Word Vectors; Gradient Descent

LING 282/482: Deep Learning for Computational Linguistics

C.M. Downey

Fall 2024

Today's Plan

- Word Vectors
- Machine Learning Terminology / Notation
- Gradient Descent

Word Vectors, Intro

Distributional Similarity

- "You shall know a word by the company it keeps!" (*Firth, 1957*)

Distributional Similarity

- “You shall know a word by the company it keeps!” (*Firth, 1957*)
  - A bottle of ***tezgüino*** is on the table.

Distributional Similarity

- “You shall know a word by the company it keeps!” (*Firth, 1957*)
  - A bottle of ***tezgüino*** is on the table.
  - Everybody likes ***tezgüino***.

Distributional Similarity

- "You shall know a word by the company it keeps!" (*Firth, 1957*)
  - A bottle of ***tezgüino*** is on the table.
  - Everybody likes ***tezgüino***.
  - ***Tezgüino*** makes you drunk.

Distributional Similarity

- “You shall know a word by the company it keeps!” (*Firth, 1957*)
  - A bottle of ***tezgüino*** is on the table.
  - Everybody likes ***tezgüino***.
  - ***Tezgüino*** makes you drunk.
  - We make ***tezgüino*** from corn.

Distributional Similarity

- “You shall know a word by the company it keeps!” (*Firth, 1957*)
  - A bottle of ***tezgüino*** is on the table.
  - Everybody likes ***tezgüino***.
  - ***Tezgüino*** makes you drunk.
  - We make ***tezgüino*** from corn.
- Tezguino; corn-based alcoholic beverage. (From *Lin, 1998a*)

Distributional Similarity

- How can we represent the “company” of a word?

Distributional Similarity

- How can we represent the “company” of a word?
- How can we make similar words have similar representations?

Why use word vectors?

- With words, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires exact same word to be in training and test
- One-hot vectors:
  - “terrible”: [0 0 0 0 0 0 1 0 0 0 … 0]
  - “Sparse” vectors
  - length = size of vocabulary
  - All words are as different from each other
    - e.g. “terrible” is as different from “bad” as from “awesome”

Why use word vectors?

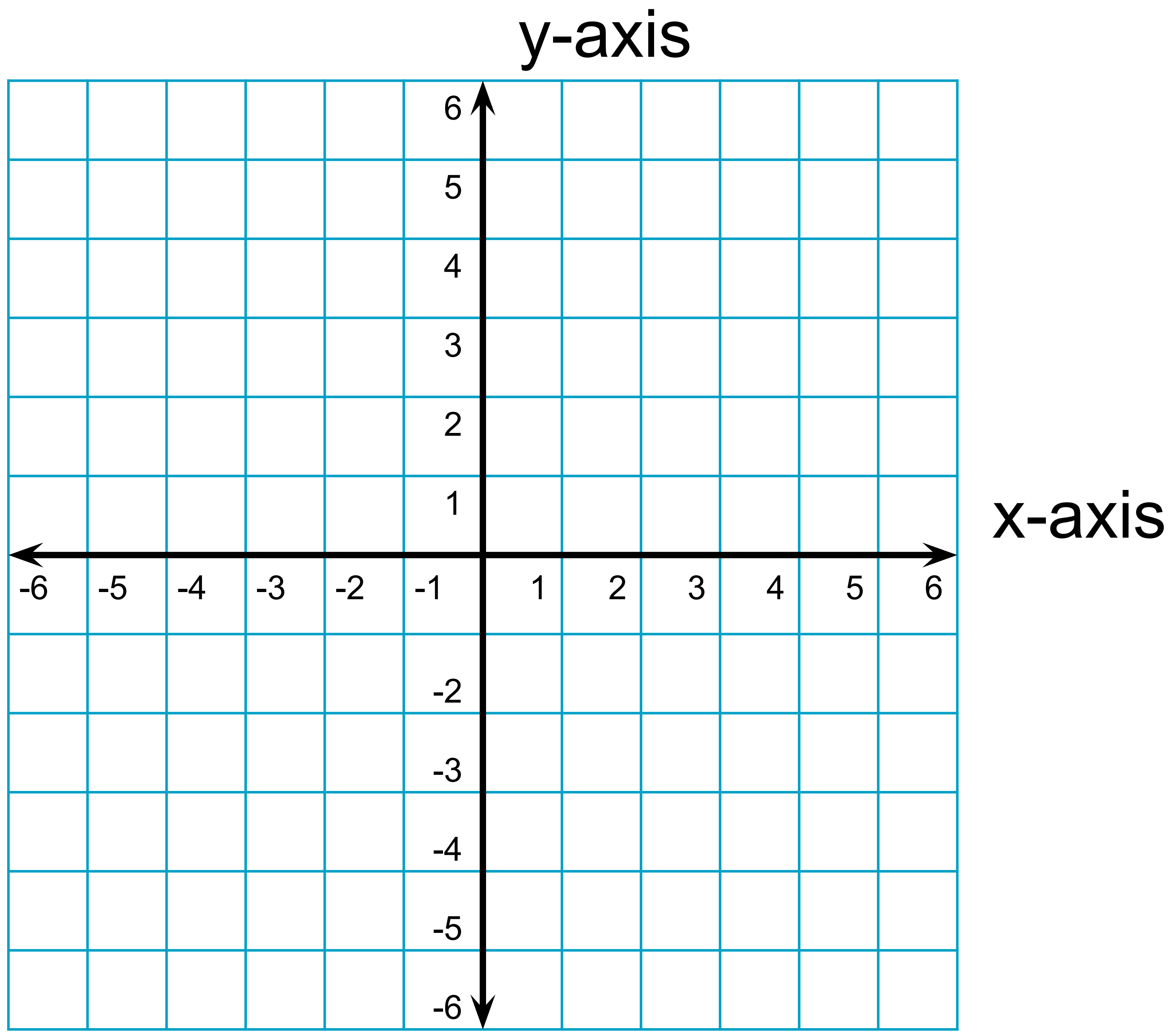
- With embeddings:
  - “Dense” vectors
  - “The previous word was vector [35,22,17, …]”
  - Now in the test set we might see a similar vector [34,21,14, …]
  - We can generalize to similar but unseen words!

Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”

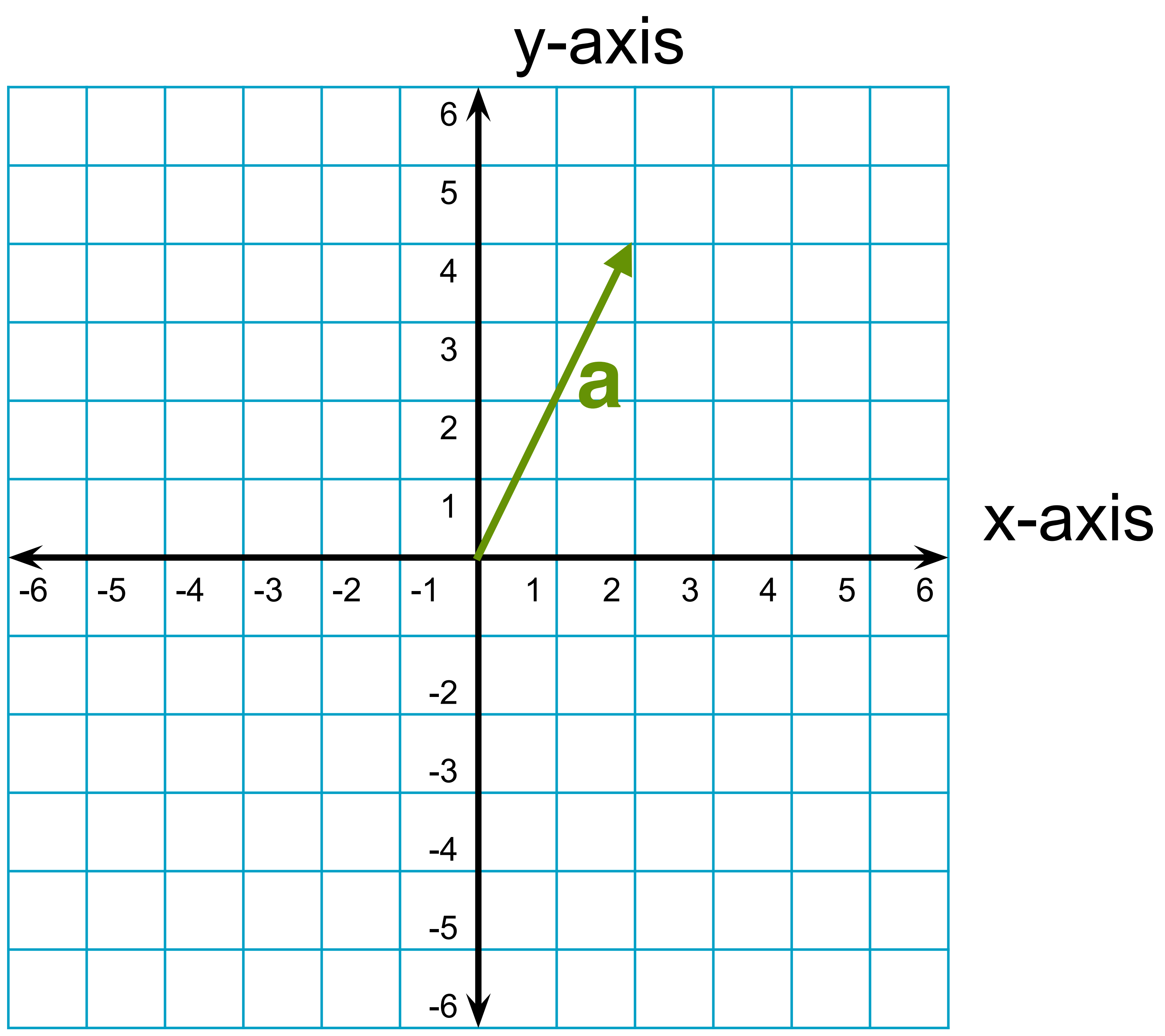
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”



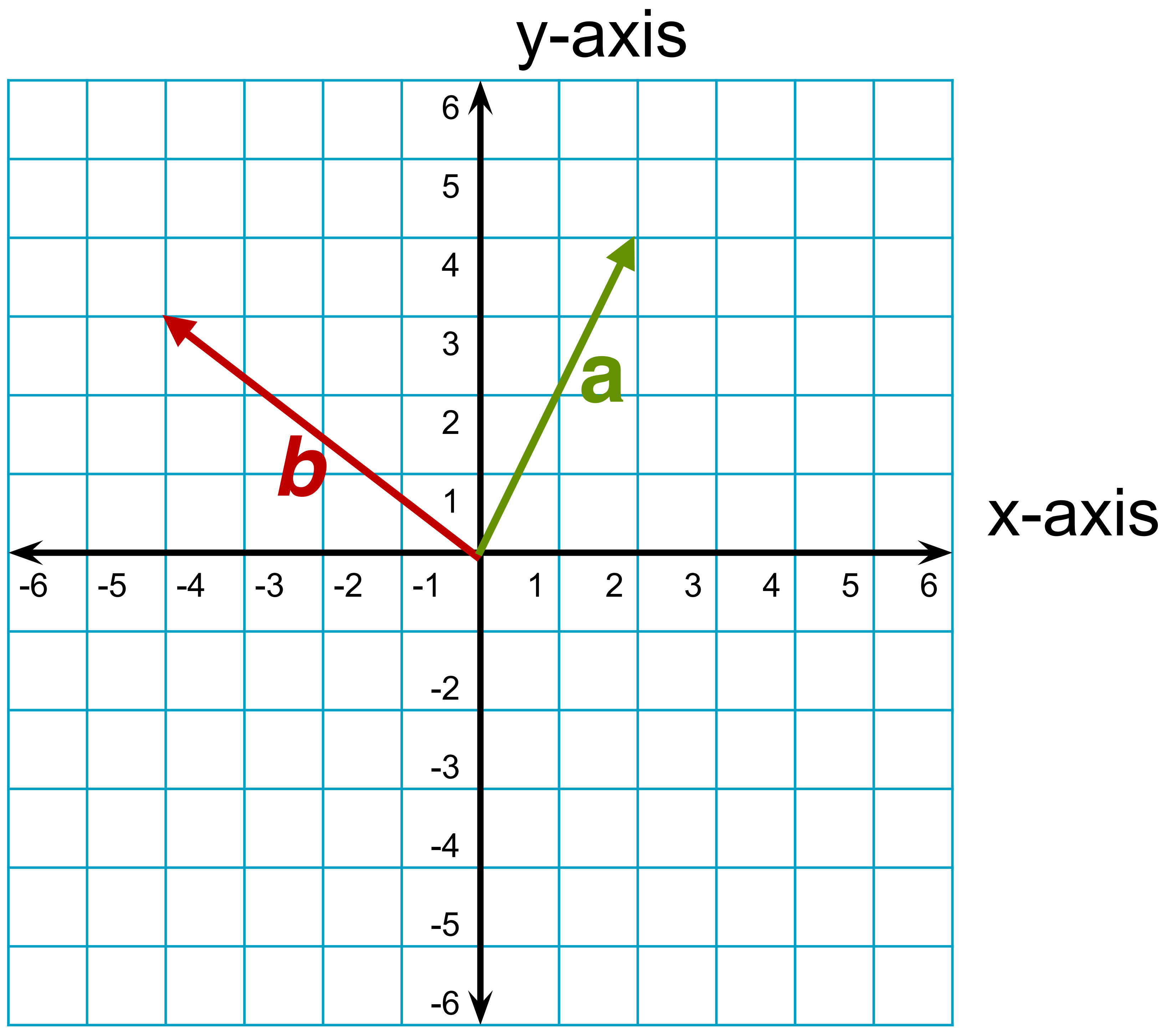
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”
  - $\vec{a} = \langle 2,4 \rangle$



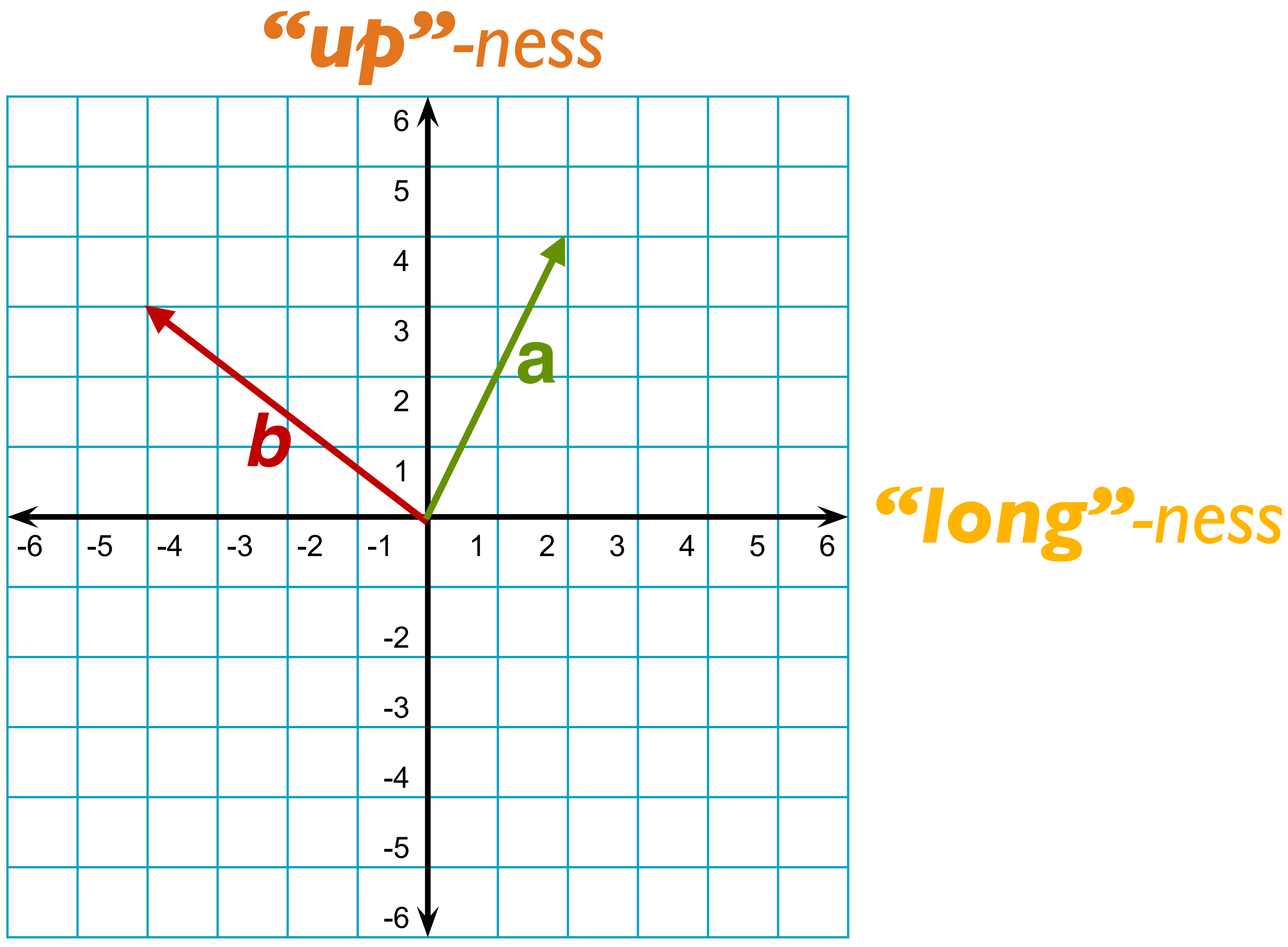
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”
  - $\vec{a}$ = ⟨2,4⟩
  - $\vec{b}$ = ⟨-4,3⟩



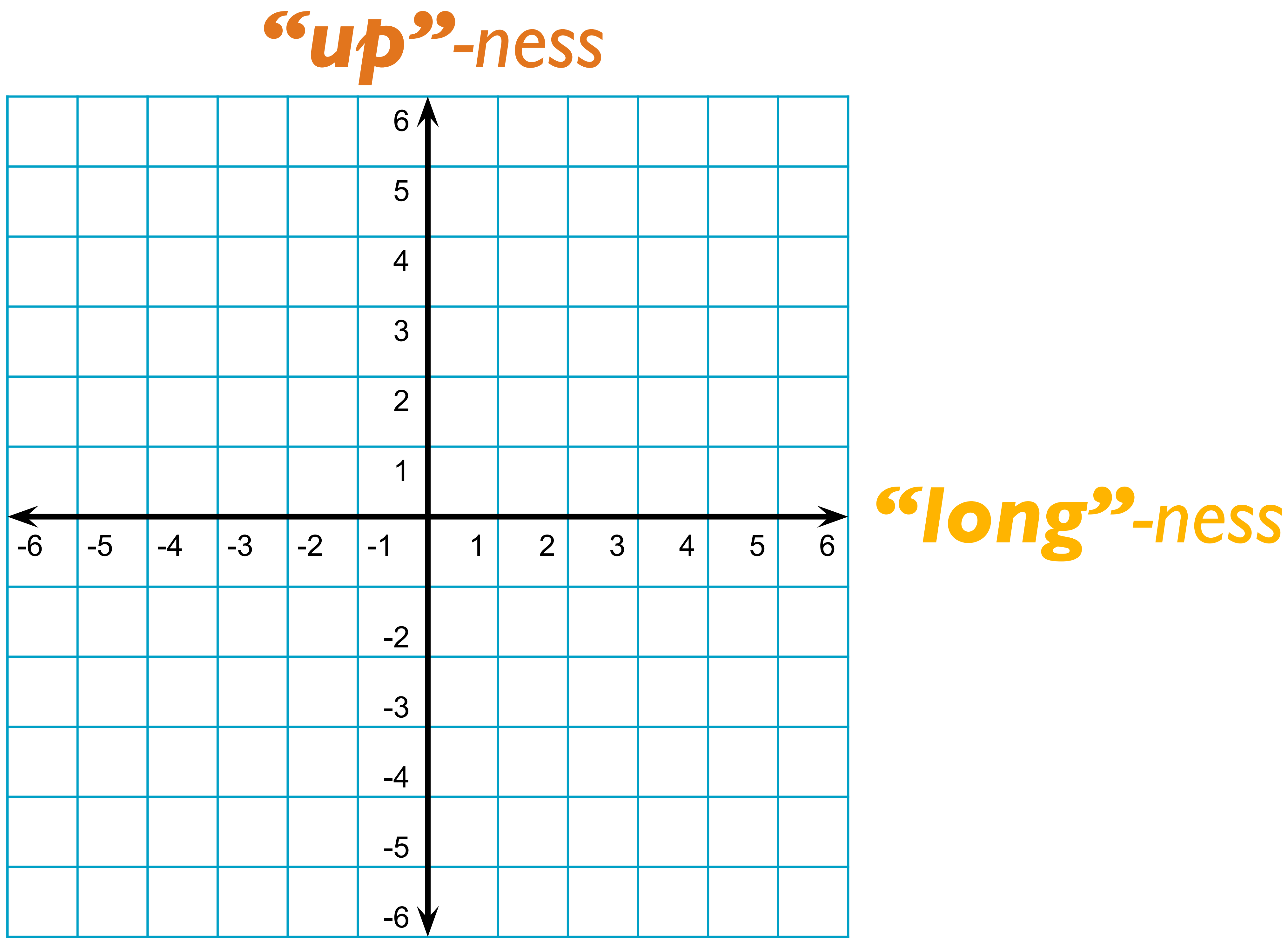
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”
  - $\vec{a} = \langle 2,4 \rangle$
  - $\vec{b} = \langle -4,3 \rangle$



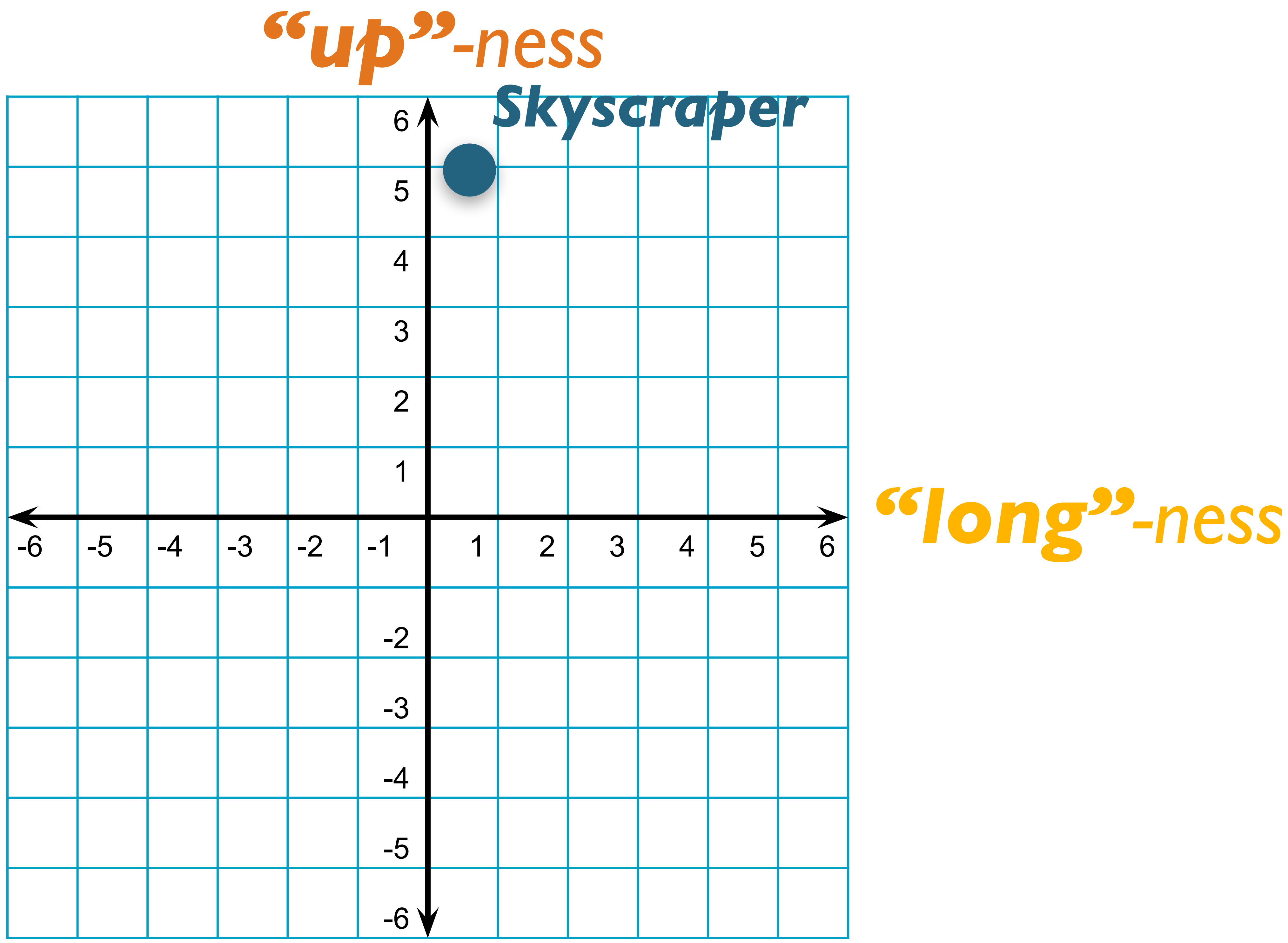
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a "dimension"
  - $\vec{a} = \langle 2,4 \rangle$
  - $\vec{b} = \langle -4,3 \rangle$



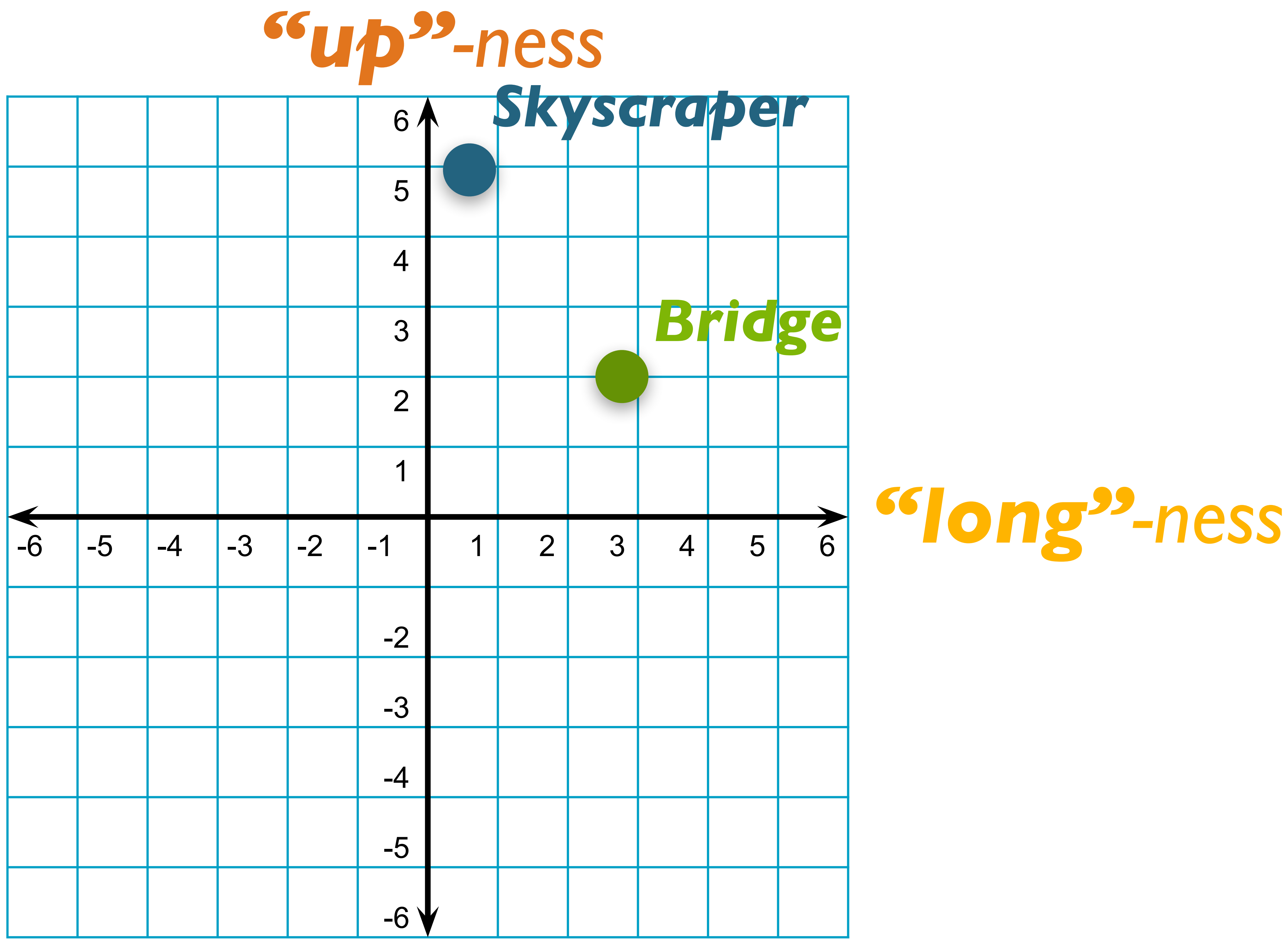
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”
  - $\vec{a} = \langle 2,4 \rangle$
  - $\vec{b} = \langle -4,3 \rangle$



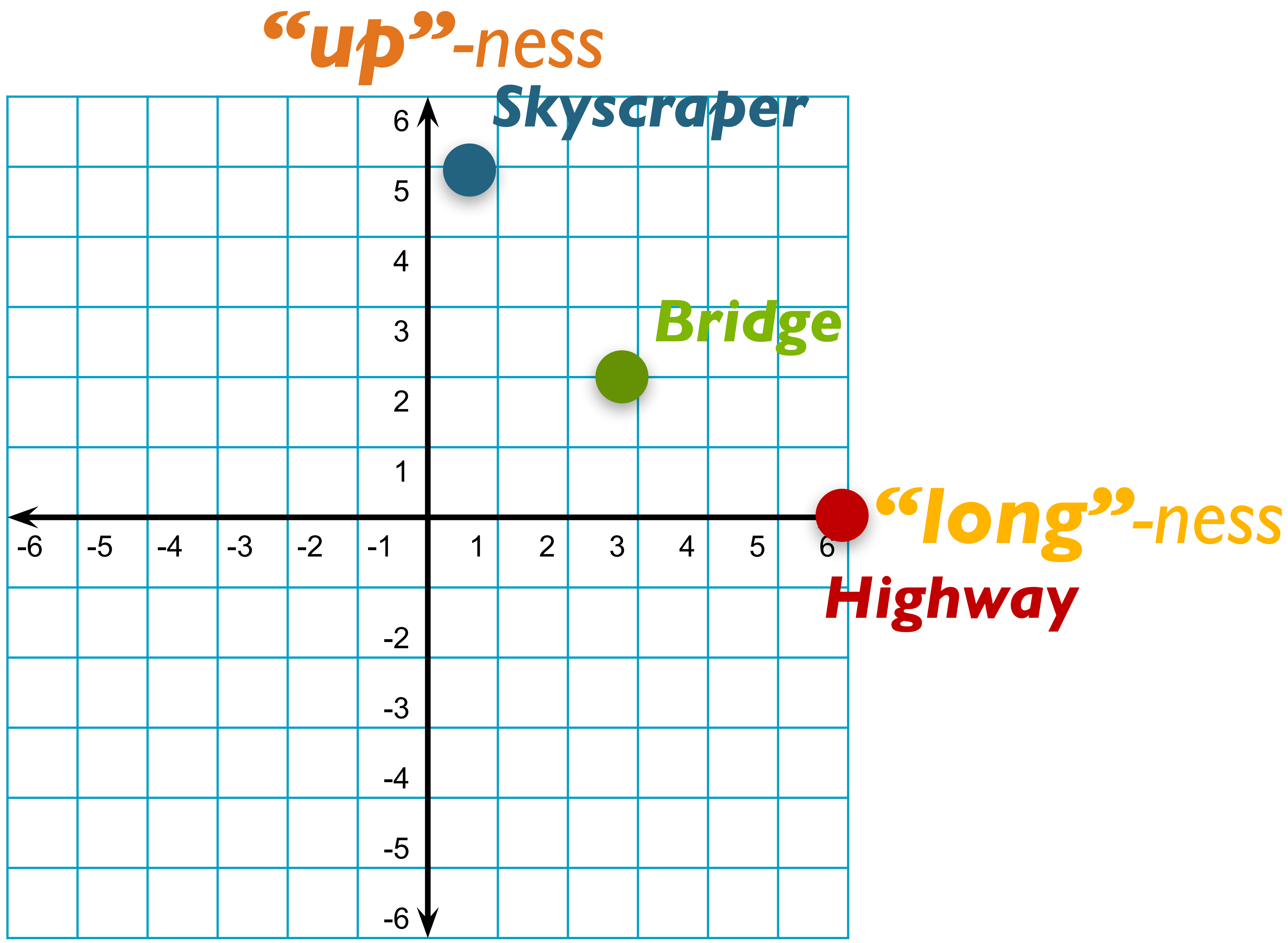
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a “dimension”
  - $\vec{a} = \langle 2,4 \rangle$
  - $\vec{b} = \langle -4,3 \rangle$



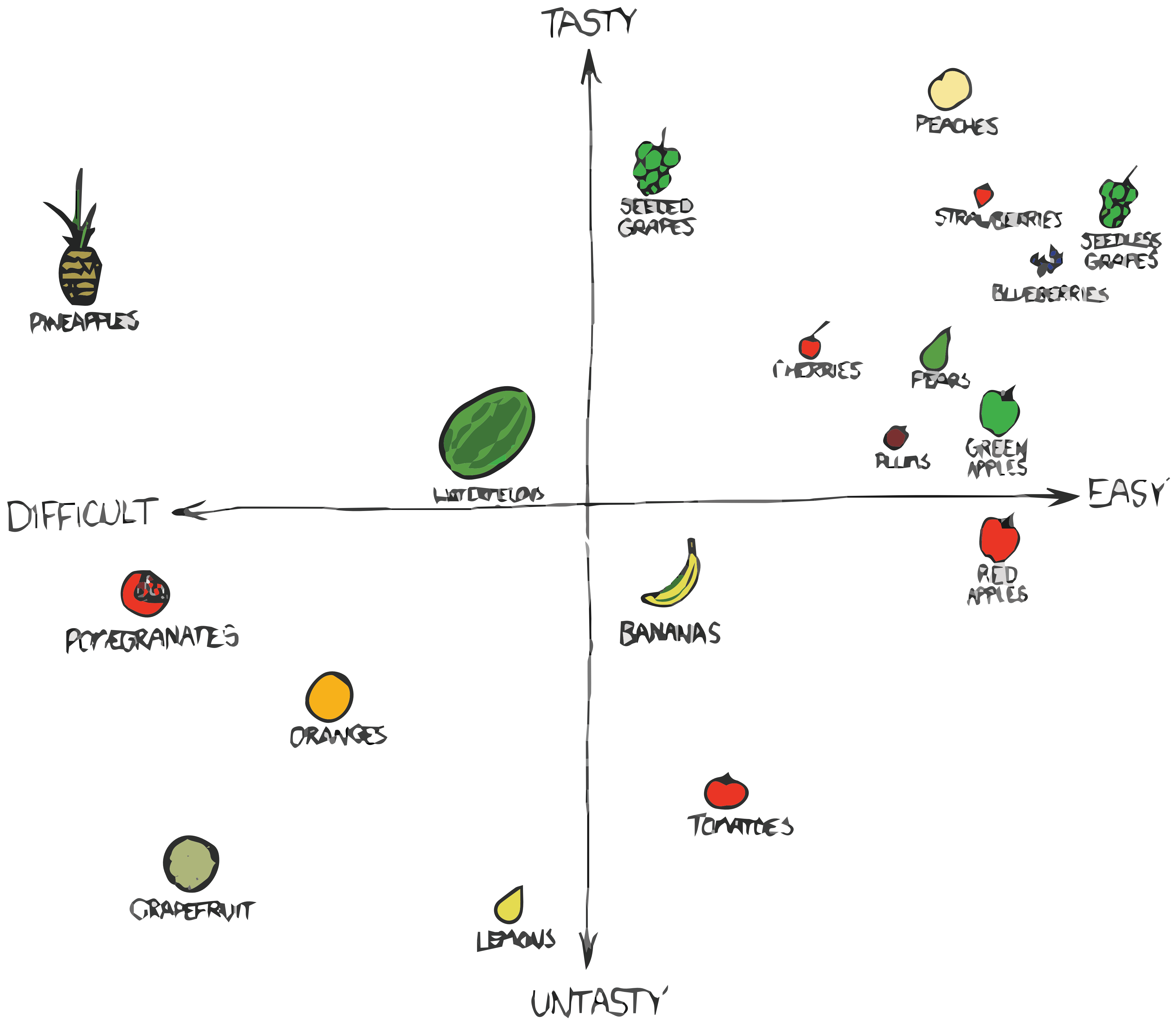
Vectors as information

- A vector is a list of numbers
- Each number can be thought of as representing a "dimension"
  - $\vec{a} = \langle 2,4 \rangle$
  - $\vec{b} = \langle -4,3 \rangle$



Vectors as information

xkcd.com/388



Vector Length

- A vector's length is equal to the *square root of the dot product with itself*

$$\text{length}(x) = \|x\| = \sqrt{x \cdot x}$$

Vector Distances: Manhattan & Euclidean

- **Manhattan Distance**
  - Distance as cumulative horizontal + vertical moves
- **Euclidean Distance**
  - Our normal notion of distance
- Both are too sensitive to extreme values

$$d_{\text{manhattan}}(x, y) = \sum_i |x_i - y_i|$$

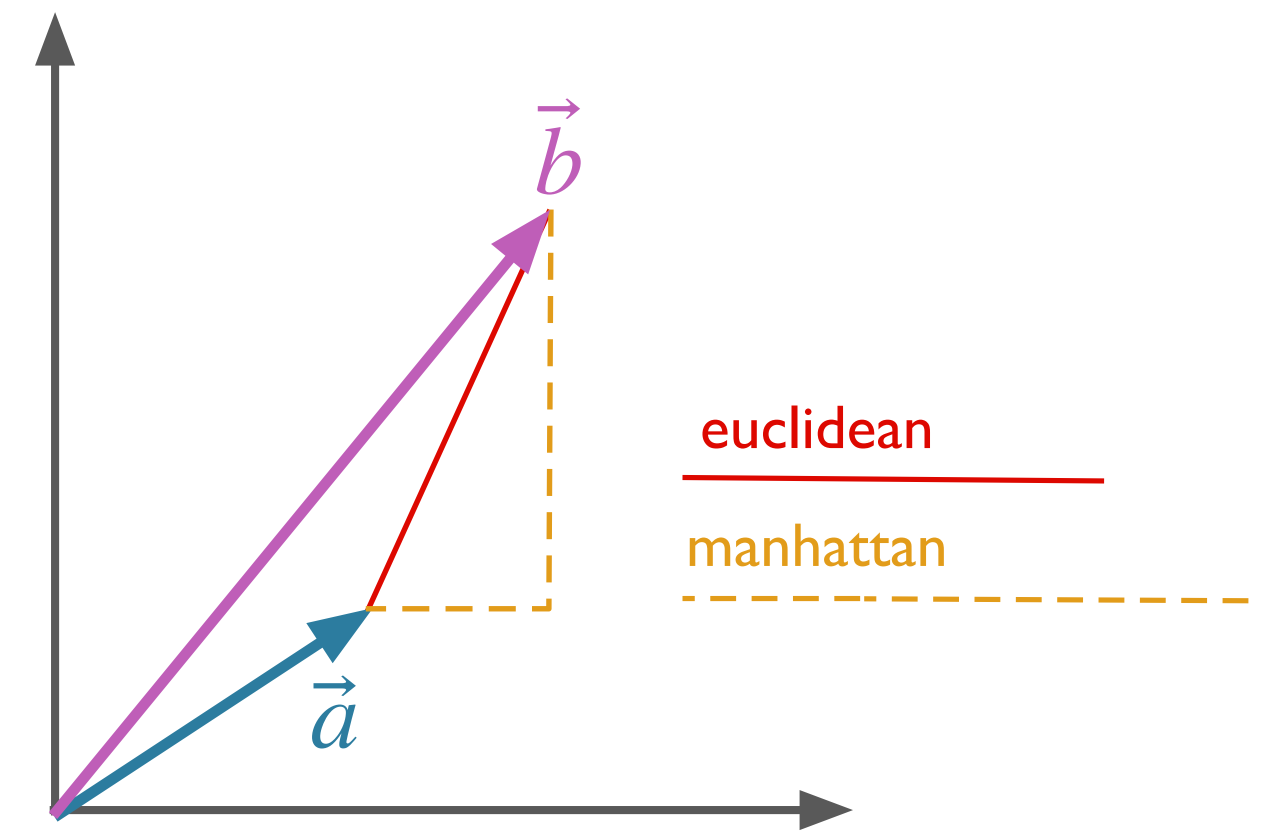
$$d_{\text{euclidian}}(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$$

Vector Distances: Manhattan & Euclidean

- **Manhattan Distance**
  - Distance as cumulative horizontal + vertical moves
- **Euclidean Distance**
  - Our normal notion of distance
- Both are too sensitive to extreme values

$$d_{\text{manhattan}}(x, y) = \sum_i |x_i - y_i|$$

$$d_{\text{euclidian}}(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$$



Vector Similarity: Dot Product

- Produces real number scalar from product of vectors' components
- Gives **higher similarity** to **longer** vectors

$$\text{sim}_{\text{dot}}(x, y) = x \cdot y = \sum_i x_i y_i$$

Vector Similarity: Cosine

- If you normalize the dot product for vector magnitude…
- …result is same as **cosine of angle** between the vectors

$$\text{sim}_{\text{cosine}}(x, y) = \frac{x \cdot y}{\|x\| \|y\|} = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2} \sqrt{\sum_i y_i^2}}$$

Bag of Words Vectors

- Represent ‘company’ of word such that similar words will have similar representations
  - ‘Company’ = context

Bag of Words Vectors

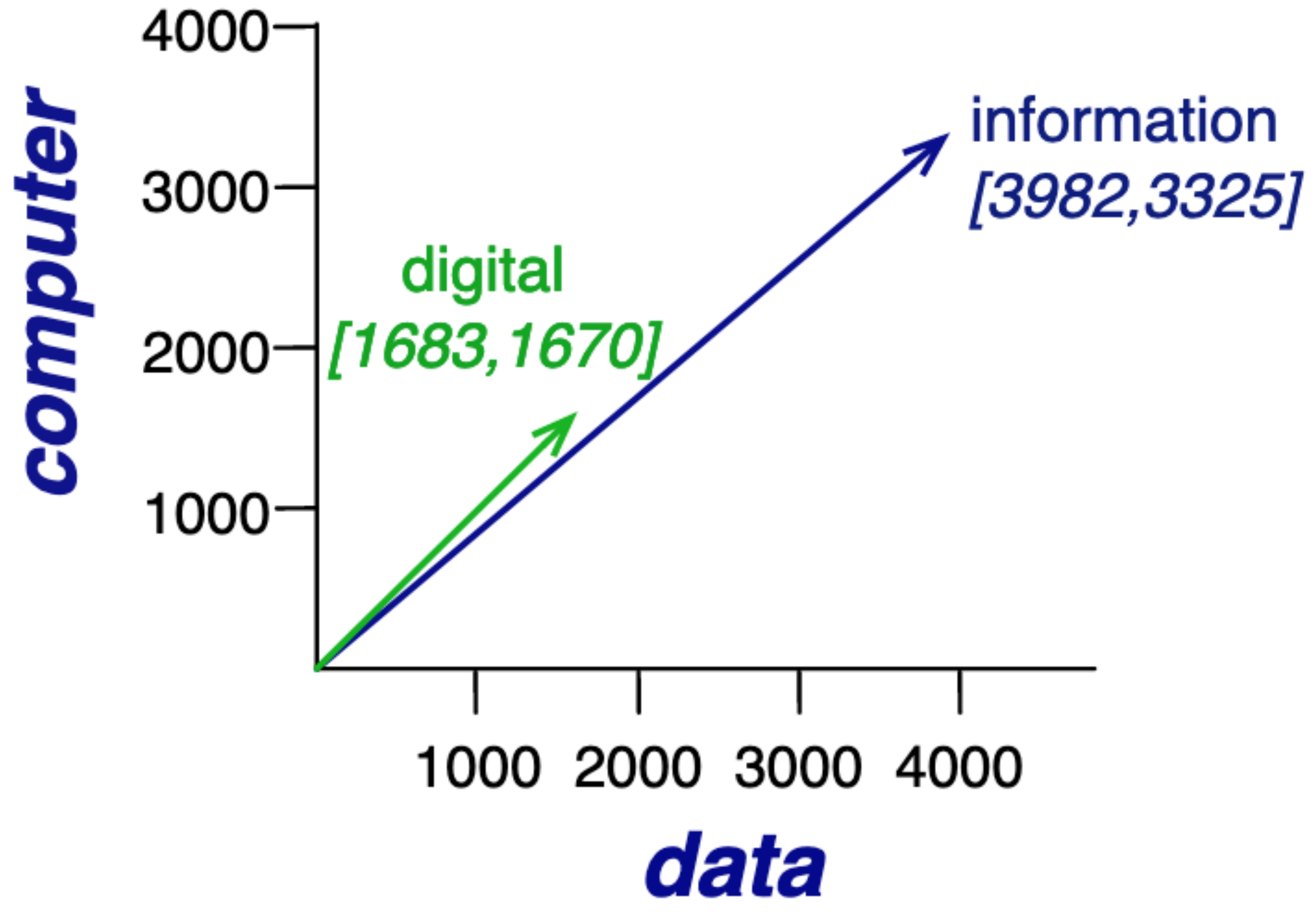
- Represent ‘company’ of word such that similar words will have similar representations
  - ‘Company’ = context
- Word represented by **context feature vector**

Bag of Words Vectors

- Represent 'company' of word such that similar words will have similar representations
  - 'Company' = context
- Word represented by **context feature vector**
- Initial representation:
  - "**Bag of words**" feature vector
  - Feature vector length *N*, where *N* is size of vocabulary
    - f_i+=1 if $word_i$ within window size *w* of *word*

Bag of Words Vectors

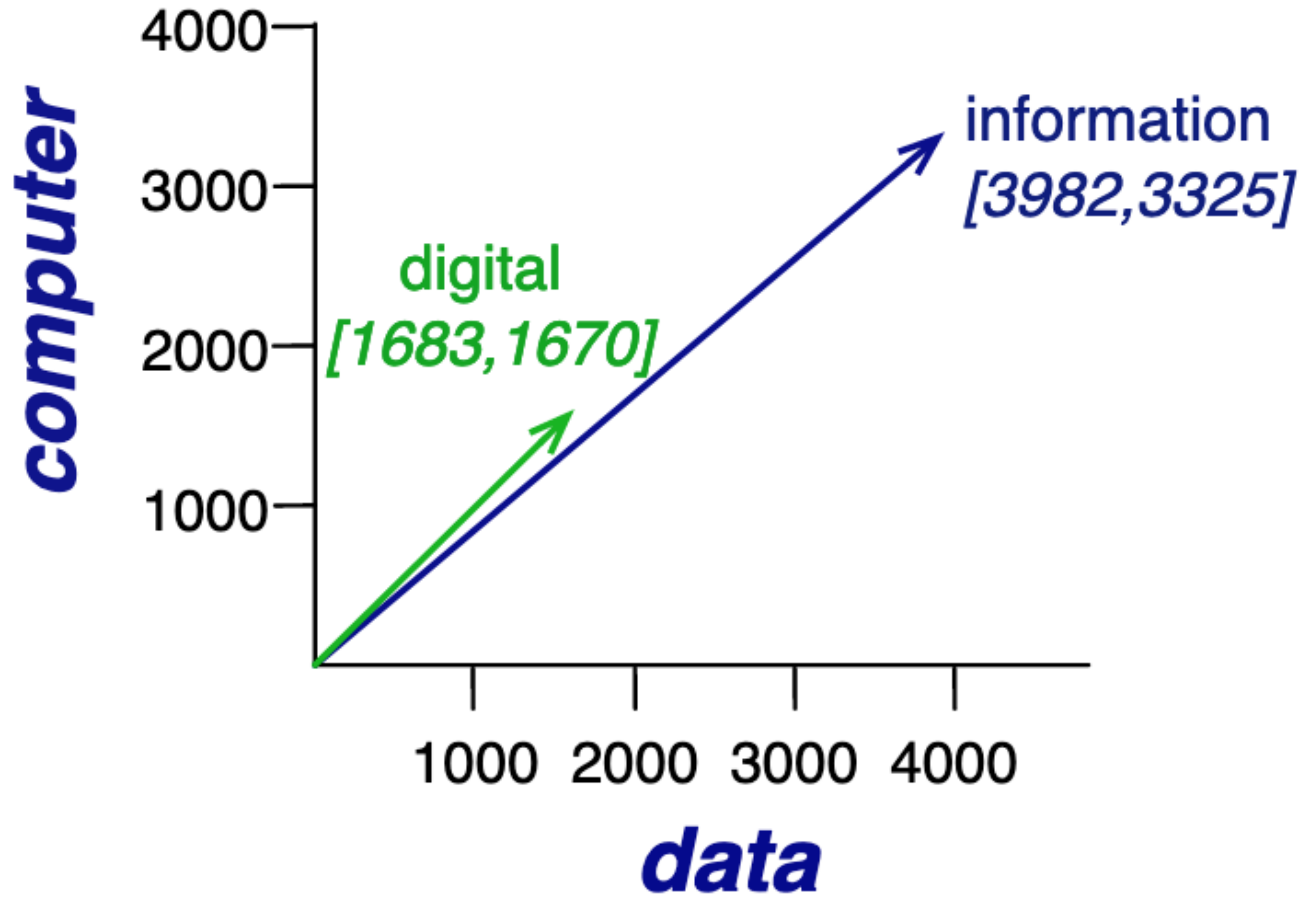
	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...



Bag of Words Vectors

- Usually re-weighted by some algorithm
  - (e.g. tf-idf, ppmi)
- Still sparse
- Very high-dimensional: |V|

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...



Gradient Descent

Supervised Learning

Supervised Learning

- Given: a dataset $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example

Supervised Learning

- Given: a dataset $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
    - $x_i \in X$: input for i-th example
    - $y_i \in Y$: output for i-th example
- Example: sentiment analysis
    - Input: bag of words representation of “This movie was great.”
    - Output: 4 [on a scale 1-5]

Supervised Learning

- Given: a dataset $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example
- Example: sentiment analysis
  - Input: bag of words representation of “This movie was great.”
  - Output: 4 [on a scale 1-5]
- Example: language modeling
  - Input: “This movie was”
  - Output: “great”

Supervised Learning

Supervised Learning

- Given: a dataset $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example

Supervised Learning

- Given: a dataset $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example
- Goal: learn a function $f : X \to Y$ which:
  - "Does well" on the given data D
  - Generalizes well to unseen data

Parameterized Functions

Parameterized Functions

- A learning algorithm searches for a function f amongst a space of **possible functions**

Parameterized Functions

- A learning algorithm searches for a function f amongst a space of **possible functions**
- Parameters define a **family** of functions
  - θ: general symbol for parameters
  - $\hat{y} = f(x; \theta)$: input x, parameters θ; model/function output $\hat{y}$

Parameterized Functions

- A learning algorithm searches for a function f amongst a space of **possible functions**
- Parameters define a **family** of functions
  - θ: general symbol for parameters
  - $\hat{y} = f(x; \theta)$: input x, parameters θ; model/function output $\hat{y}$
- Example: the **family of linear functions** $f(x) = mx + b$
  - $\theta = \{m, b\}$

Parameterized Functions

- A learning algorithm searches for a function f amongst a space of **possible functions**
- Parameters define a **family** of functions
  - θ: general symbol for parameters
  - $\hat{y} = f(x; \theta)$: input x, parameters θ; model/function output $\hat{y}$
- Example: the **family of linear functions** $f(x) = mx + b$
  - $\theta = \{m, b\}$
- Later: neural network architecture defines the family of functions

Loss Minimization

- General form of **optimization** problem

$$\mathscr{L}(\hat{Y}, Y) = \frac{1}{|Y|} \sum_i \ell(\hat{y}(x_i), y_i)$$

- $\mathscr{L}(\hat{Y}, Y)$: “loss function” / “objective function”
  - **How close** are the model outputs to the desired output?
  - $\ell(\hat{y}, y)$: local (per-instance) loss, averaged over training instances
  - Choice of loss function **depends on task**
- View the loss as a **function of the model’s parameters**

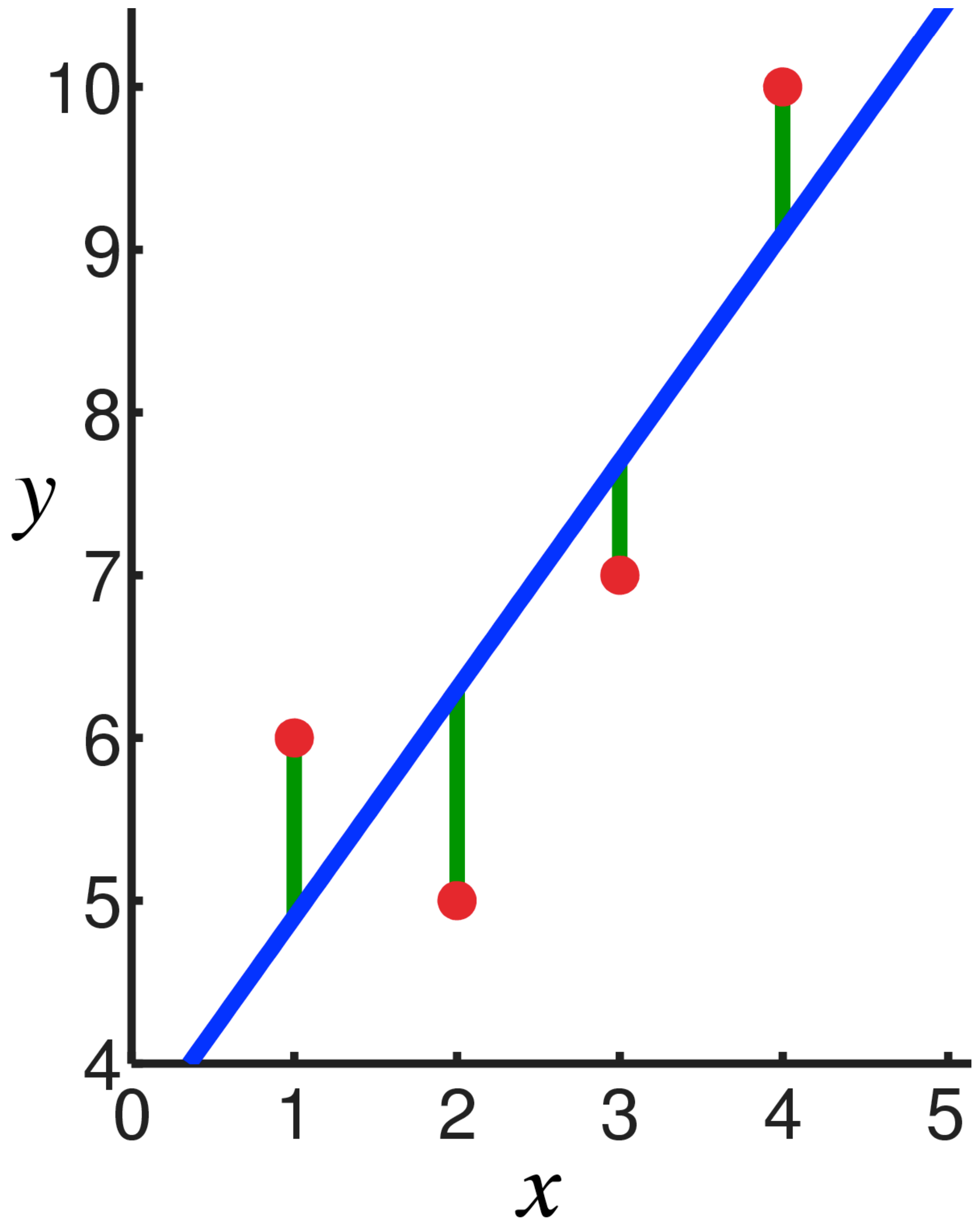
$$\mathscr{L}(\theta) := \mathscr{L}(\hat{Y}, Y) = \mathscr{L}(f(X; \theta), Y)$$

Loss Minimization

- The optimization problem:

$$\theta^* = \arg\min_{\theta} \mathcal{L}(\theta)$$

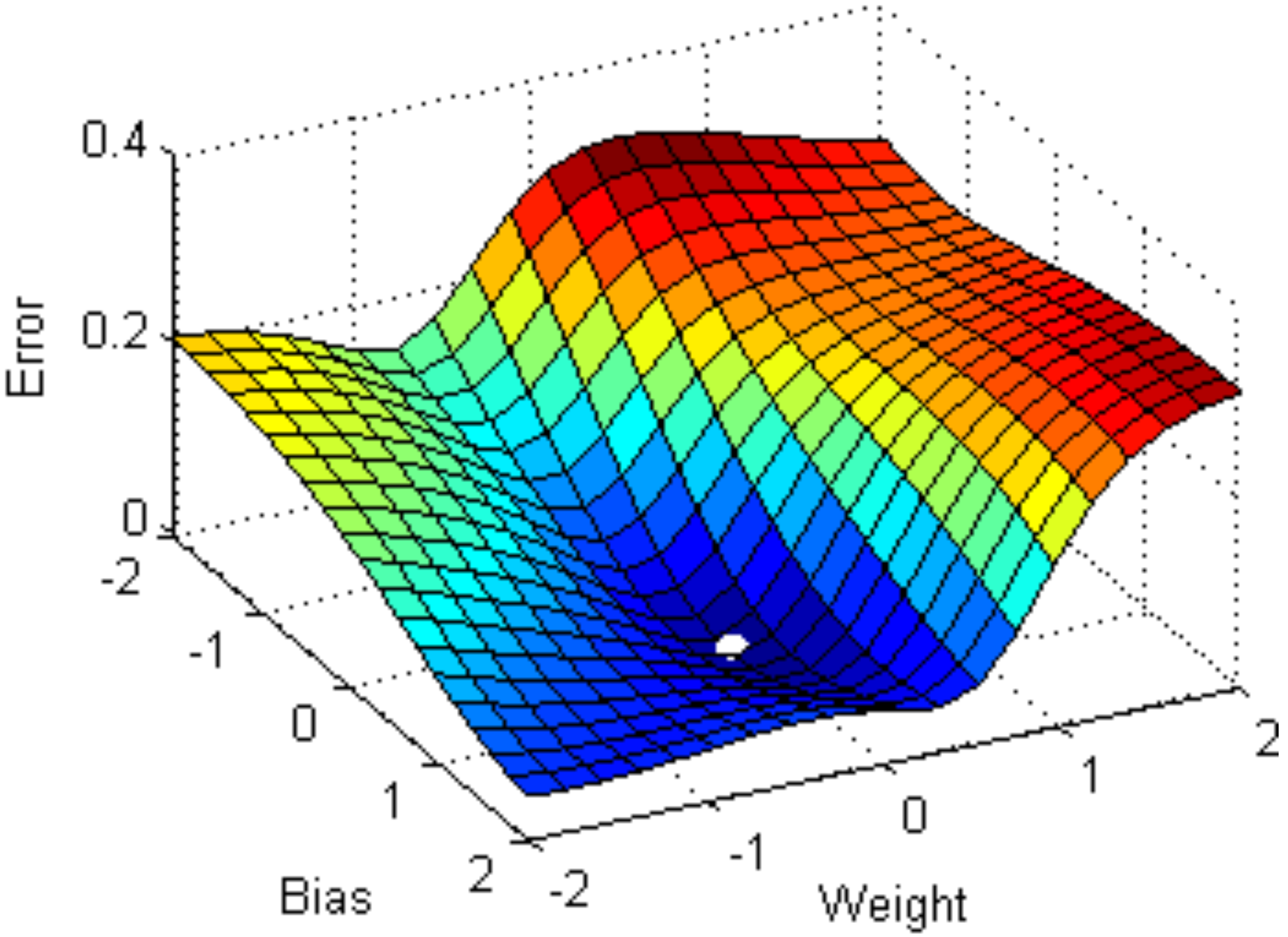
- Example: (least-squares) linear regression
  - $\ell(\hat{y}, y) = (\hat{y} - y)^2$



$$m^*, b^* = \arg\min_{m,b} \sum_i ((mx_i + b) - y_i)^2$$

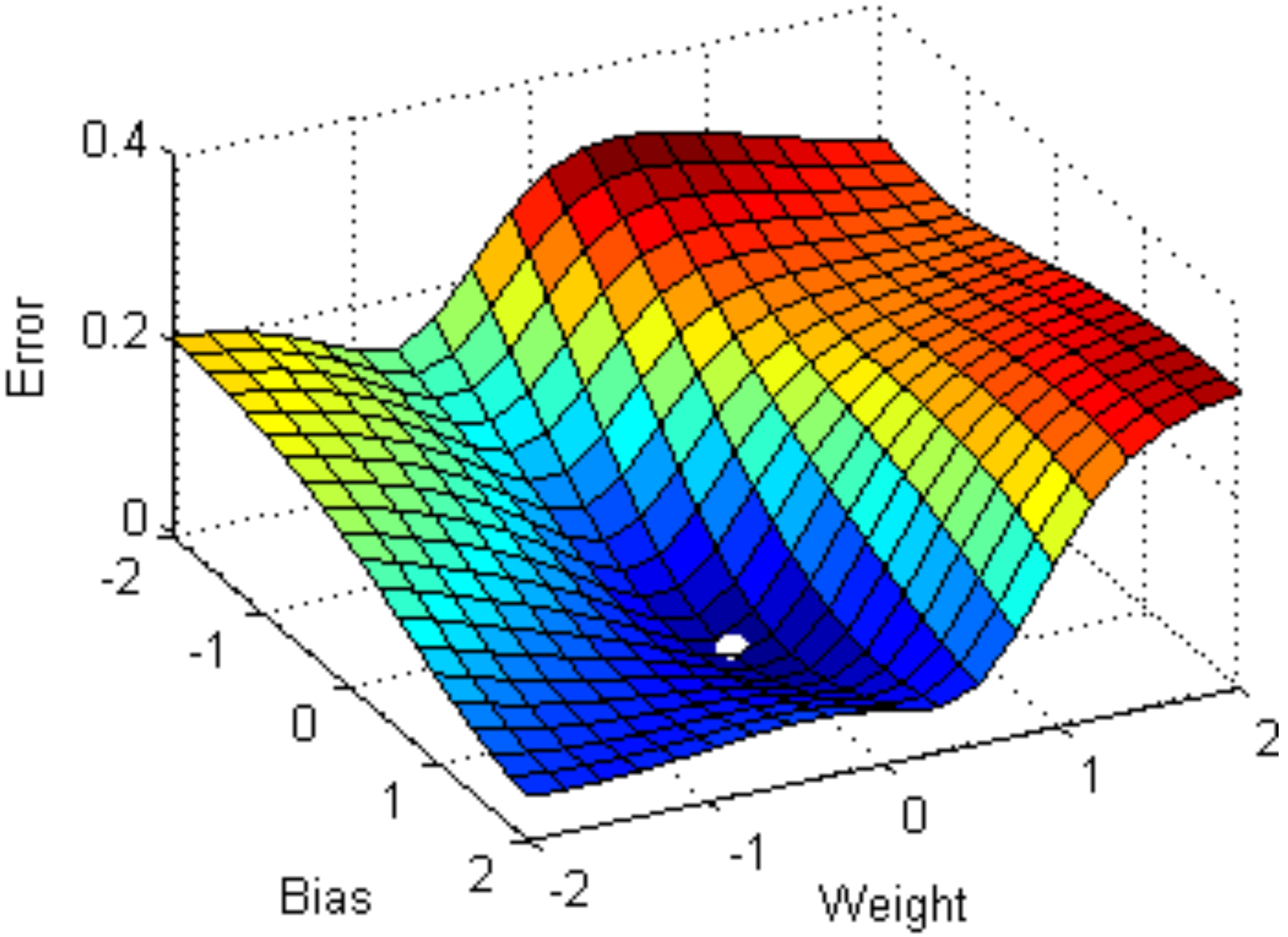
Learning: (Stochastic) Gradient Descent

Gradient Descent: Basic Idea



source

Gradient Descent: Basic Idea



source

Gradient Descent: Basic Idea

- We use the **gradient of the loss with respect to the parameters**
  - Tells **which direction** in parameter space to “walk” to **make the loss smaller***
  - i.e. to improve model outputs
- **Guaranteed** to find **optimal solution** for a **linear** model
  - Can **get stuck** in local minima for **non-linear** functions, like **NNs**
  - More precisely: if loss is a ***convex*** function of the parameters, gradient descent is guaranteed to find an optimal solution.
  - For non-linear functions, the loss will generally **not** be convex

Derivatives

- The **derivative** of a function measures how much the **output changes** with respect to a **change in the input**
- Intuition: the **slope** of a function at any given point
- Calculus 1 covers rules for finding derivatives. Review if necessary

Derivatives

- The **derivative** of a function measures how much the **output changes** with respect to a **change in the input**
- Intuition: the **slope** of a function at any given point
- Calculus 1 covers rules for finding derivatives. Review if necessary

$$f(x) = x^2 + 35x + 12$$

$$\frac{df}{dx} = 2x + 35$$

Derivatives

- The **derivative** of a function measures how much the **output changes** with respect to a **change in the input**
- Intuition: the **slope** of a function at any given point
- Calculus 1 covers rules for finding derivatives. Review if necessary

$$f(x) = x^2 + 35x + 12$$
$$\frac{df}{dx} = 2x + 35$$

$$f(x) = e^x$$
$$\frac{df}{dx} = e^x$$

Partial Derivatives

- A **partial derivative** is needed for a function with **multiple input variables**
- Each measures slope respect **one variable**, with the others held constant

Partial Derivatives

- A **partial derivative** is needed for a function with **multiple input variables**
- Each measures slope respect **one variable**, with the others held constant

$$f(x, y) = 10x^3y^2 + 5xy^3 + 4x + y$$

Partial Derivatives

- A **partial derivative** is needed for a function with **multiple input variables**
- Each measures slope respect **one variable**, with the others held constant

$$f(x, y) = 10x^3y^2 + 5xy^3 + 4x + y$$

$$\frac{\partial f}{\partial x} = 30x^2y^2 + 5y^3 + 4$$

Partial Derivatives

- A **partial derivative** is needed for a function with **multiple input variables**
- Each measures slope respect **one variable**, with the others held constant

$$f(x, y) = 10x^3y^2 + 5xy^3 + 4x + y$$

$$\frac{\partial f}{\partial x} = 30x^2y^2 + 5y^3 + 4$$

$$\frac{\partial f}{\partial y} = 20x^3y + 15xy^2 + 1$$

Gradient

Gradient

- The gradient of a function $f(x_1, x_2, \ldots x_n)$ is a **vector**, consisting of **all partial derivatives**

Gradient

- The gradient of a function $f(x_1, x_2, ... x_n)$ is a **vector**, consisting of **all partial derivatives**

$$\nabla f = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$$

Gradient

- The gradient of a function $f(x_1, x_2, ... x_n)$ is a **vector**, consisting of **all partial derivatives**

$$\nabla f = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$$

$$f(x, y) = 4x^2 + y^2$$

$$\nabla f = \langle 8x, 2y \rangle$$

Gradient

- The gradient of a function $f(x_1, x_2, ... x_n)$ is a **vector**, consisting of **all partial derivatives**

$$\nabla f = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$$

$$f(x, y) = 4x^2 + y^2$$

$$\nabla f = \langle 8x, 2y \rangle$$

- The gradient is perpendicular to the *level curve* at a point (next slide)

Gradient

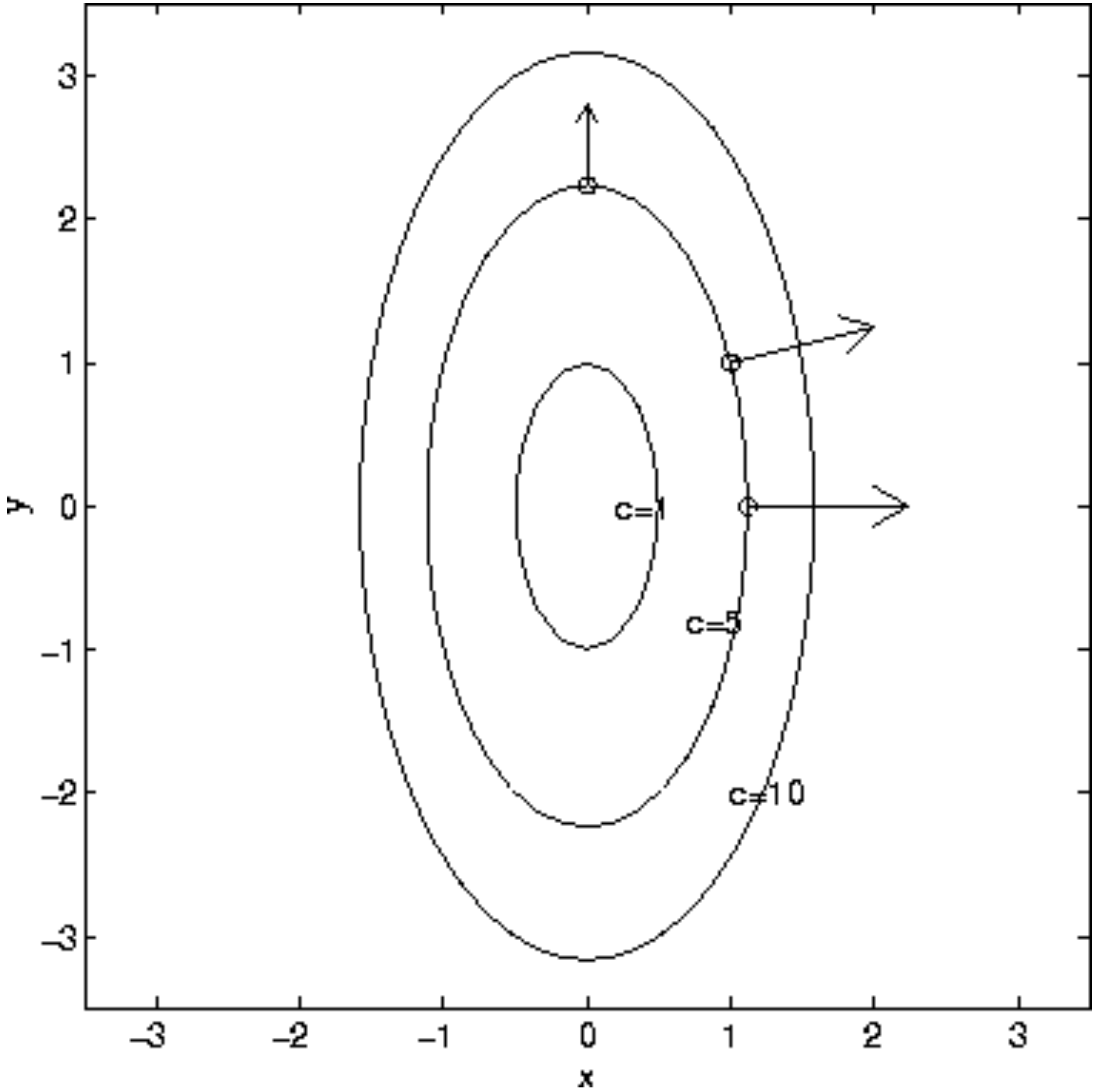
- The gradient of a function $f(x_1, x_2, \ldots x_n)$ is a **vector**, consisting of **all partial derivatives**

$$\nabla f = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$$

$$f(x, y) = 4x^2 + y^2$$
$$\nabla f = \langle 8x, 2y \rangle$$

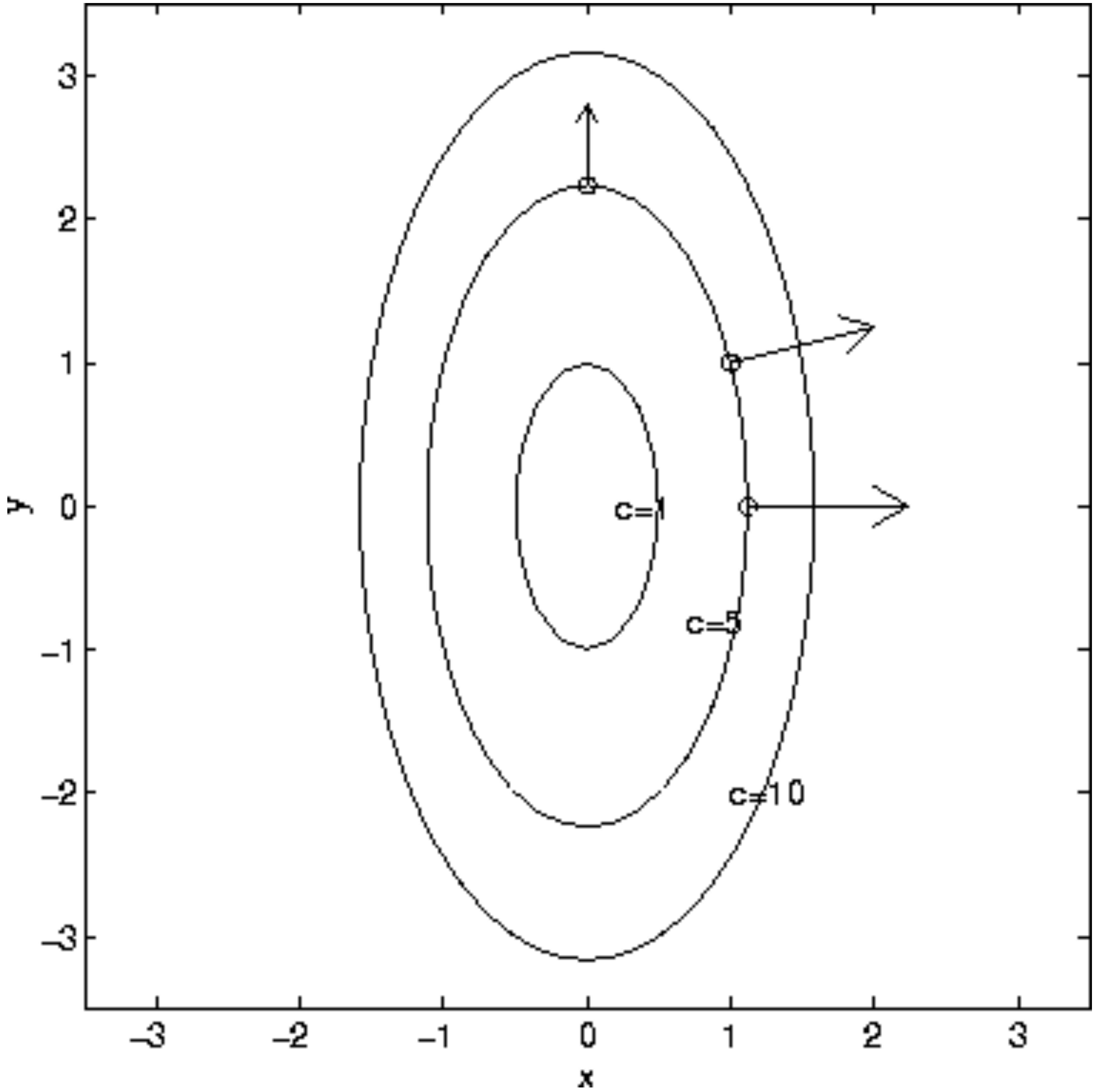
- The gradient is perpendicular to the *level curve* at a point (next slide)
- The gradient points in the direction of **greatest increase** of f

Gradient and Level Curves



Level curves: $f(x, y) = c$

Gradient and Level Curves

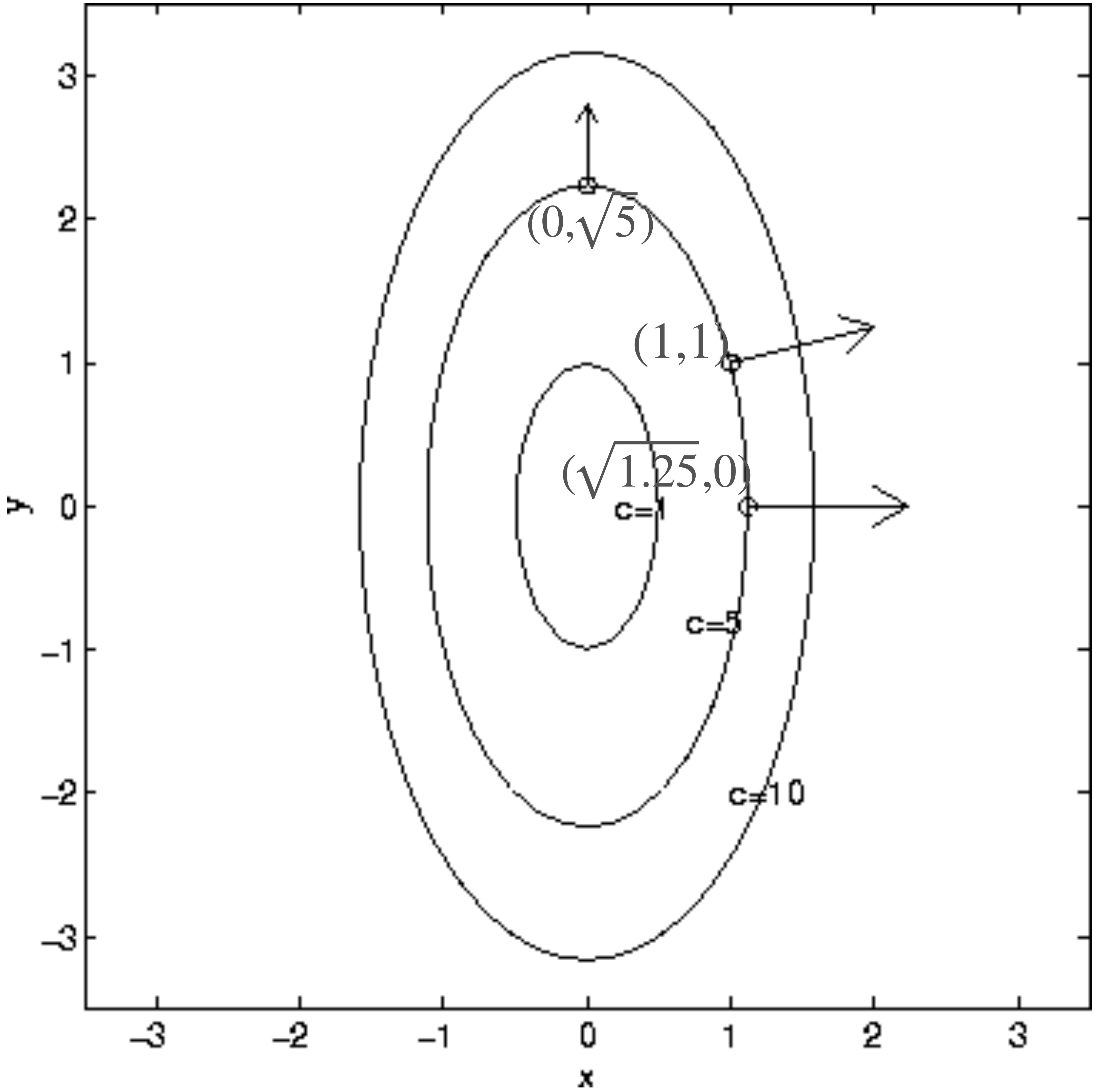


$$f(x,y) = 4x^2 + y^2$$

$$\nabla f = \langle 8x, 2y \rangle$$

Level curves: $f(x,y) = c$

Gradient and Level Curves

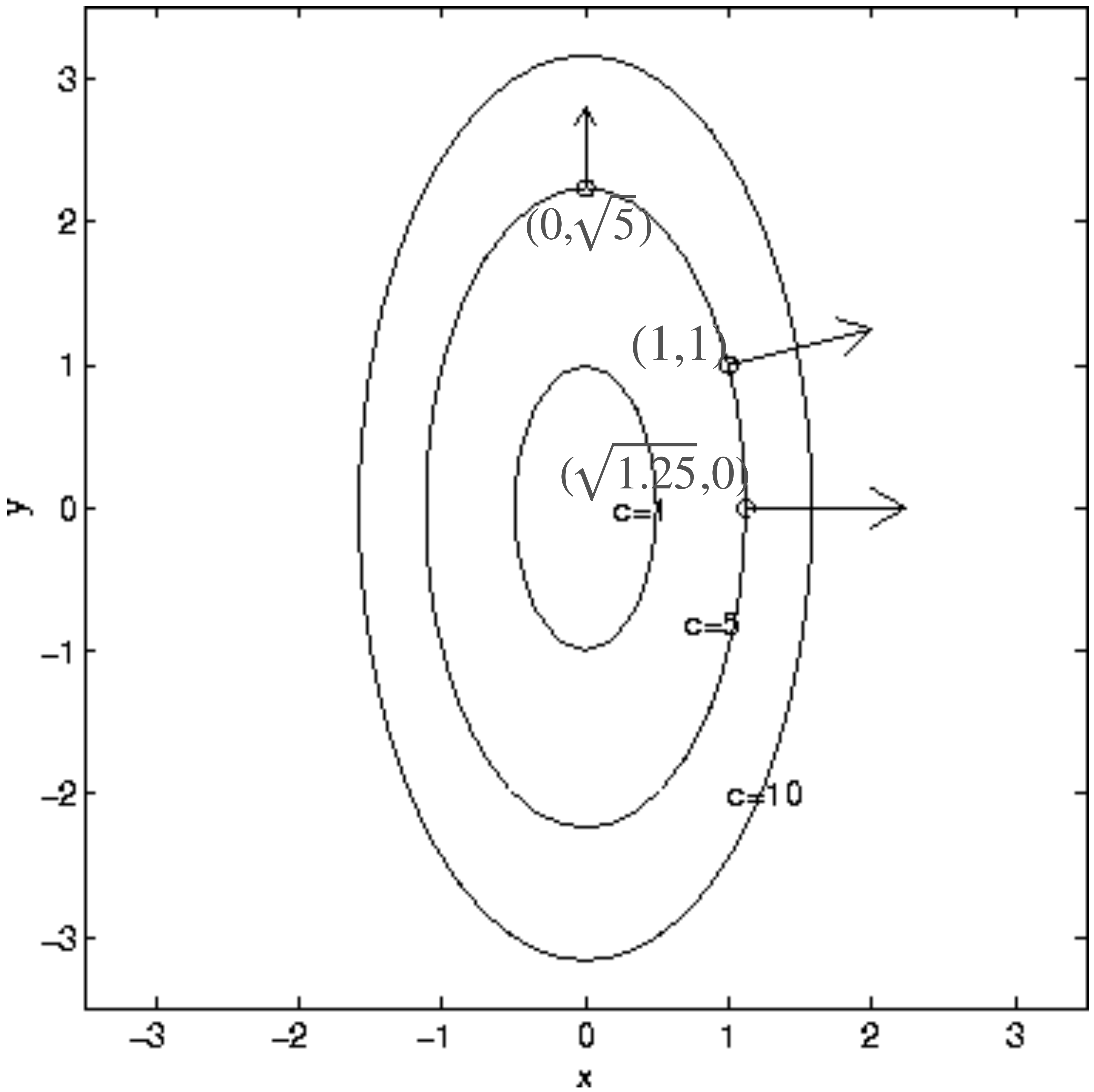


$$f(x, y) = 4x^2 + y^2$$

$$\nabla f = \langle 8x, 2y \rangle$$

Level curves: $f(x, y) = c$

Gradient and Level Curves



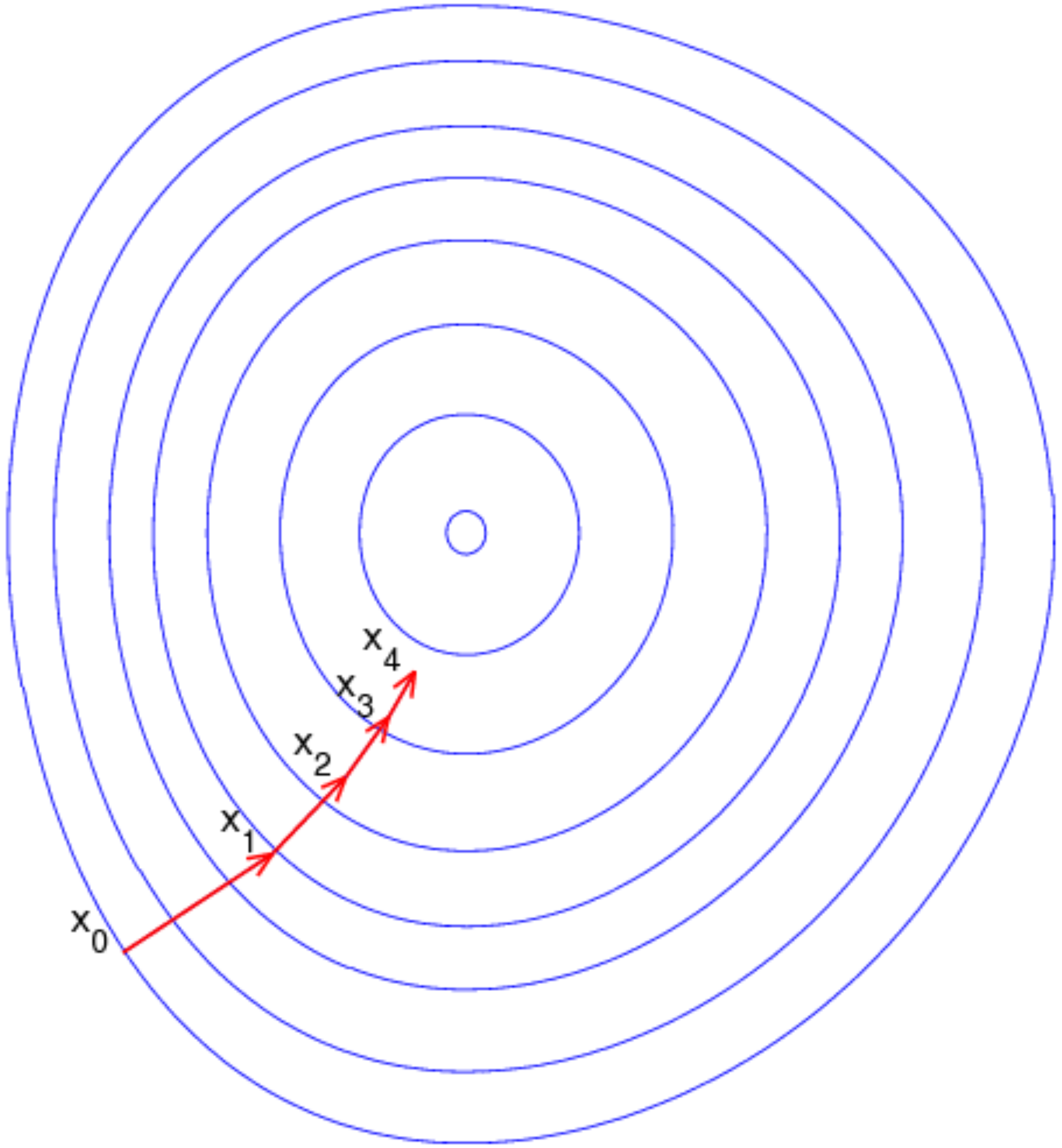
$$f(x, y) = 4x^2 + y^2$$

$$\nabla f = \langle 8x, 2y \rangle$$

Level curves: $f(x, y) = c$

Q: what are the actual gradients at those points?

Gradient Descent and Level Curves



source

Gradient Descent Algorithm

- Initialize θ_0
- Repeat until convergence:

$$\theta_{n+1} = \theta_n - \alpha \nabla \mathscr{L}(\hat{Y}(\theta_n), Y)$$

- **High learning rate**: big steps, may bounce and **“overshoot”** the target
- **Low learning rate**: small steps, smoother minimization of loss, but can be slow or **get stuck**

Gradient Descent Algorithm

- Initialize θ_0
- Repeat until convergence:

$$\theta_{n+1} = \theta_n - \alpha \nabla \mathscr{L}(\hat{Y}(\theta_n), Y)$$

Learning rate

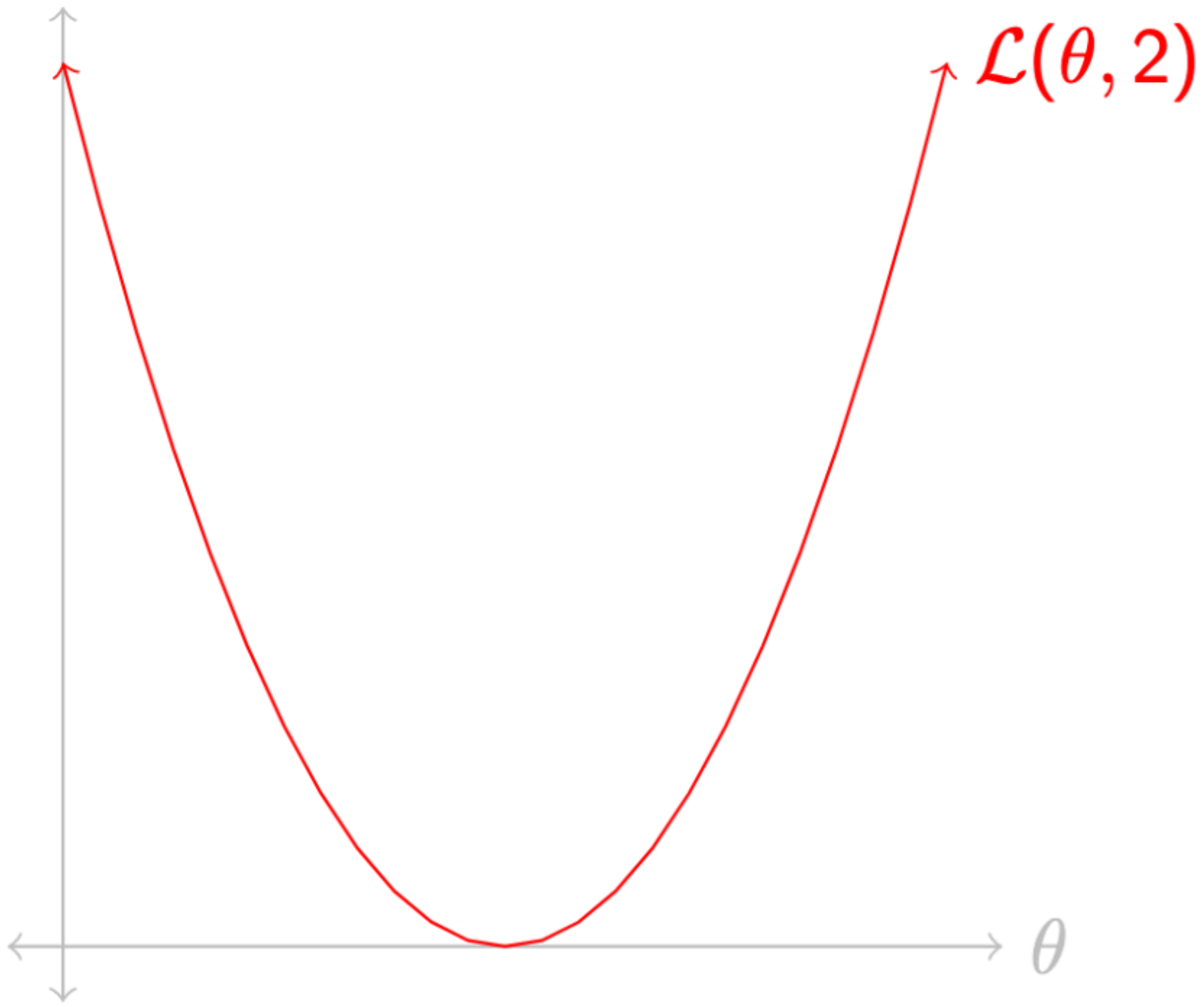
- **High learning rate**: big steps, may bounce and **"overshoot"** the target
- **Low learning rate**: small steps, smoother minimization of loss, but can be slow or **get stuck**

Gradient Descent: Minimal Example

- Task: predict a target/true value $y = 2$
- "Model": $\hat{y}(\theta) = \theta$
  - A single parameter: the actual guess
- Loss: Euclidean distance

$$\mathcal{L}(\hat{y}(\theta), y) = (\hat{y} - y)^2 = (\theta - y)^2$$

Gradient Descent: Minimal Example



$$\frac{\partial}{\partial \theta}\mathcal{L}(\theta, y) = 2(\theta - y)$$

$$\theta_{t+1} = \theta_t - \alpha \cdot \frac{\partial}{\partial \theta}\mathcal{L}(\theta, y)$$

Stochastic Gradient Descent

Stochastic Gradient Descent

- The above is called **batch gradient descent**
  - Updates **based on entire dataset at once**
  - Expensive, and slow; **does not scale well**

Stochastic Gradient Descent

- The above is called **batch gradient descent**
  - Updates **based on entire dataset at once**
  - Expensive, and slow; **does not scale well**
- Stochastic gradient descent: **single example at a time**; very noisy estimate of true gradient

Stochastic Gradient Descent

- The above is called **batch gradient descent**
  - Updates **based on entire dataset at once**
  - Expensive, and slow; **does not scale well**
- Stochastic gradient descent: **single example at a time**; very noisy estimate of true gradient
- **Mini-batch** gradient descent:
  - Break the data into “mini-batches”: **small chunks** of the data
  - Compute gradients and update parameters for each batch
  - Mini-batch of size 1 = single example = stochastic gradient descent
  - A noisy estimate of the true gradient, but works well in practice; more parameter updates

Stochastic Gradient Descent

- The above is called **batch gradient descent**
  - Updates **based on entire dataset at once**
  - Expensive, and slow; **does not scale well**
- Stochastic gradient descent: **single example at a time**; very noisy estimate of true gradient
- **Mini-batch** gradient descent:
  - Break the data into “mini-batches”: **small chunks** of the data
  - Compute gradients and update parameters for each batch
  - Mini-batch of size 1 = single example = stochastic gradient descent
  - A noisy estimate of the true gradient, but works well in practice; more parameter updates
- **Epoch**: one pass through the whole training data

Stochastic Gradient Descent

```
initialize parameters / build model

for each epoch:

 data = shuffle(data)
 batches = make_batches(data)

 for each batch in batches:

  outputs = model(batch)
  loss = loss_fn(outputs, true_outputs)
  compute gradients
  update parameters
```

Next Time

- Skip-Gram with Negative Sampling
  - How optimization framework applies to this problem
- Introduction of two tasks that we will use throughout the class
  - Language modeling
  - Text classification (sentiment analysis)