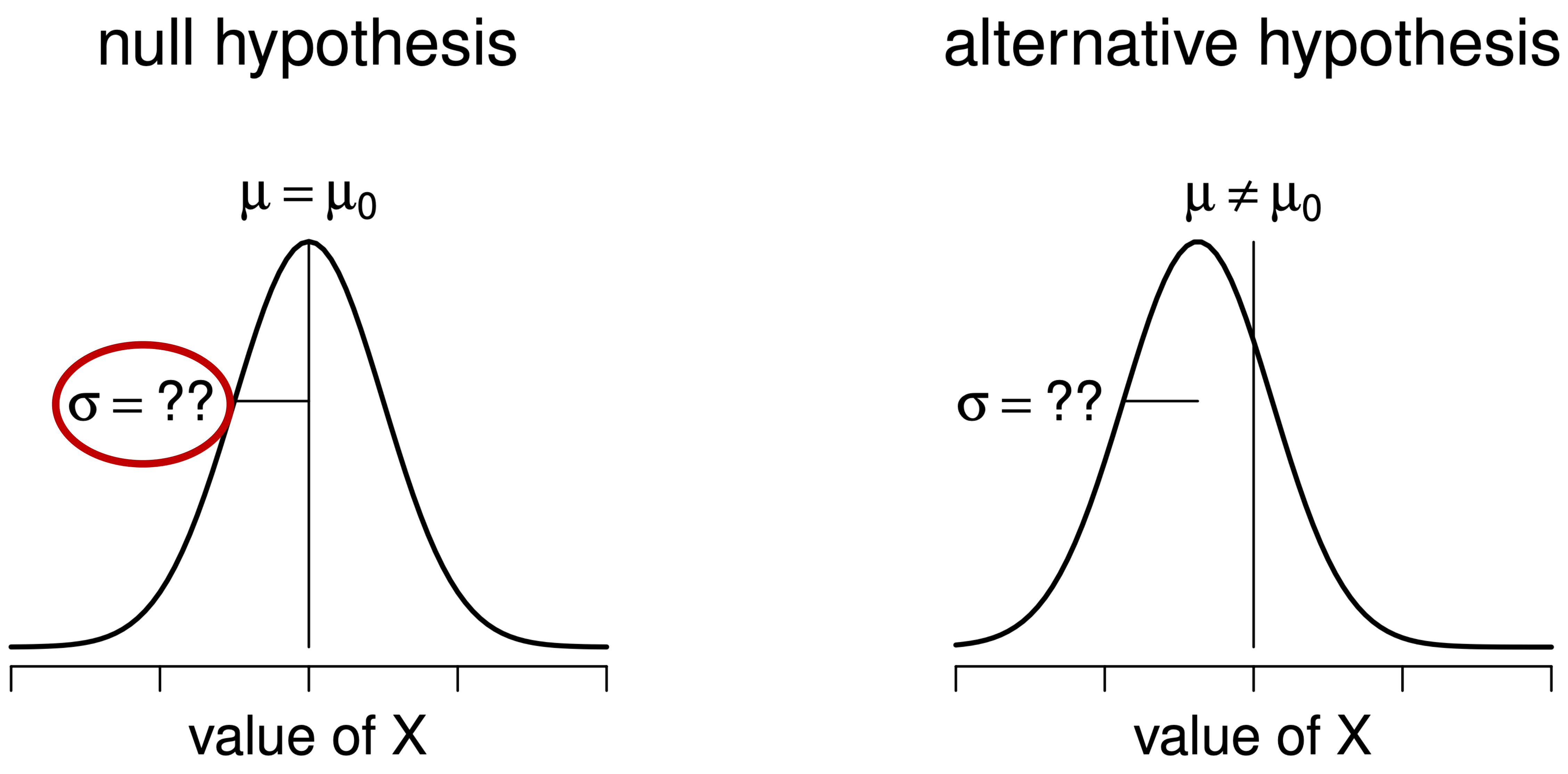

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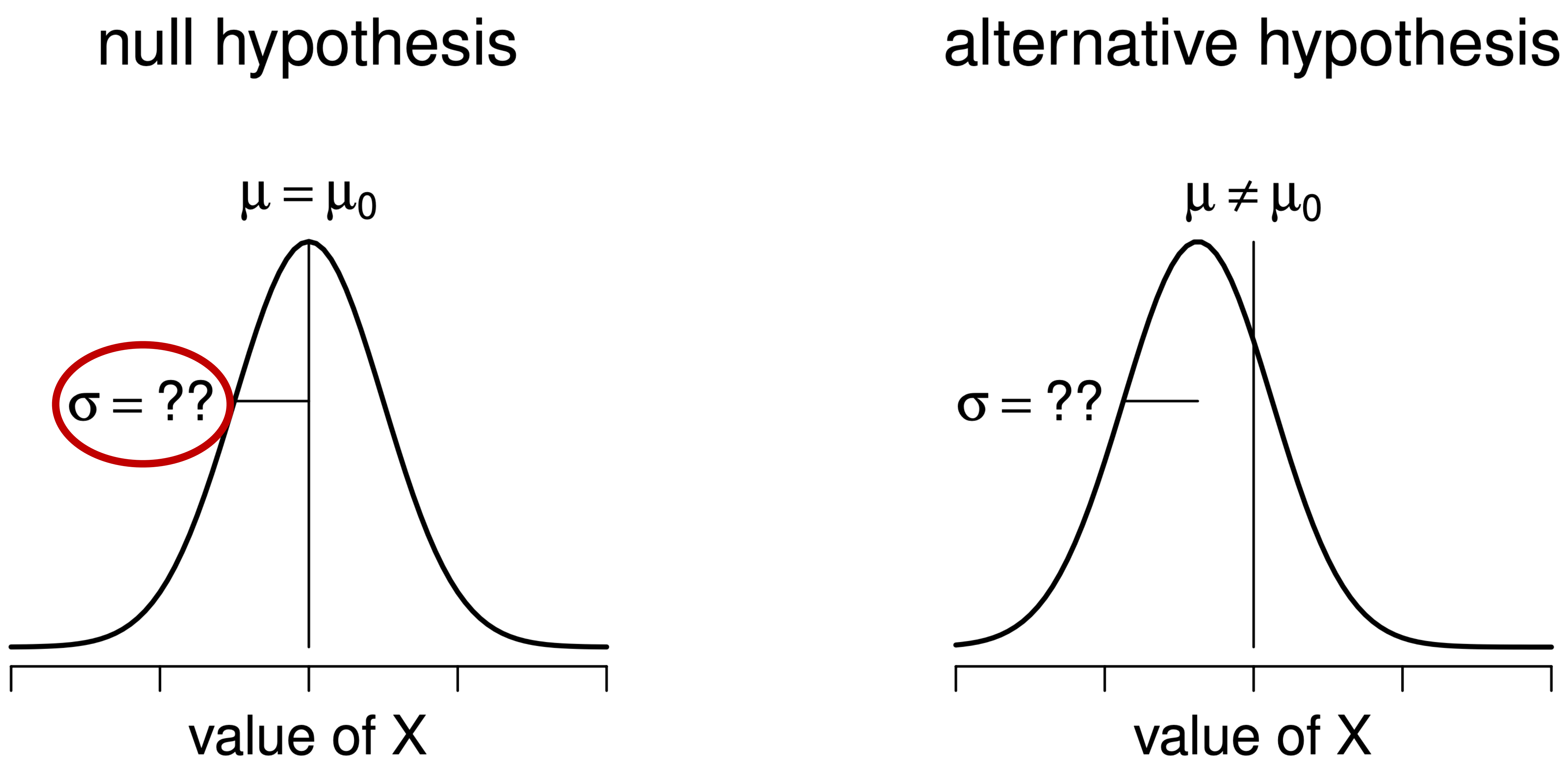

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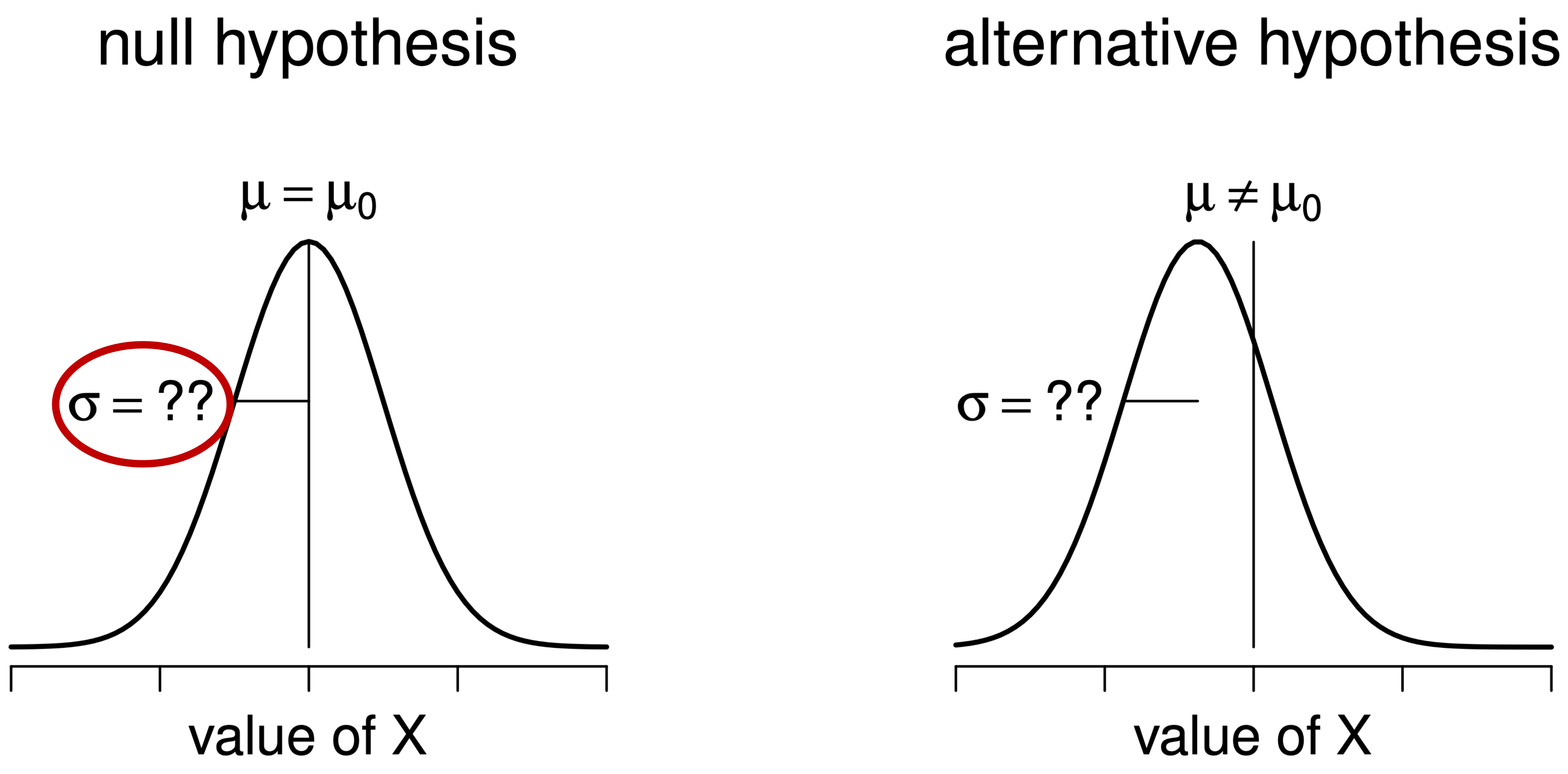
- The t-test **eliminates these assumptions**



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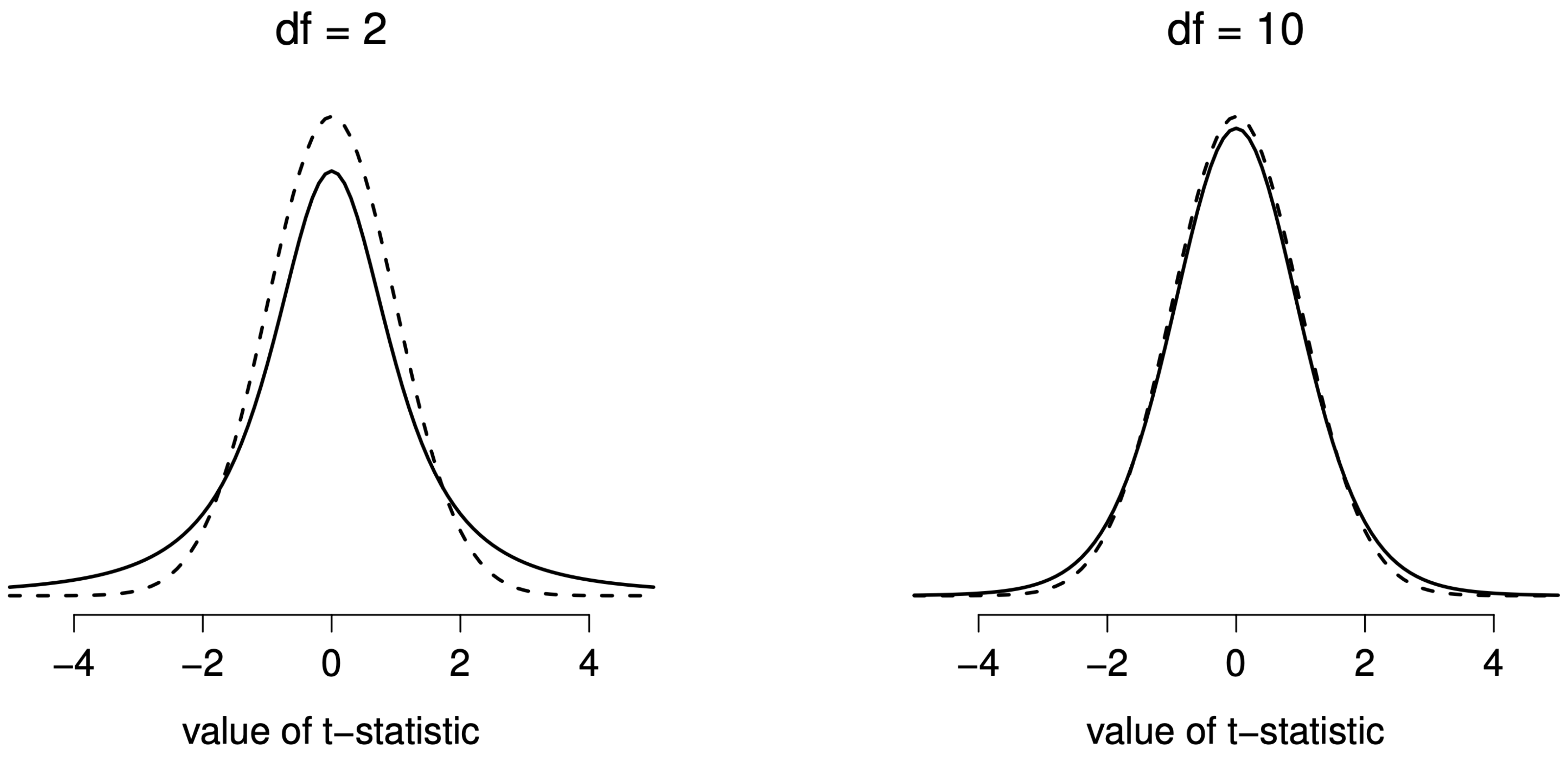
- Change 1:
  - We **don't have the true population stdev** (σ)
  - We instead **estimate it from our sample** ($\hat{\sigma} = \sigma(X)$)
- Change 2:
  - We use a **t-distribution** instead of a Normal distribution
  - What's a t-distribution?

t-distribution



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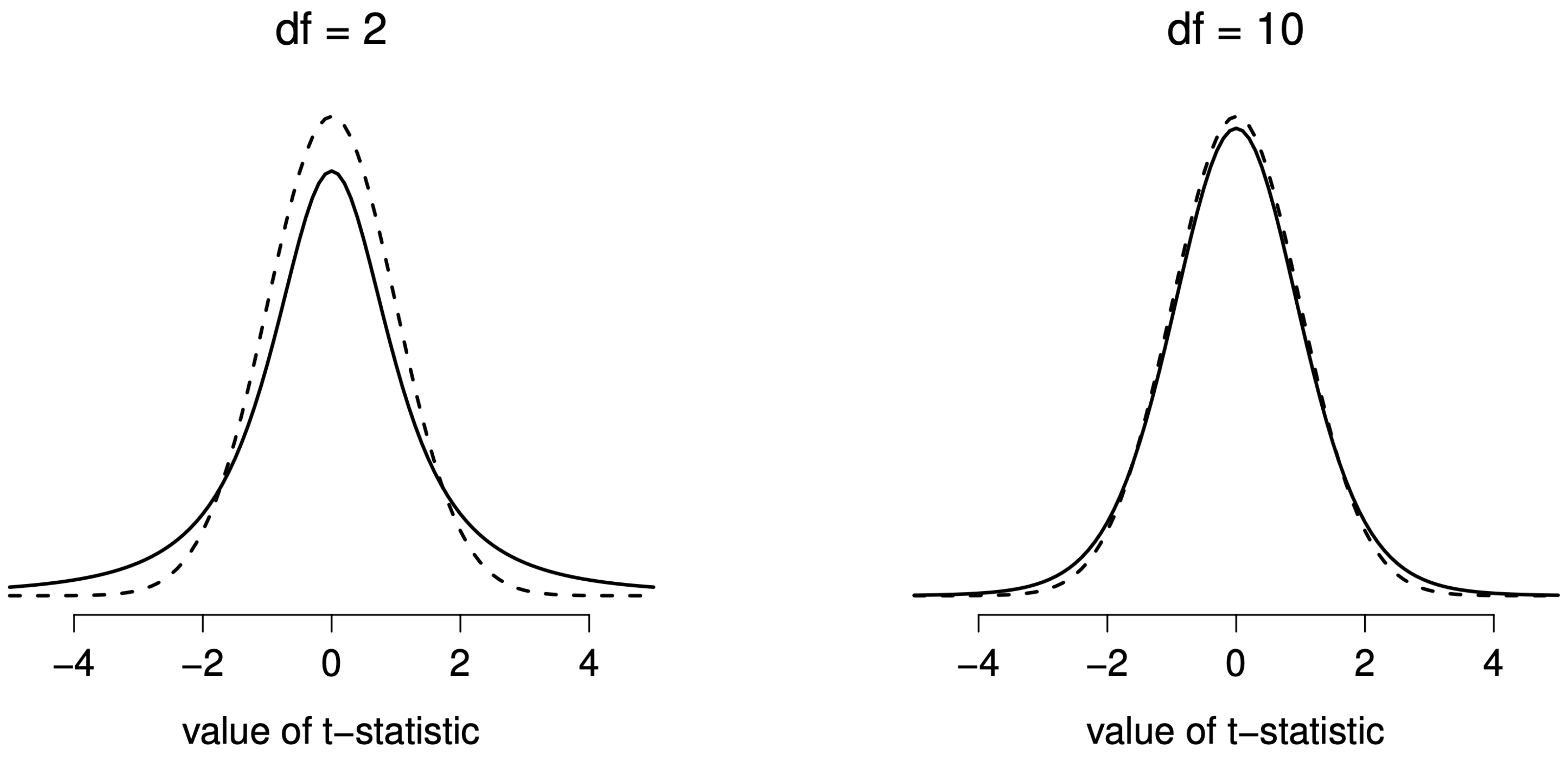

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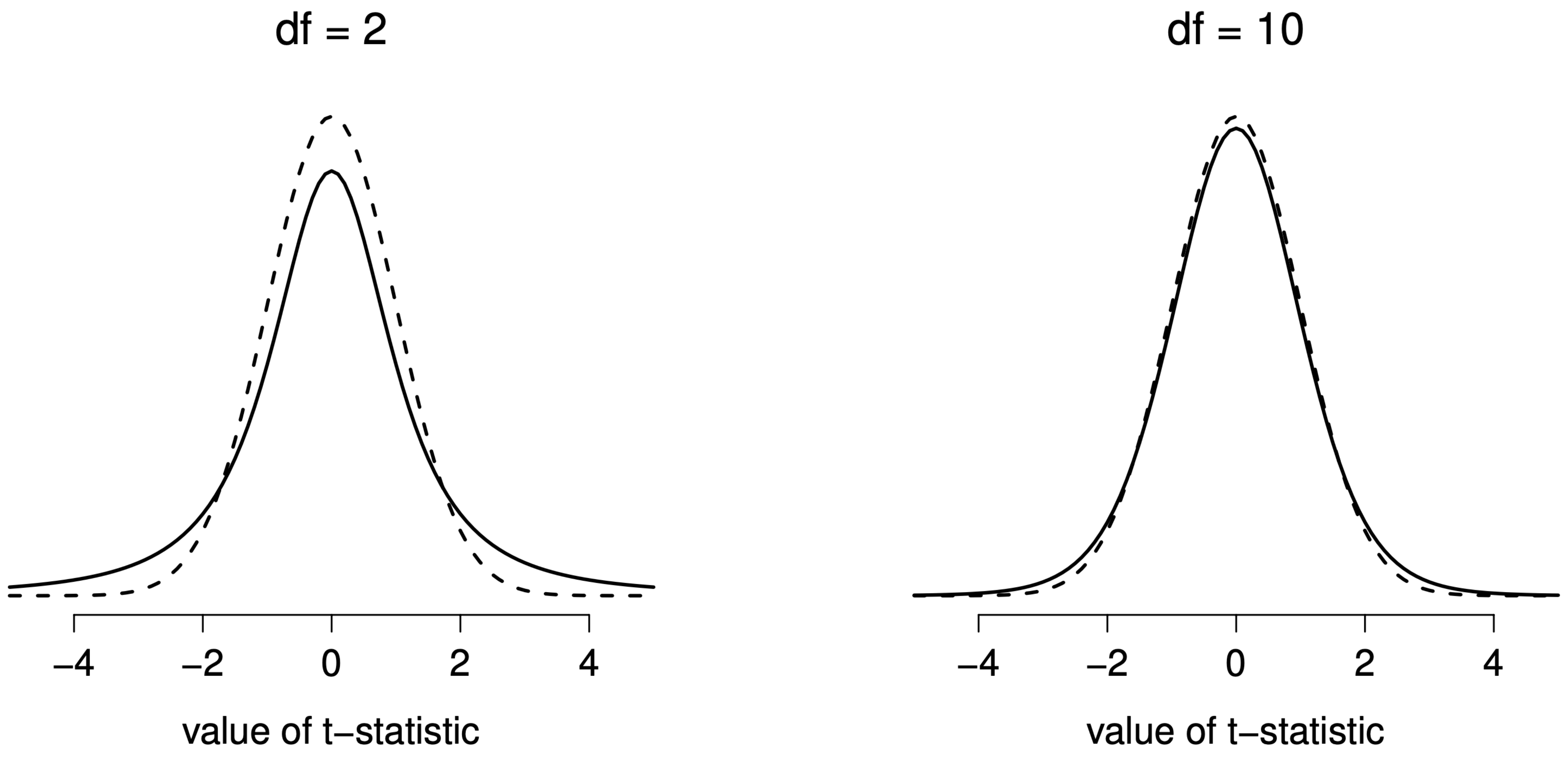

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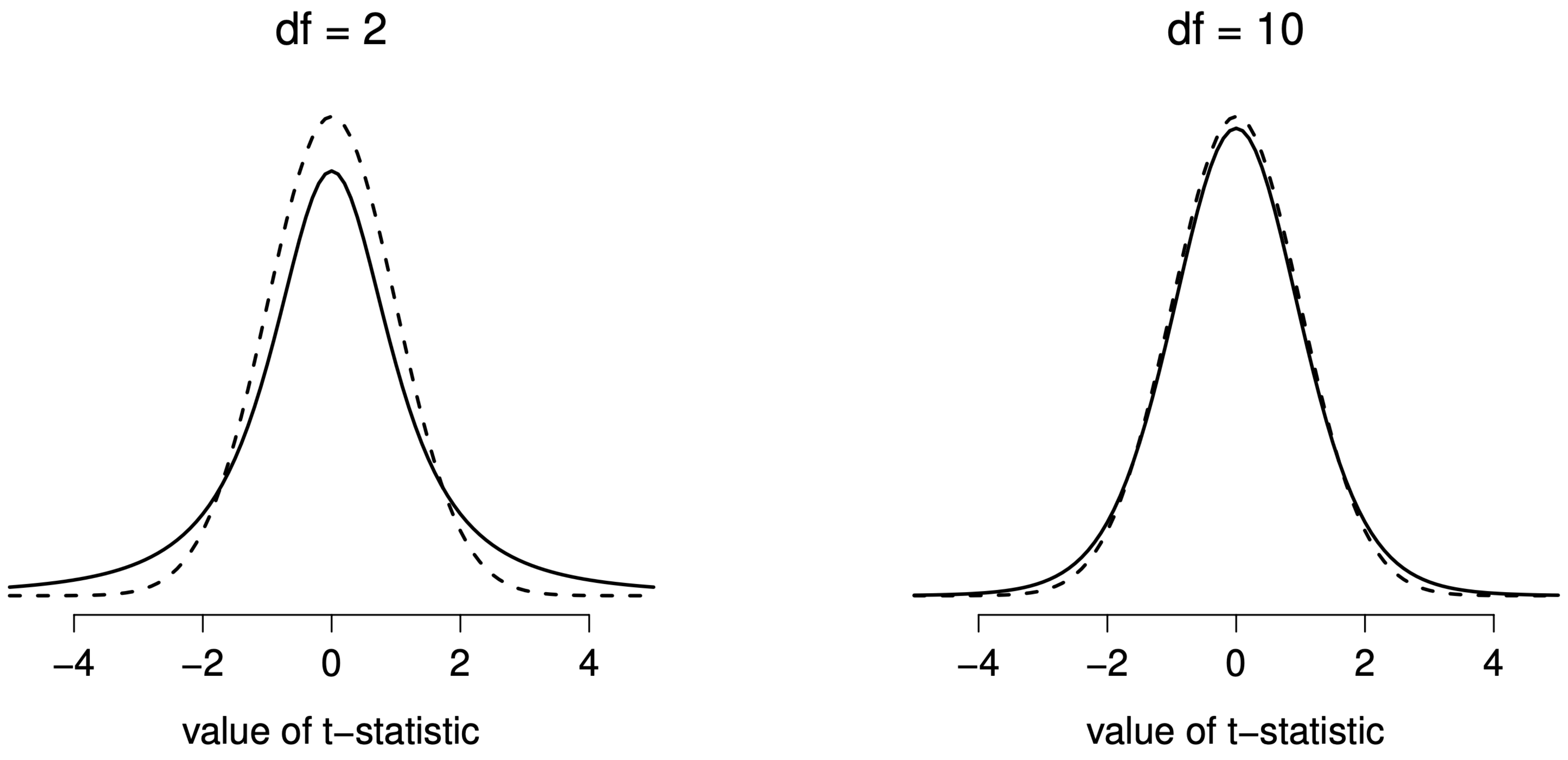

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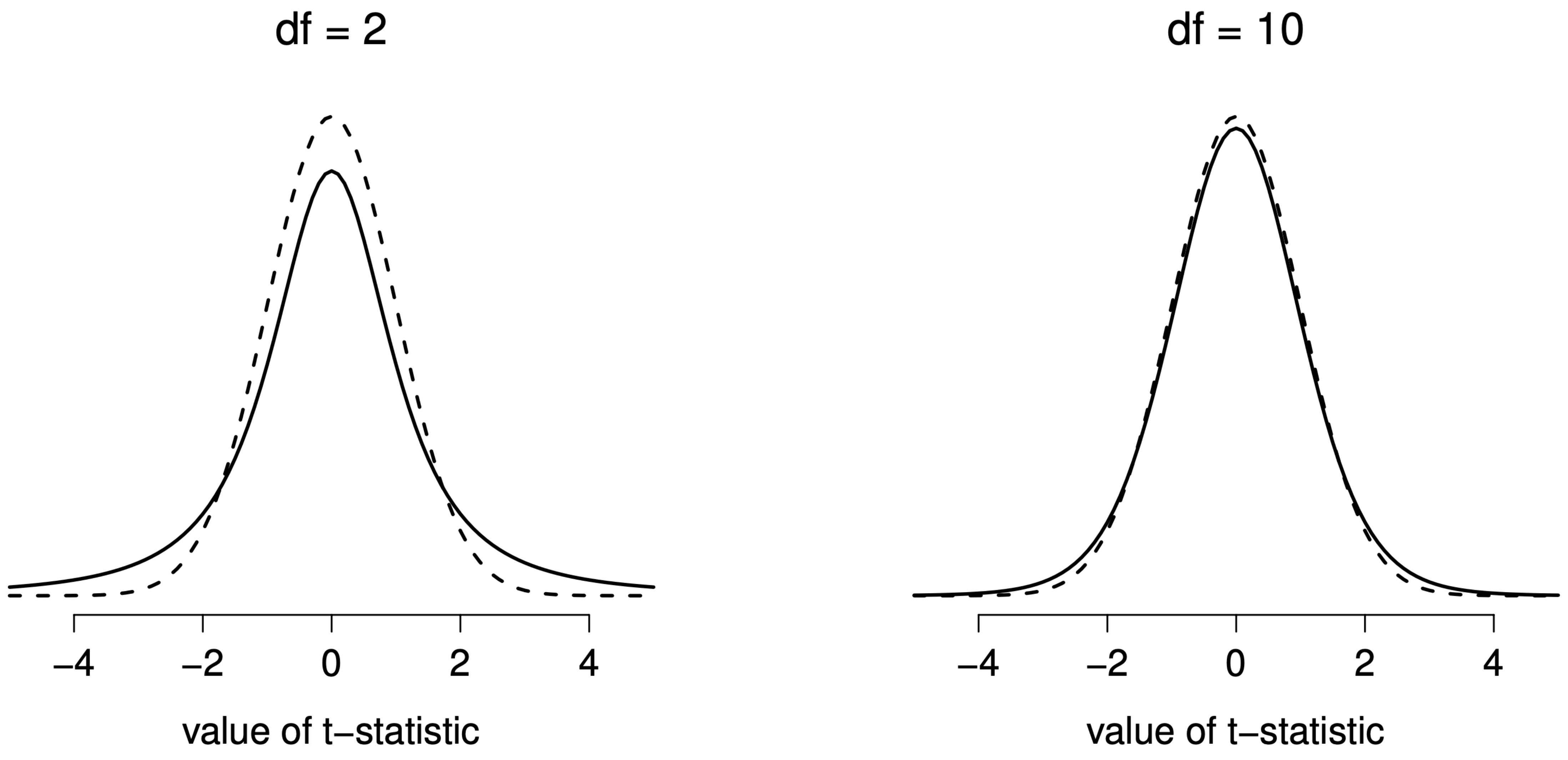

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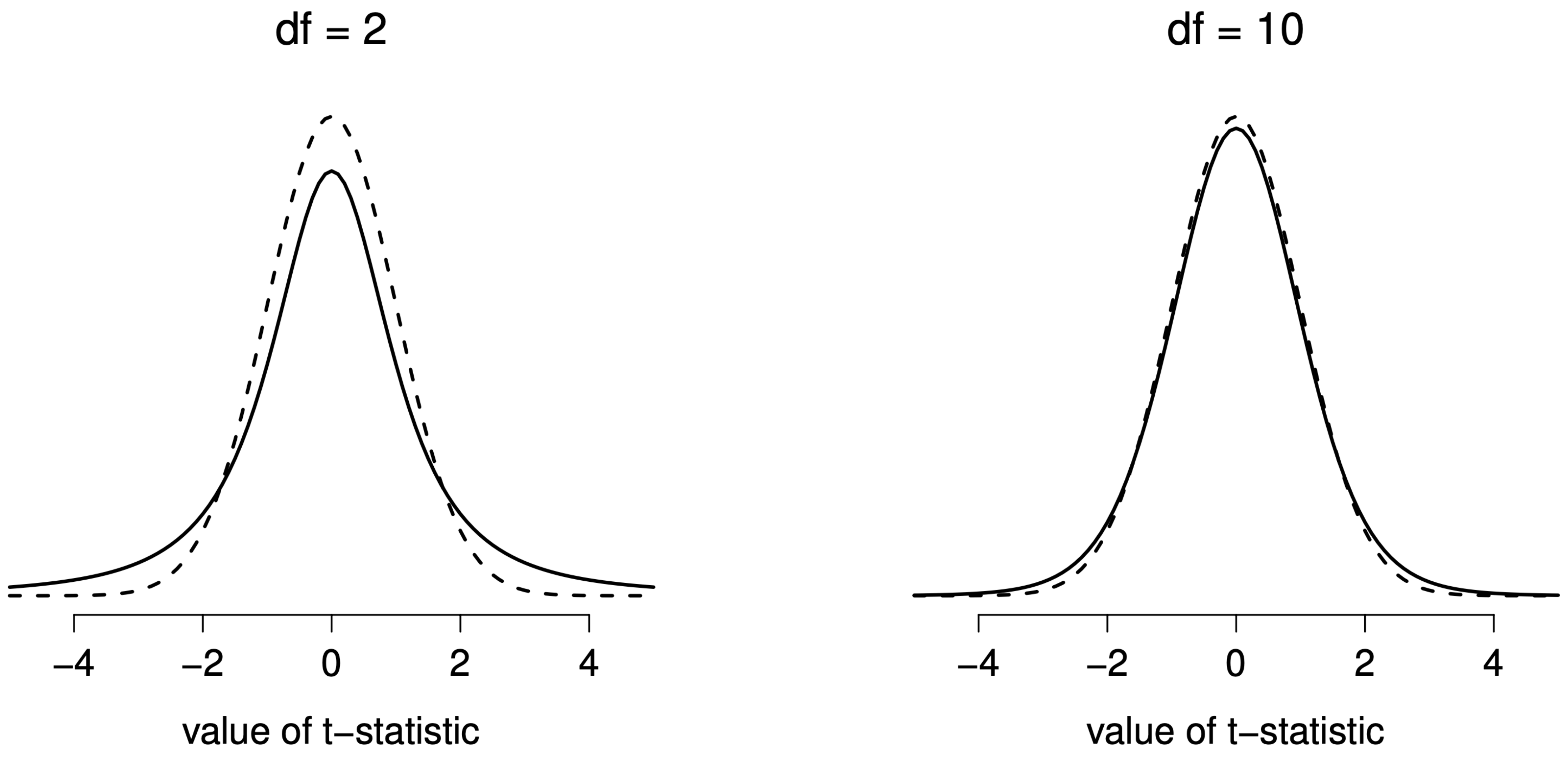

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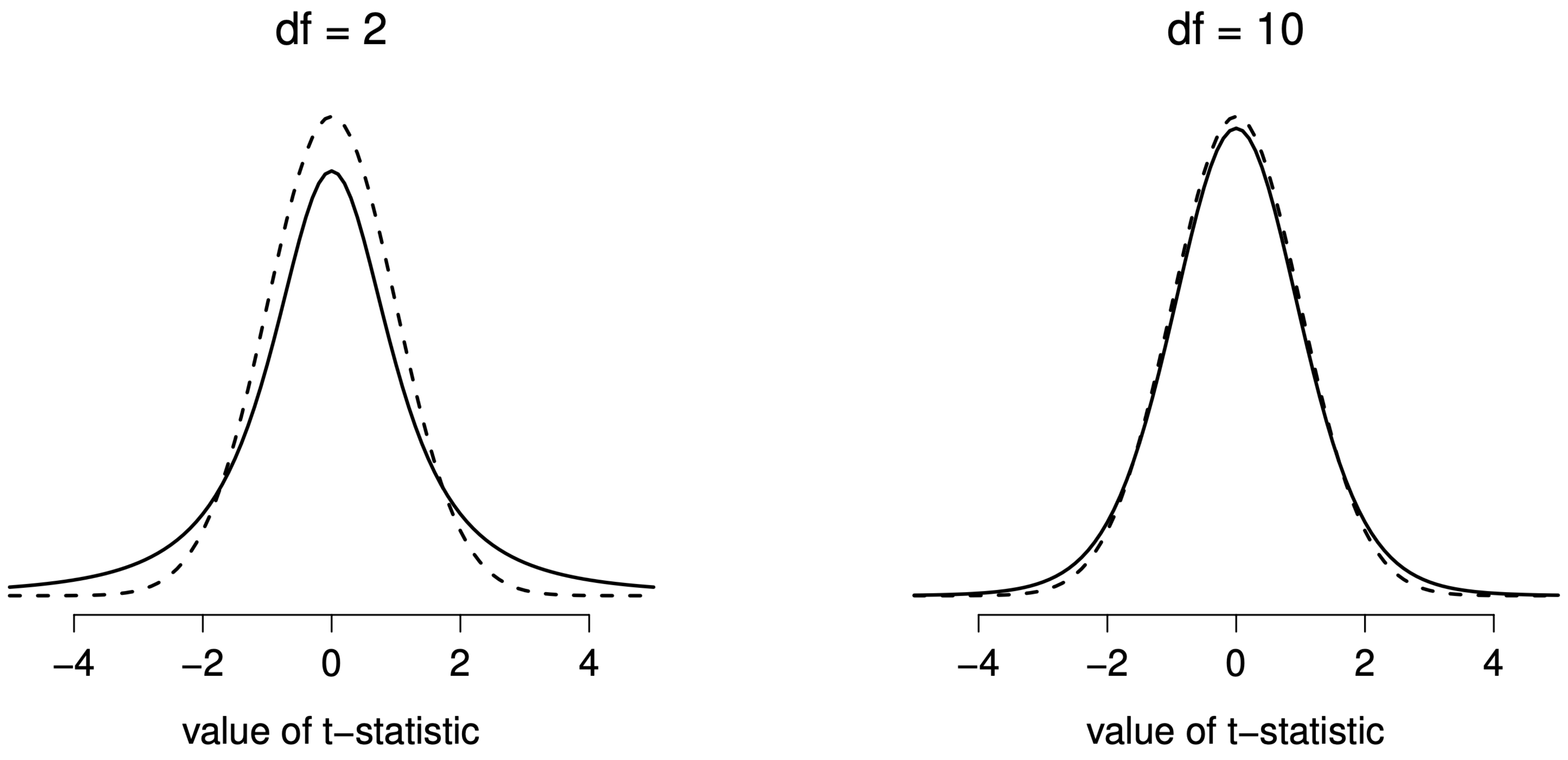

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  - Accounts for **uncertainty** in small samples
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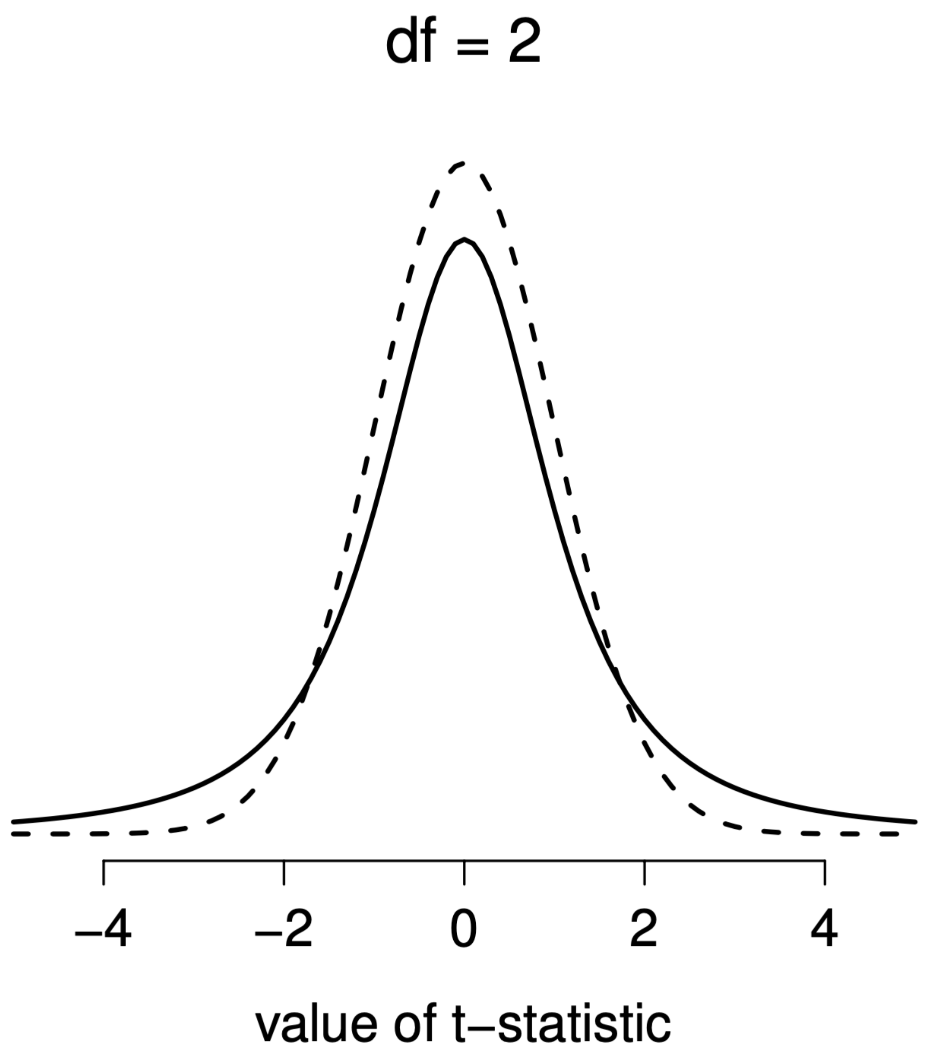
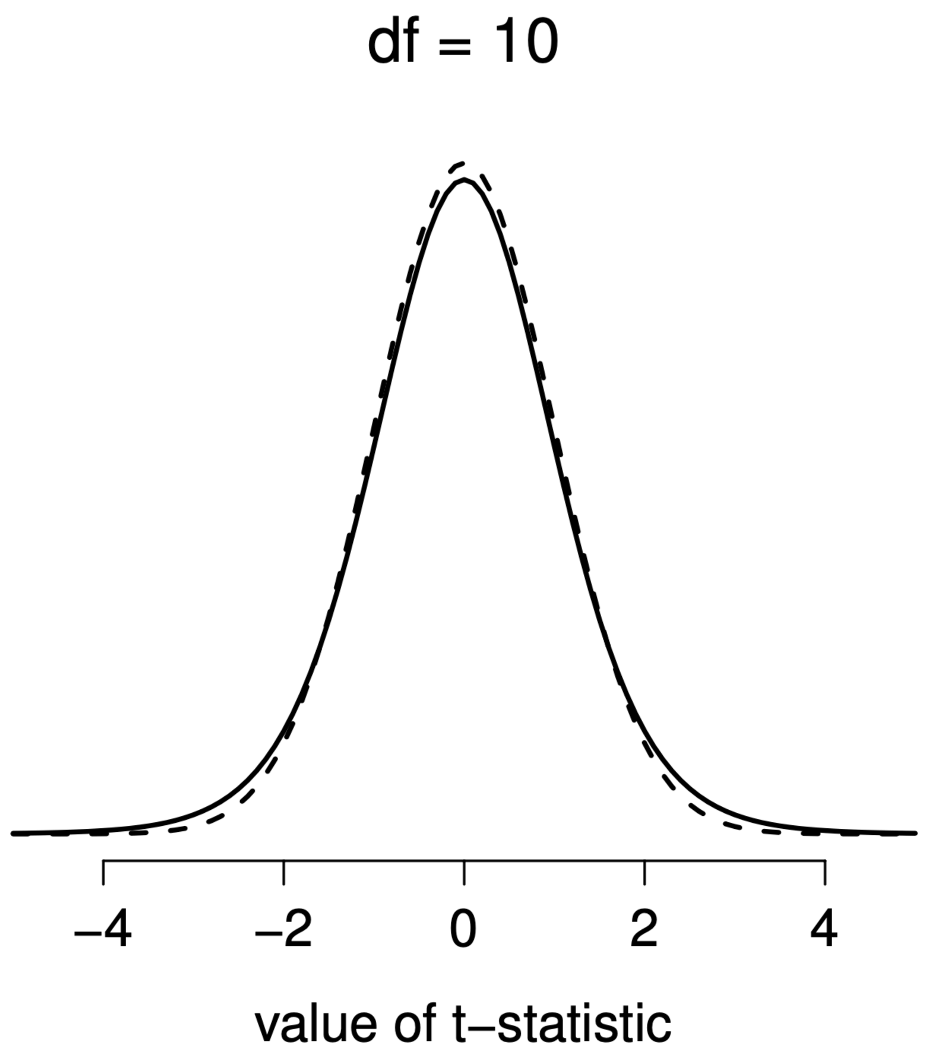
- Higher df → **closer to Normal**

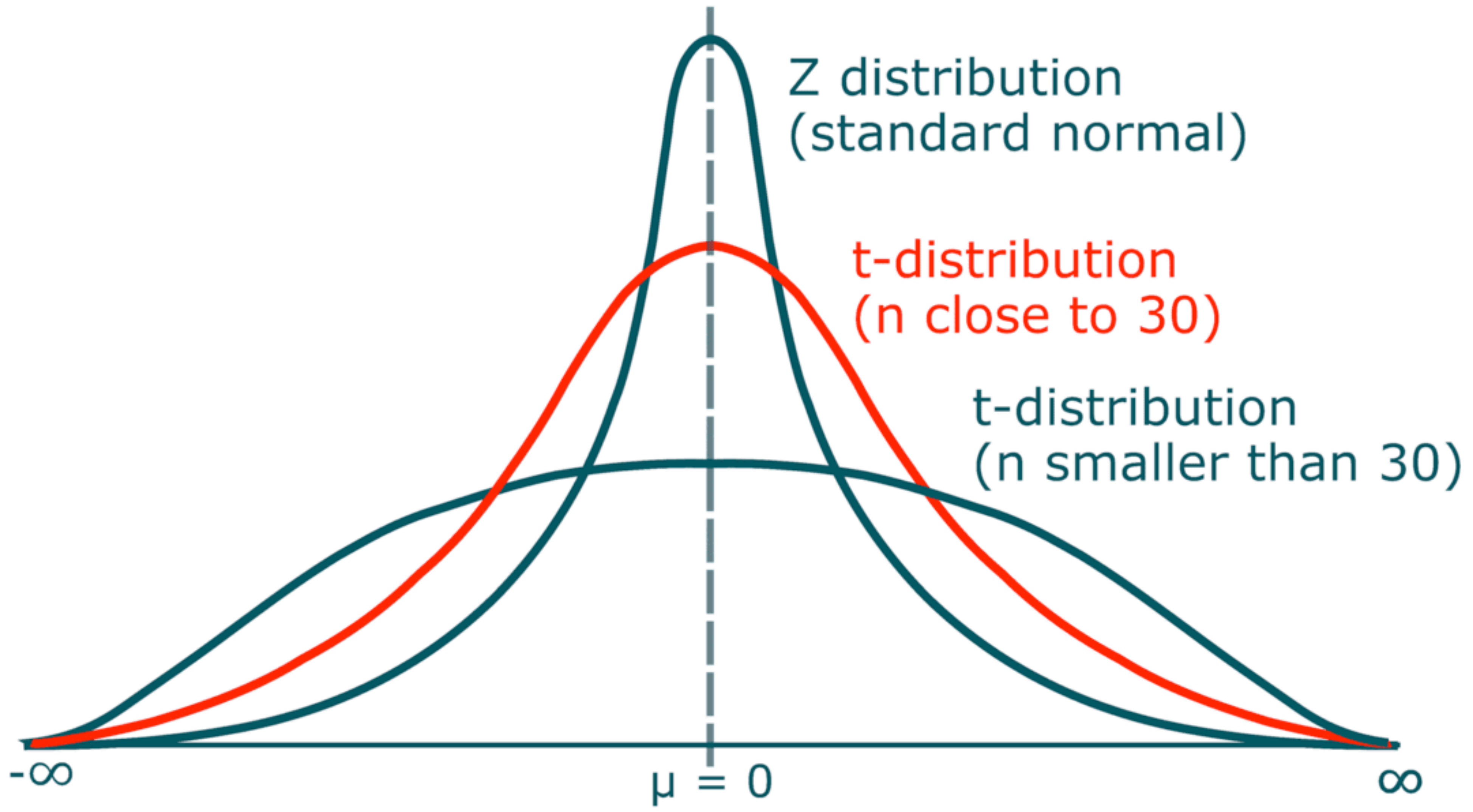




t-score

- Instead of the z-score, we calculate the **t-score**
  - **Identical** to z-score, except for the **estimated stdev**
- **Significance** found by plugging the t-score into the t-distribution
- That's it!

$$t = \frac{\bar{X} - \mu_0}{\hat{\sigma}/\sqrt{N}}$$



t-test in R (from scratch)

```
> null_mean = 67.5
> ling_mean = 72.3
> ling_stdev = 10.1
> N = 20
> # SEM using the estimated stdev from the sample
> SEM = ling_stdev / sqrt(N)
> SEM
[1] 2.258429
>
> t_score = (ling_mean - null_mean) / SEM
> t_score
[1] 2.125372
>
> upper_area = pt(t_score, df=N-1, lower.tail=FALSE)
> upper_area
[1] 0.02344606
> # This is our p-value using the t-test
```

t-test in R (the correct way)

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- You can conduct a t-test with R's `t.test()` command
  - `x`: the sample values
  - `mu`: the Null hypothesis mean
  - `alternative="greater"` makes it a **one-sided** test

```
> t.test(x=ling_grades, mu=67.5, alternative="greater")

        One Sample t-test

data:  ling_grades
t = 2.1254, df = 19, p-value = 0.02345
alternative hypothesis: true mean is greater than 67.5
95 percent confidence interval:
 68.39488      Inf
sample estimates:
mean of x
     72.3
```

t-test in R (the correct way)

- You can conduct a t-test with R's `t.test()` command
  - `x`: the sample values
  - `mu`: the Null hypothesis mean
  - `alternative="greater"` makes it a **one-sided** test
- Reminder: this is called a **One-Sample t-test**
  - We'll cover other types in less detail

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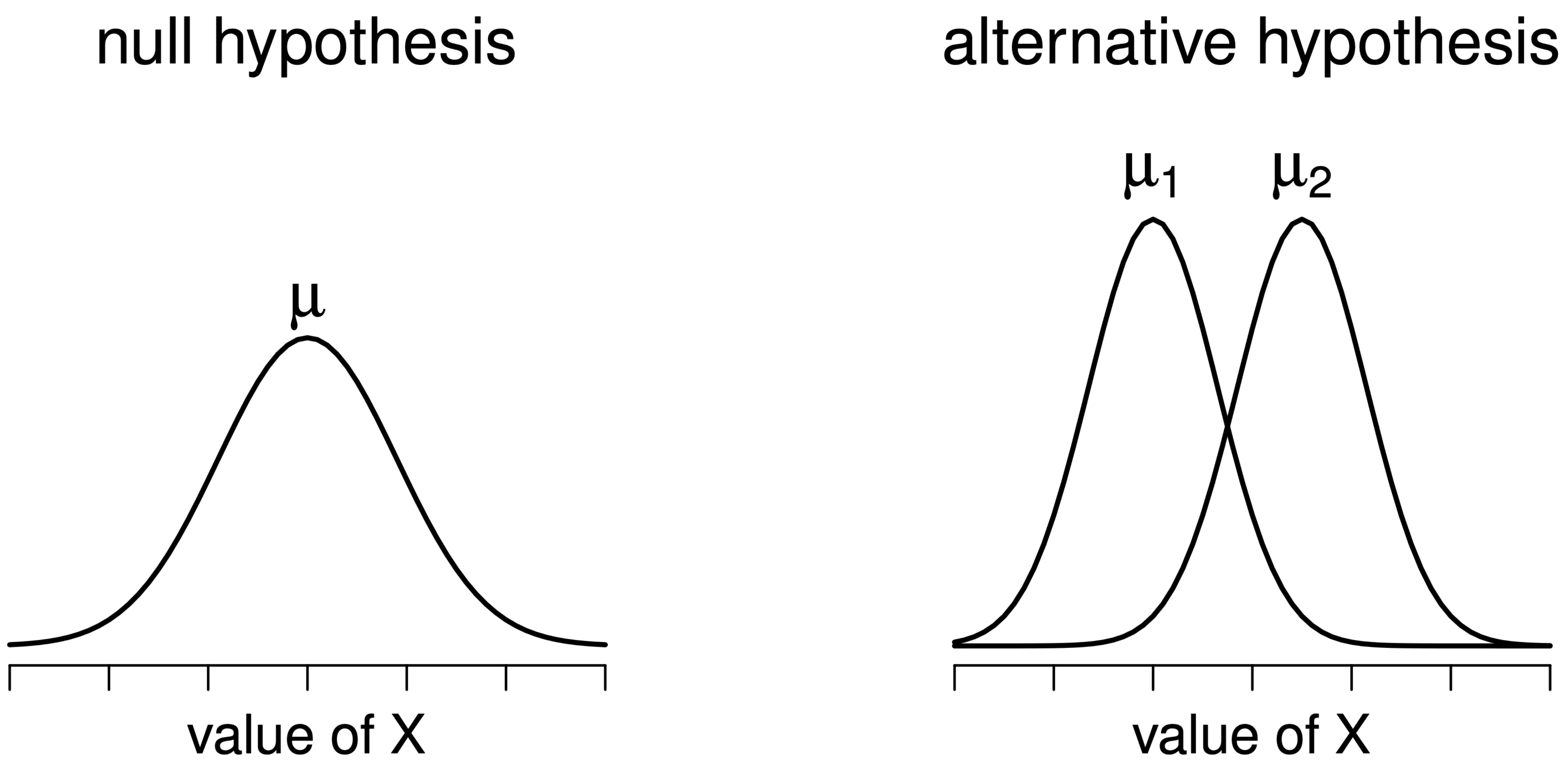
Other t-test varieties

Student's Independent Samples t-test



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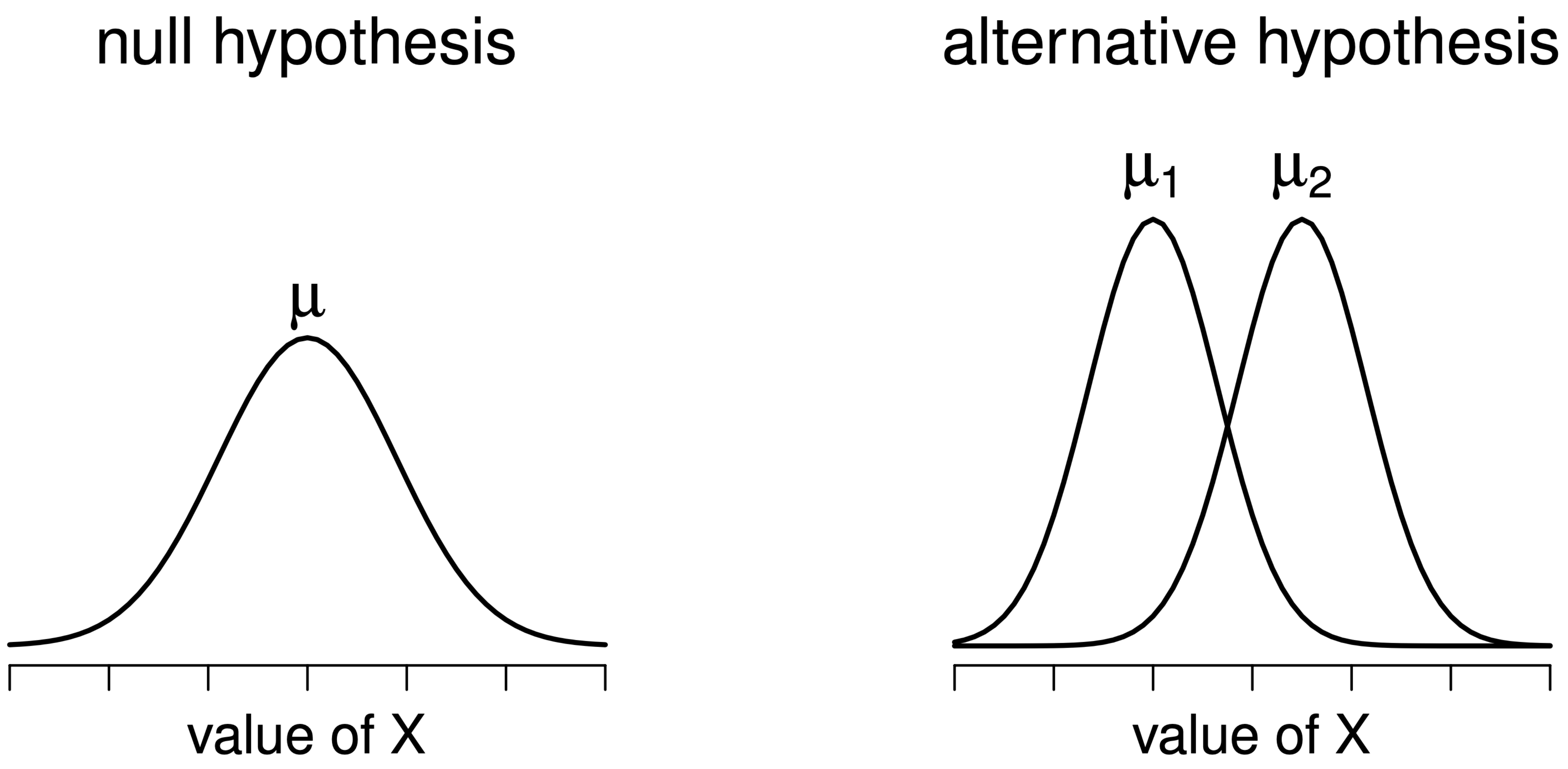

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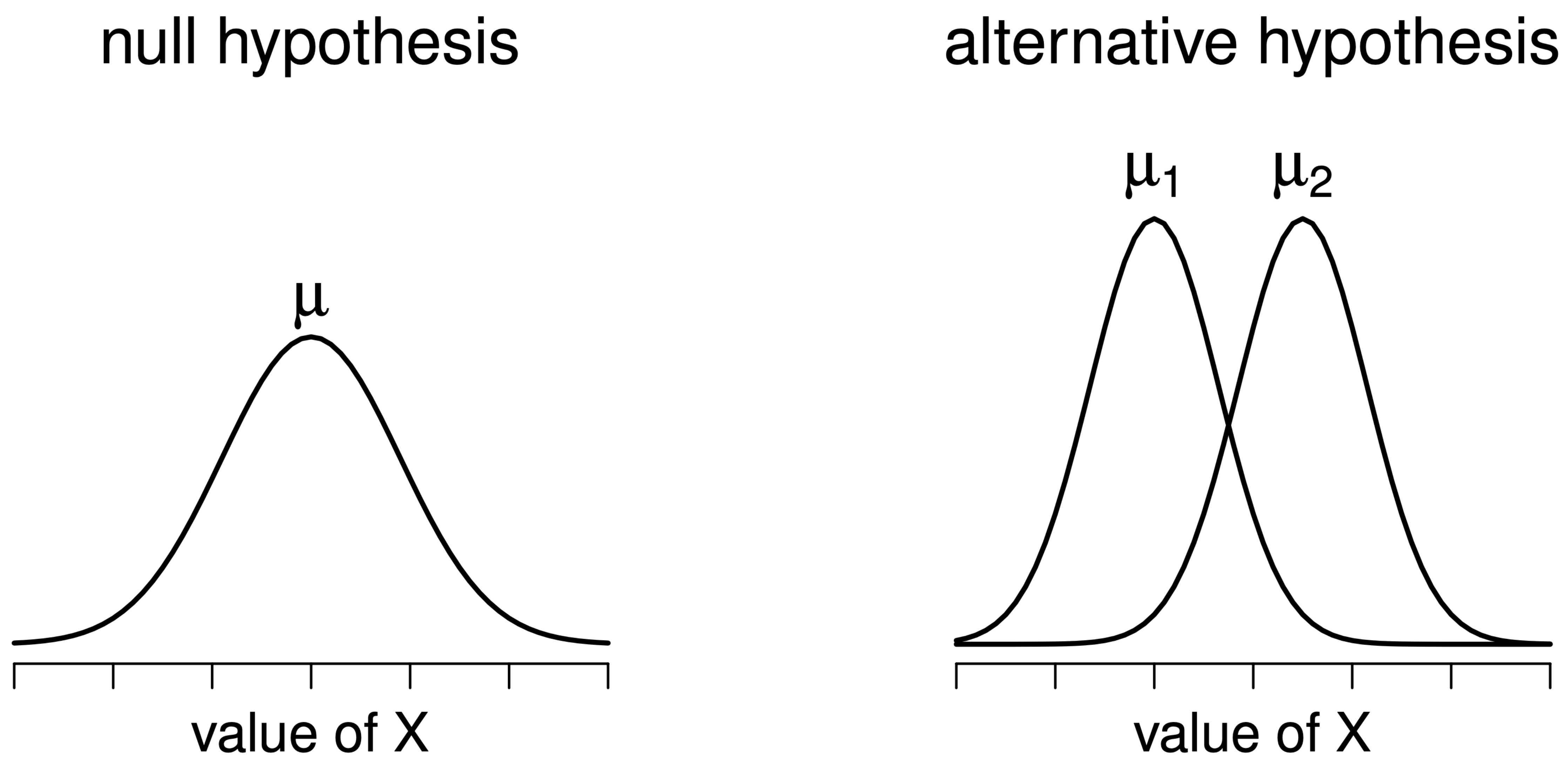

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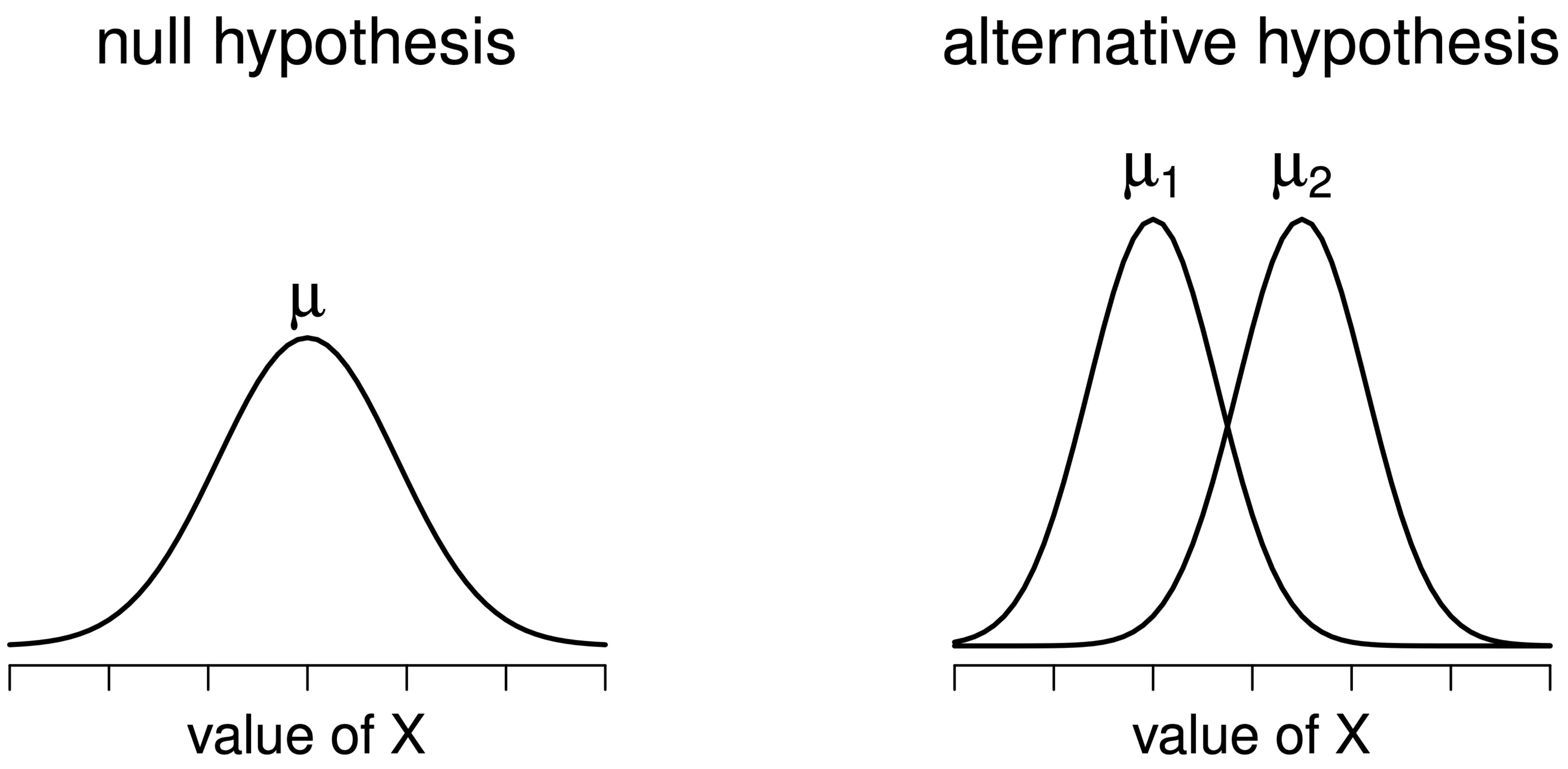

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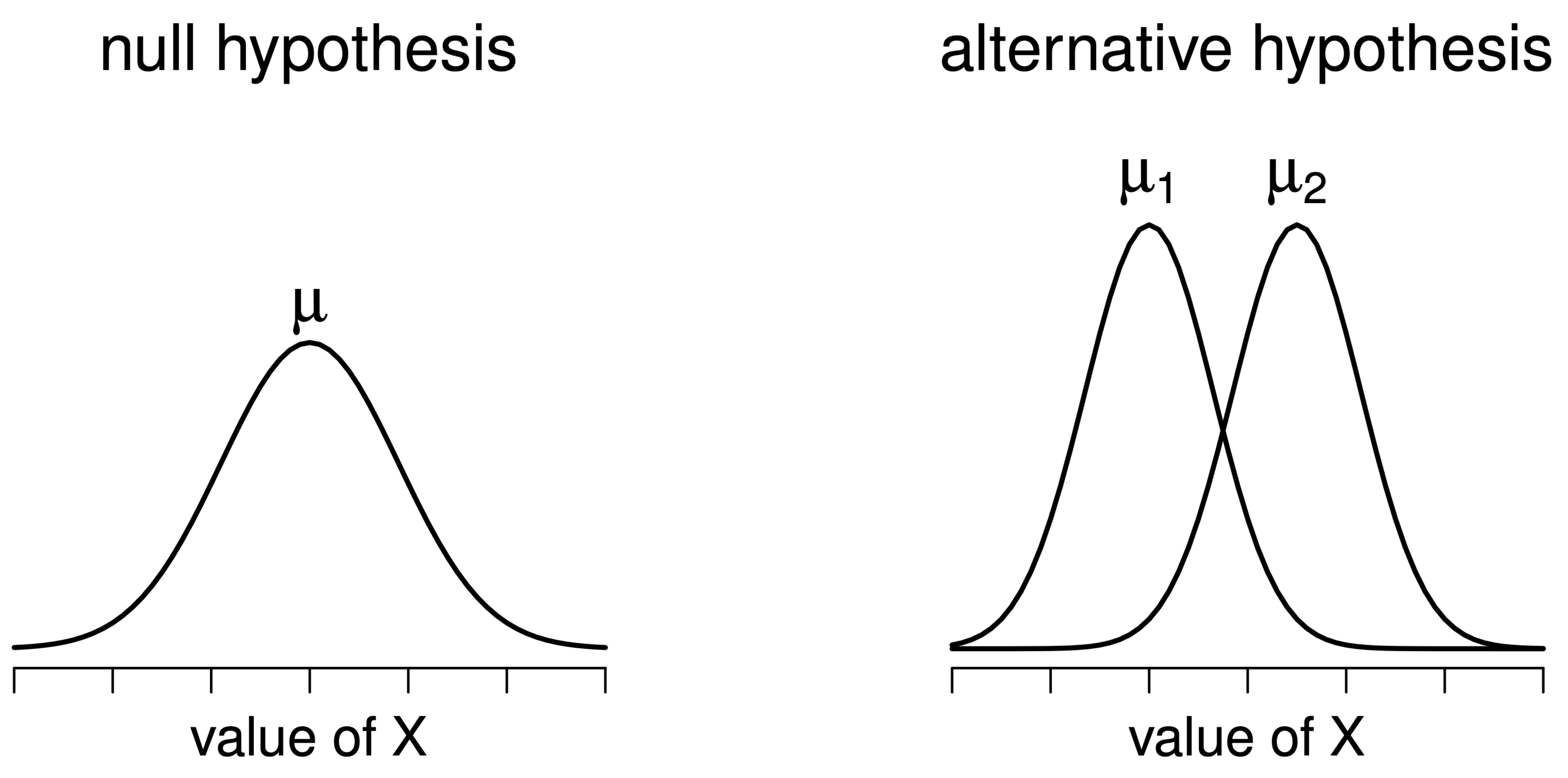

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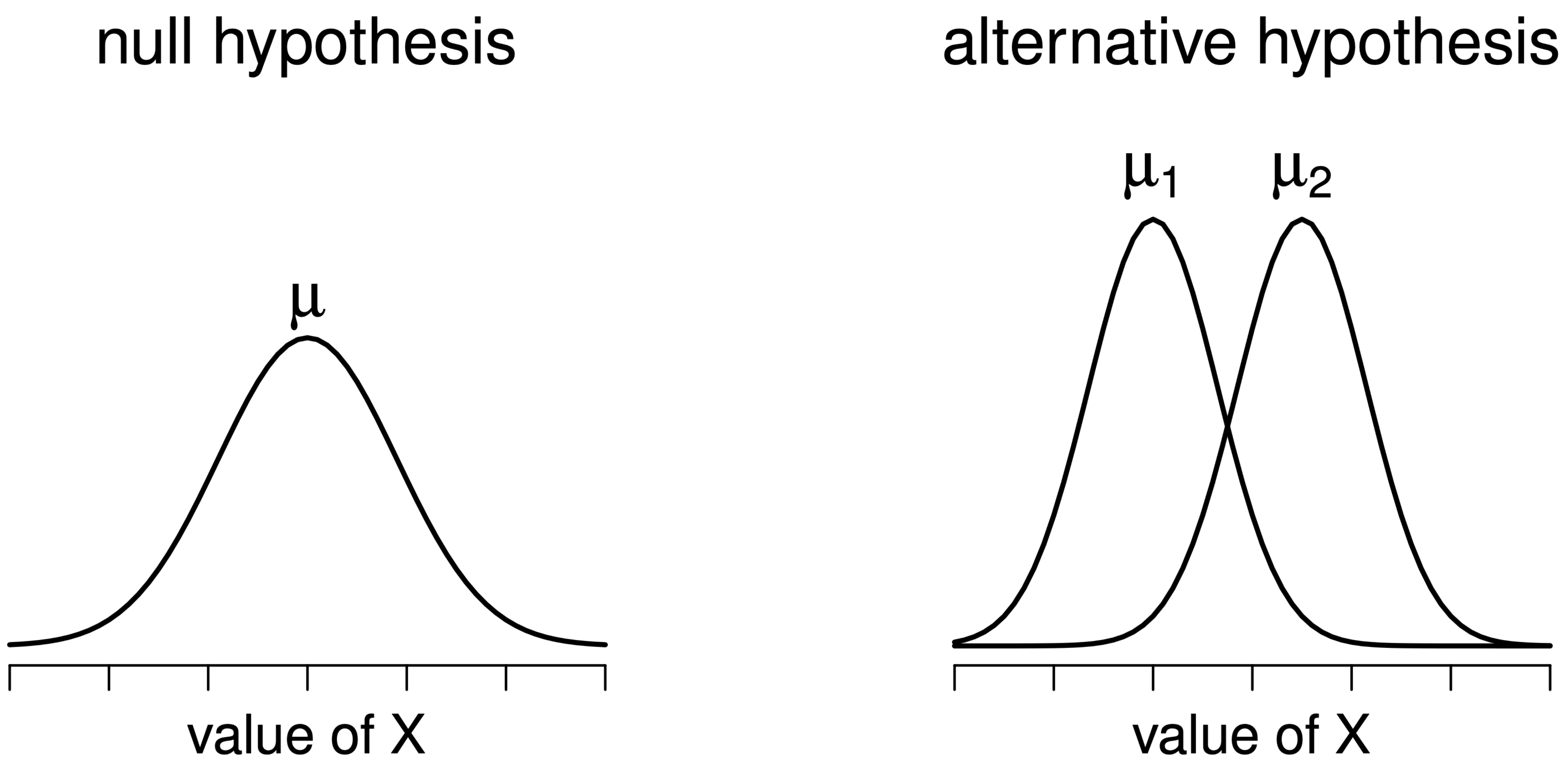

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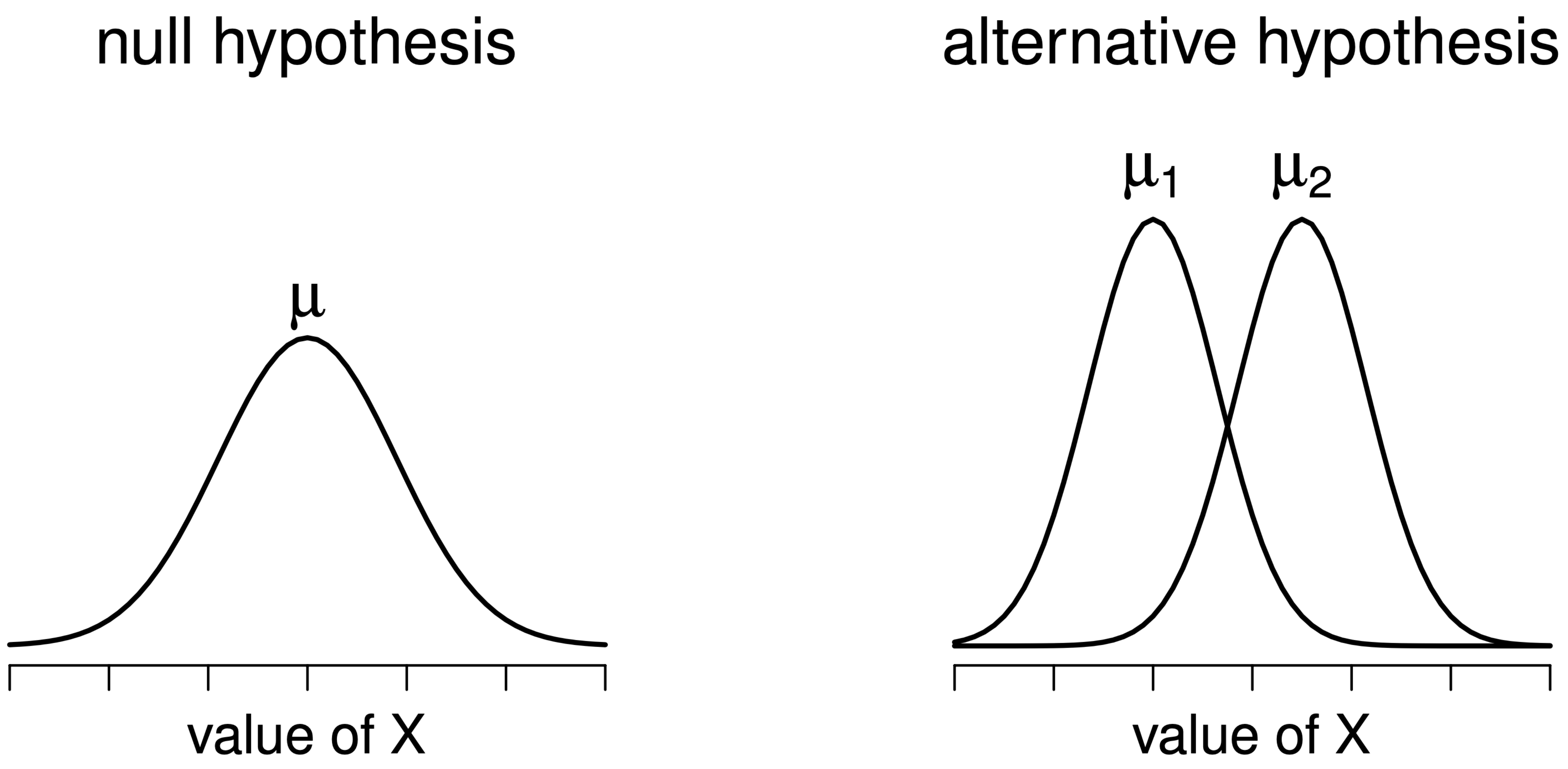

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  - $H_0 : \mu_1 = \mu_2$
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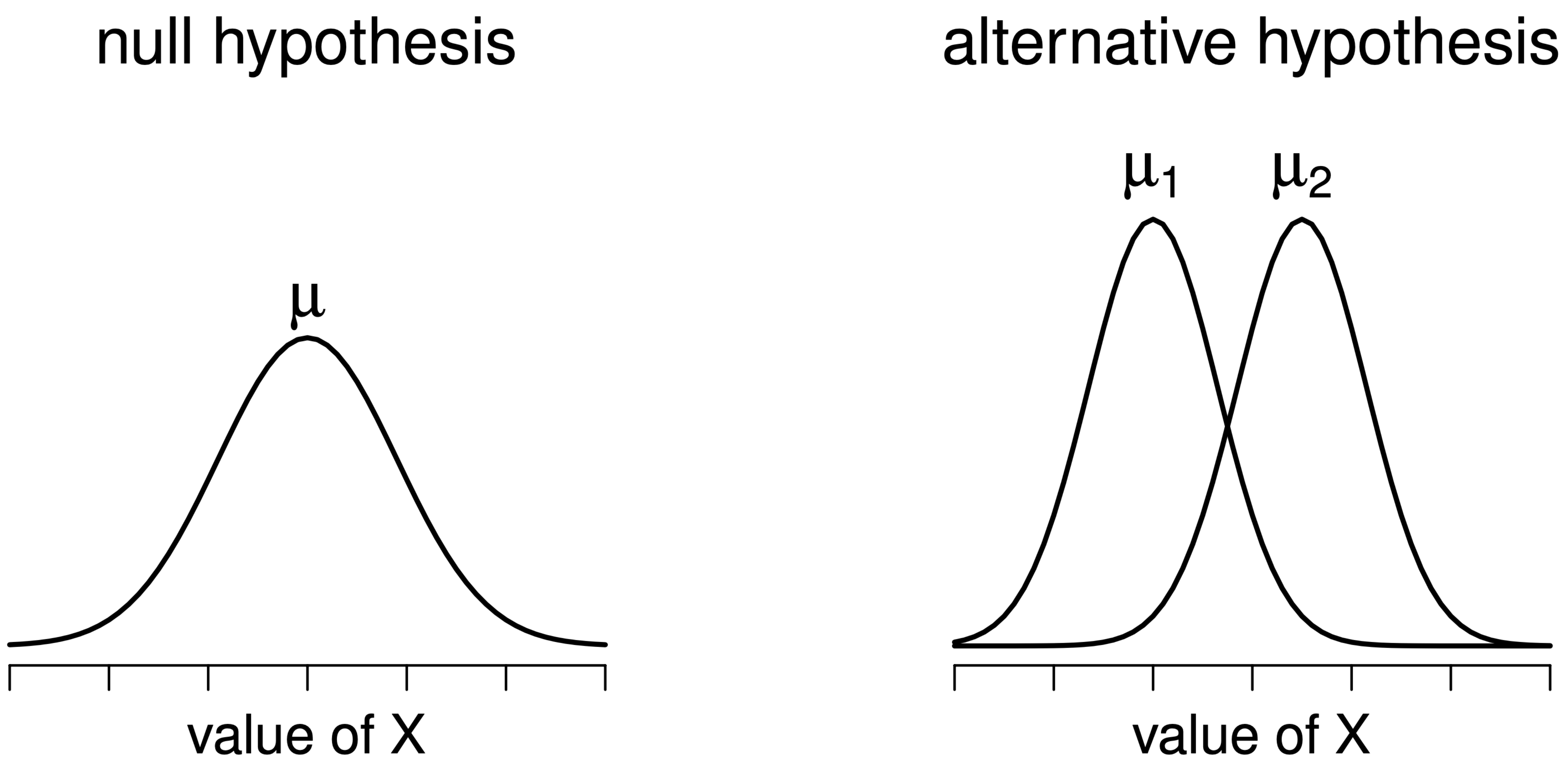
- Example: do **two tutors** get their students **significantly different grades?**



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- Student's test **assumes** the two populations have the **same standard deviation**
    - This is the key difference from **Welch's t-test**
- The two samples are **assumed** to be **independent** from each other
    - i.e. having the **same participant** in both samples would break this assumption

Student's t-test in R

```
> head(tutor_grades)
     grade     tutor
1 93.60010 Anastasia
2 84.59462 Anastasia
3 79.06581 Anastasia
4 61.45601 Anastasia
5 73.03190 Anastasia
6 73.46387 Anastasia
> t.test(formula = grade ~ tutor, data=tutor_grades, var.equal=TRUE)

	Two Sample t-test

data:  grade by tutor
t = 1.1026, df = 31, p-value = 0.2787
alternative hypothesis: true difference in means between group Anasta
sia and group Bernadette is not equal to 0
95 percent confidence interval:
 -3.053475 10.240356
sample estimates:
 mean in group Anastasia mean in group Bernadette
                72.34231                 68.74887
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- We still use `t.test()`, but the **syntax is different** from one-sample tests

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  - This makes it a **multiple-sample test**
- `var.equal = TRUE` gives us the **Student's t-test**
  - If this is left out or set to FALSE we get **Welch Independent-samples t-test**

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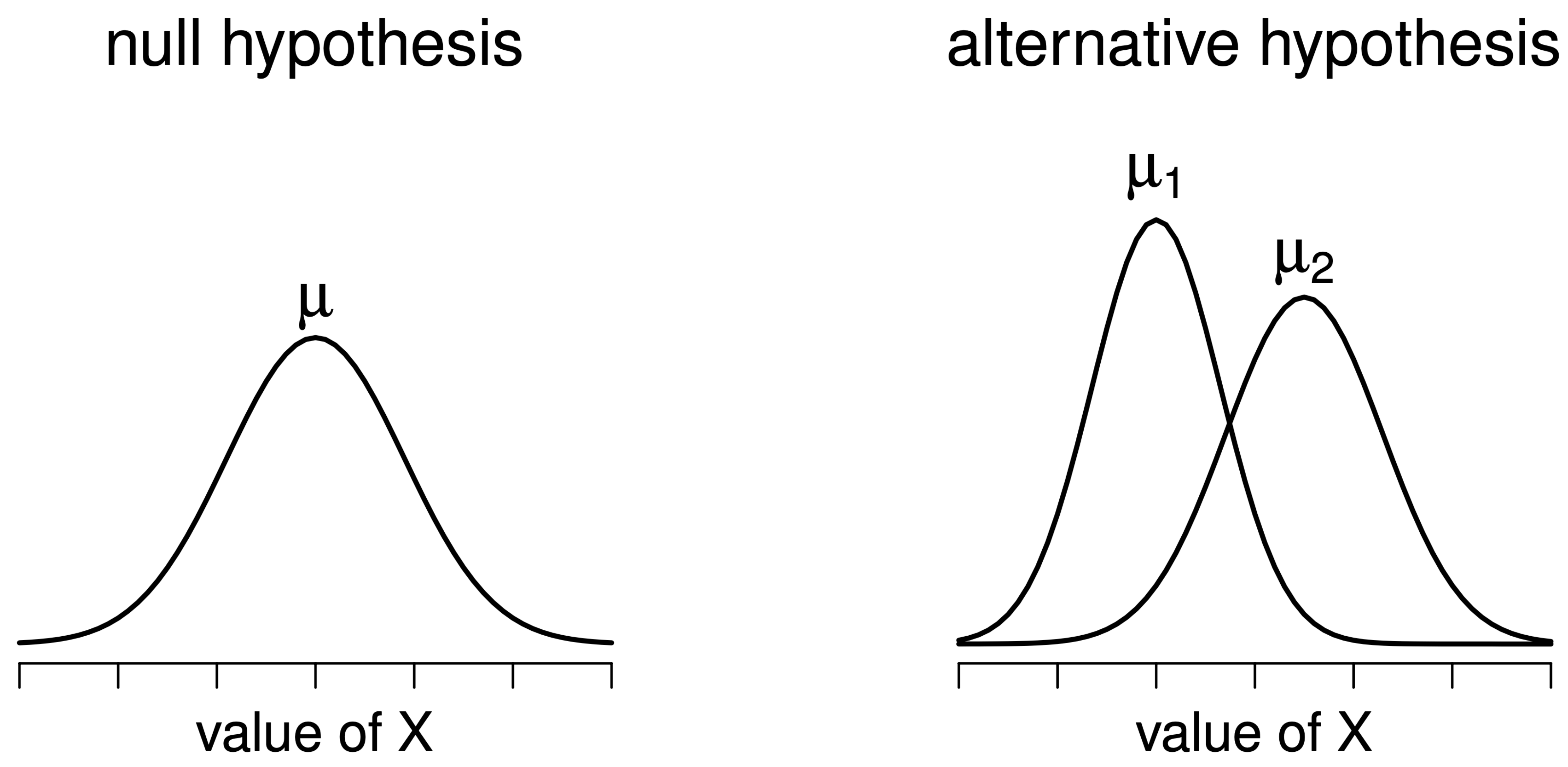
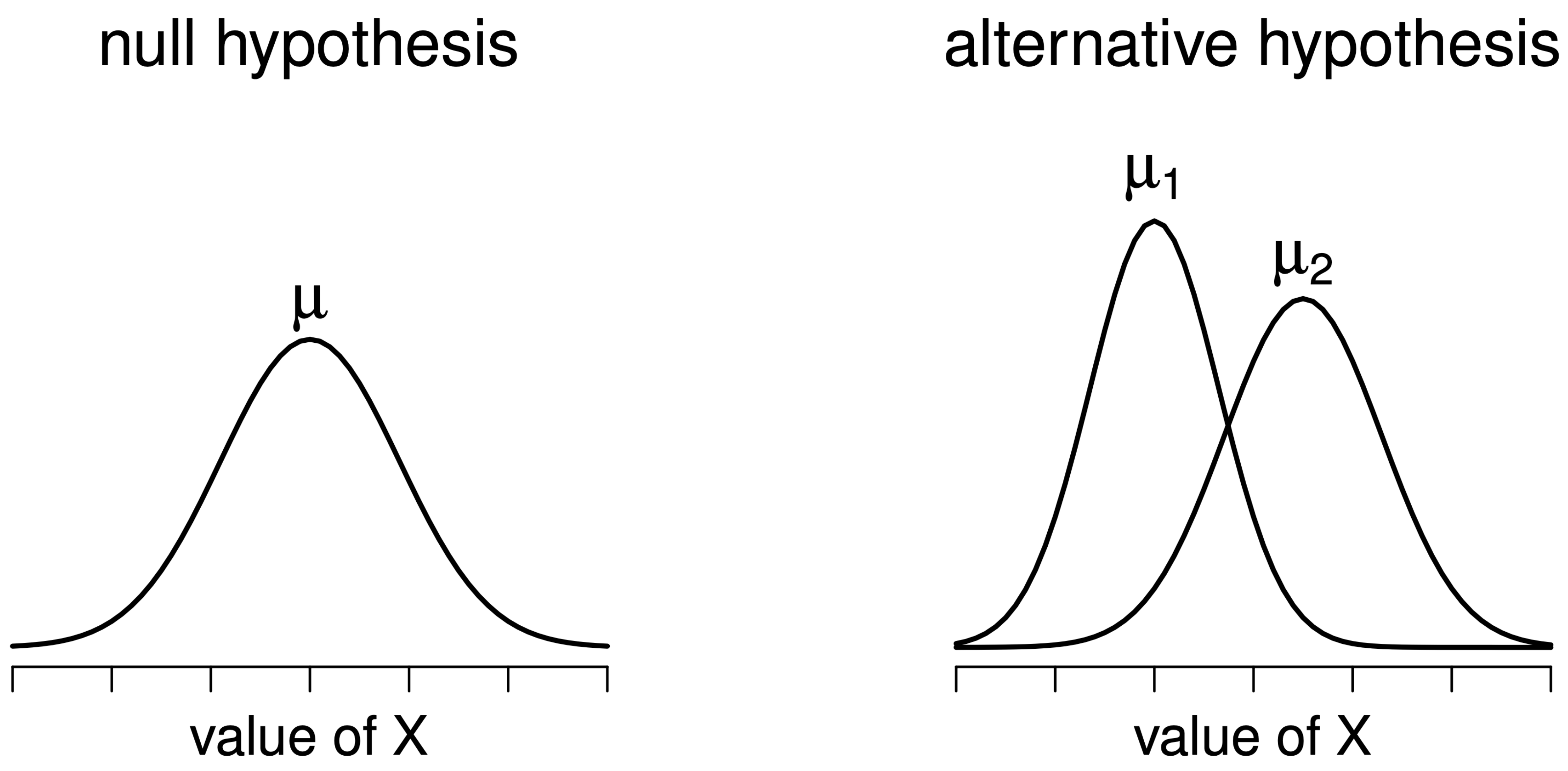


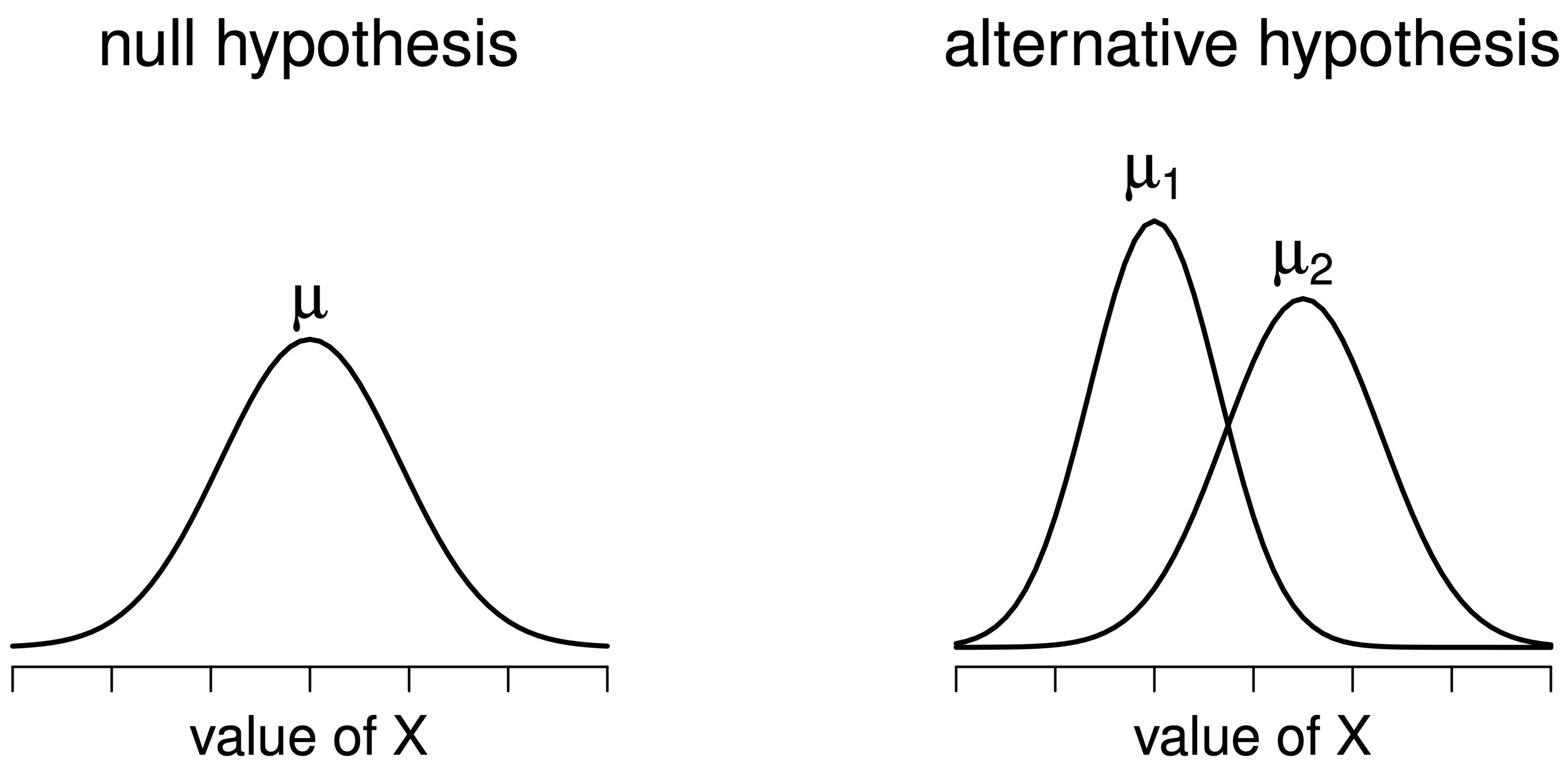
Welch Independent Samples t-test

- Welch's test is **just like Student's**, except we **don't assume the same stdev** for each population
  - Again, there's some tricky math you shouldn't worry about for this course



Welch Independent Samples t-test

- Welch's test is **just like Student's**, except we **don't assume the same stdev** for each population
  - Again, there's some tricky math you shouldn't worry about for this course
- Setting `var.equal = FALSE` in `t.test()` gets you the Welch test
  - This is the **default value**, so you can also leave the argument out



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- $t(19) = 2.25$: the t-score with 19 degrees of freedom was 2.25
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- 95% confidence interval : the range the model thinks that the Linguistics student population mean falls into (sometimes abbreviated CI_{95})