Gradient Descent

LING 282/482: Deep Learning for Computational Linguistics

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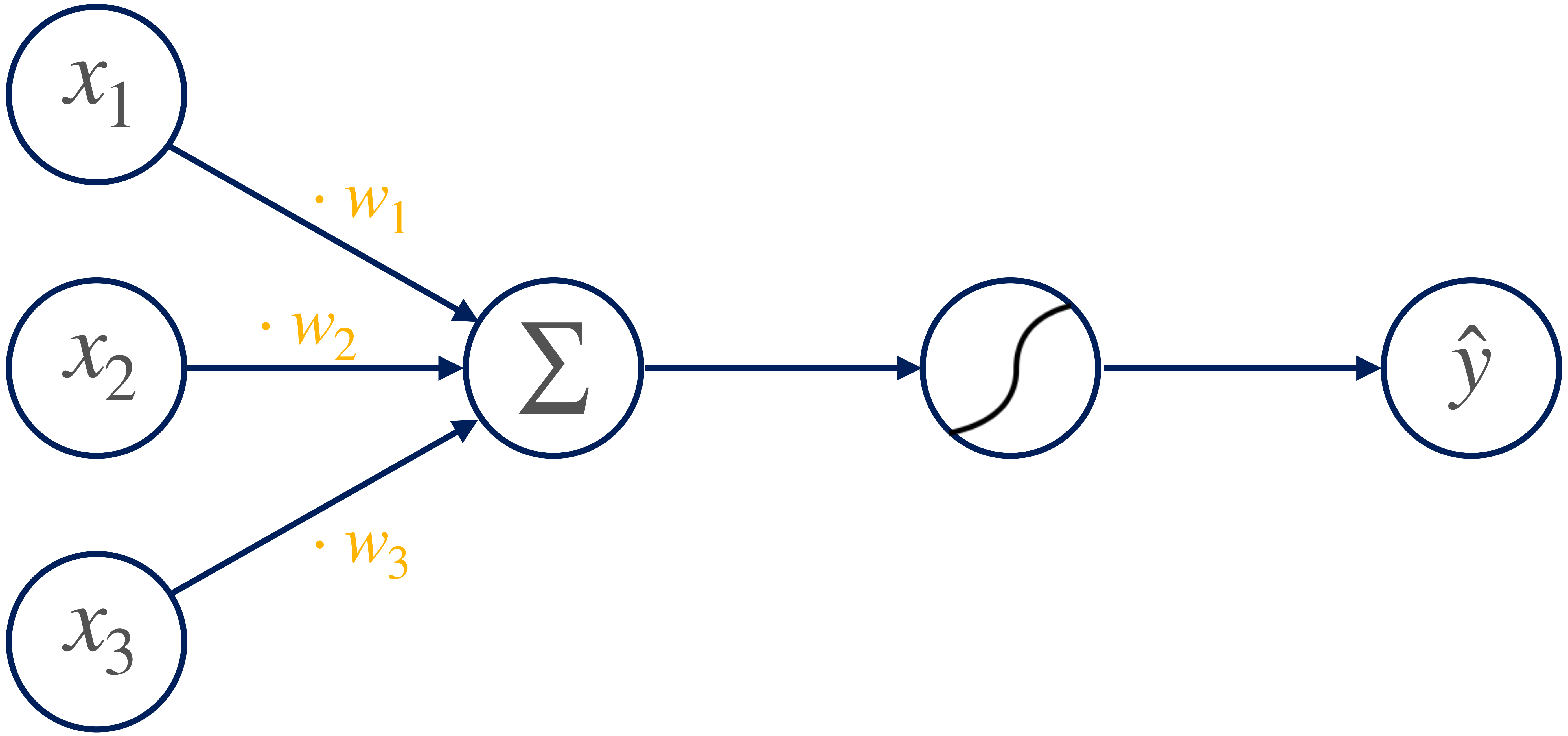
Fall 2025

Last time

- We saw **binary classification** using the Perceptron
  - Learns to **linearly separate** input examples
- Where do the **weights** come from?



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- Goal: learn the function that **best matches the dataset**

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  - Solution: learn the weights of a **parameterized function**

Parameterized Functions

Parameterized Functions

- A learning searches for a function f in a space of **possible functions**
- Parameters define a **family** of functions that share a common form
  - θ: general symbol for parameters/weights (usually represents **several**)
  - $\hat{y} = f(x; \theta)$: the function $f(x)$, **given parameters** θ
- Example: the **family of linear functions** $f(x) = mx + b$
  - $\theta = \{m, b\}$
  - This defines **all possible lines** (with different slopes and intercepts)
- Later: Neural Networks define their own family of functions

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- "Loss Function": a measure of how much the **predicted output** $\hat{y}$ **diverges** from the **true output** y
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- We always want to **minimize the loss/error**
  - This is a type of **optimization problem**, which is a huge subfield of math

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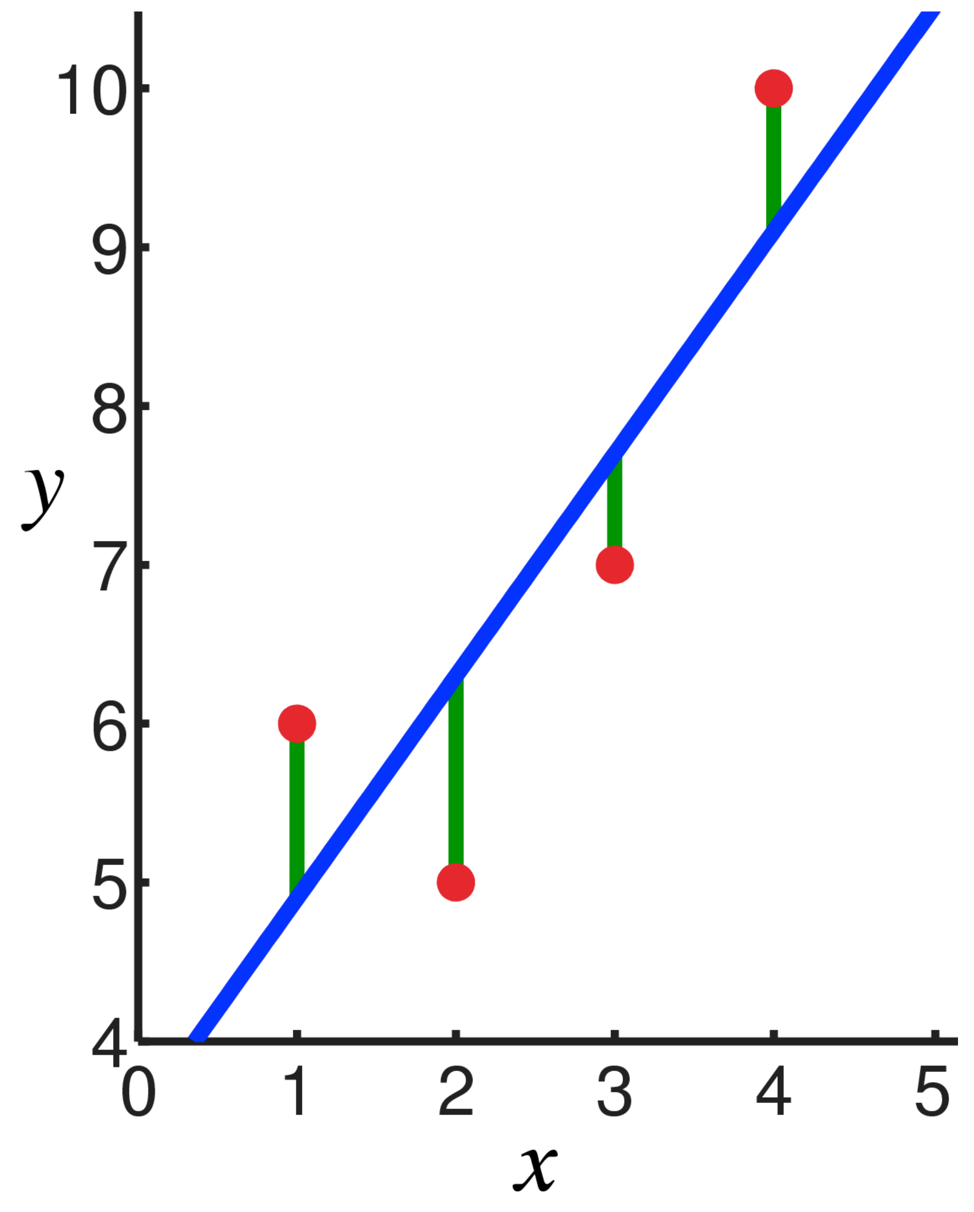
- Example: **Linear Regression** ("Least-Squares" method)

$$m^*, b^* = \arg\min_{m,b} \sum_i ((mx_i + b) - y_i)^2$$



Example: Secret Number Game

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    - $\hat{y} = f(x) = x + \theta$

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- Lastly **learn the optimal value of θ** (i.e. the value that **minimizes the loss**)

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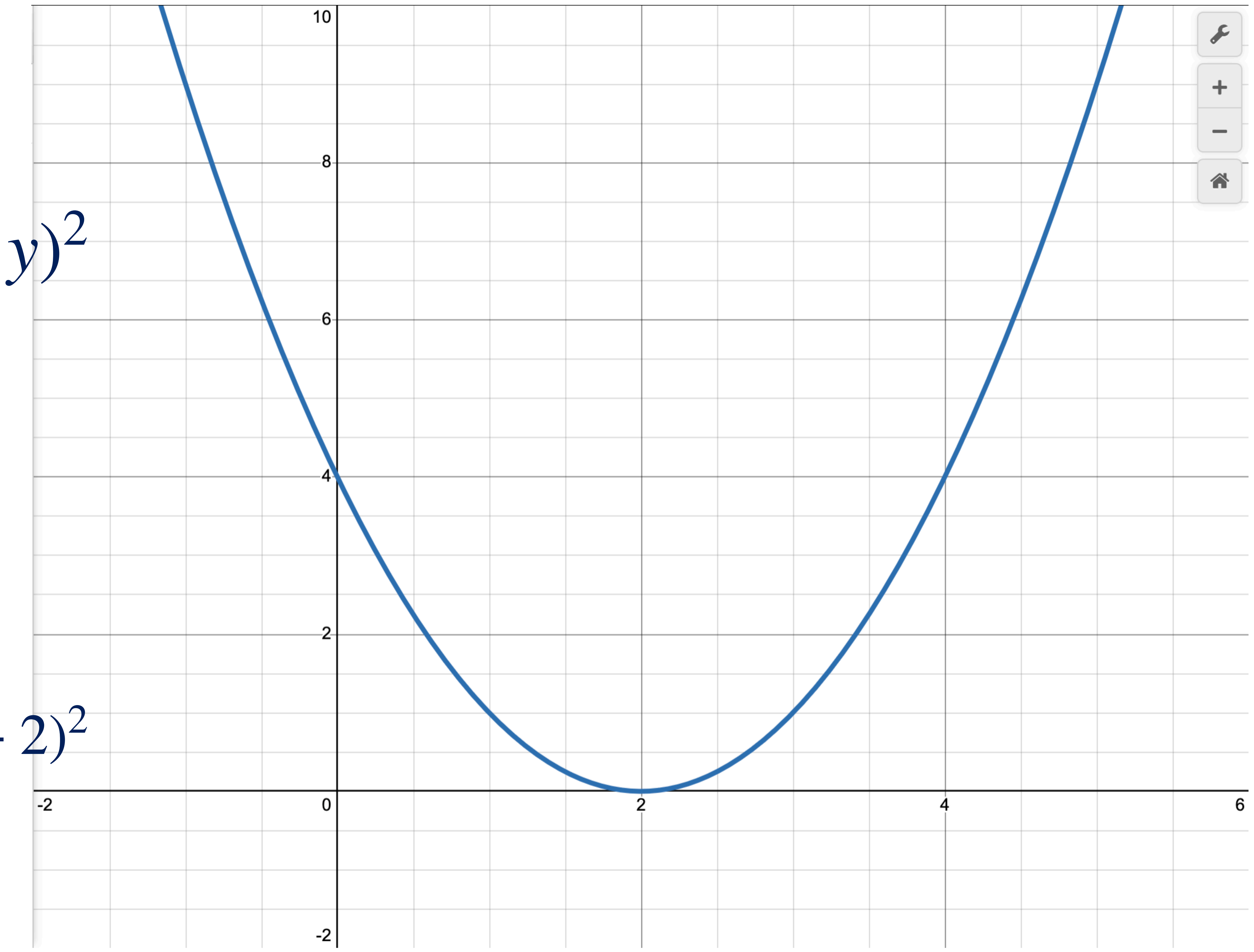

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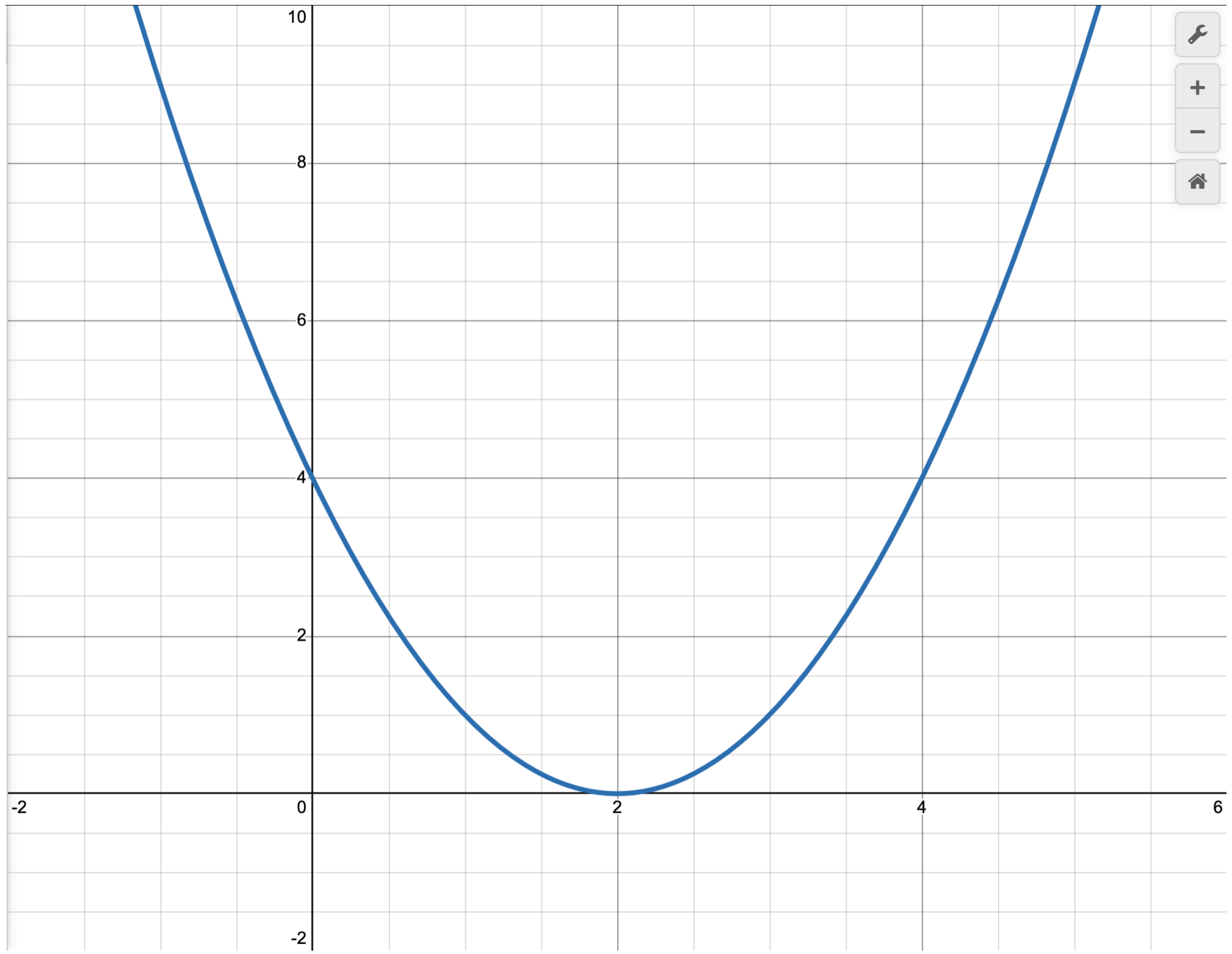

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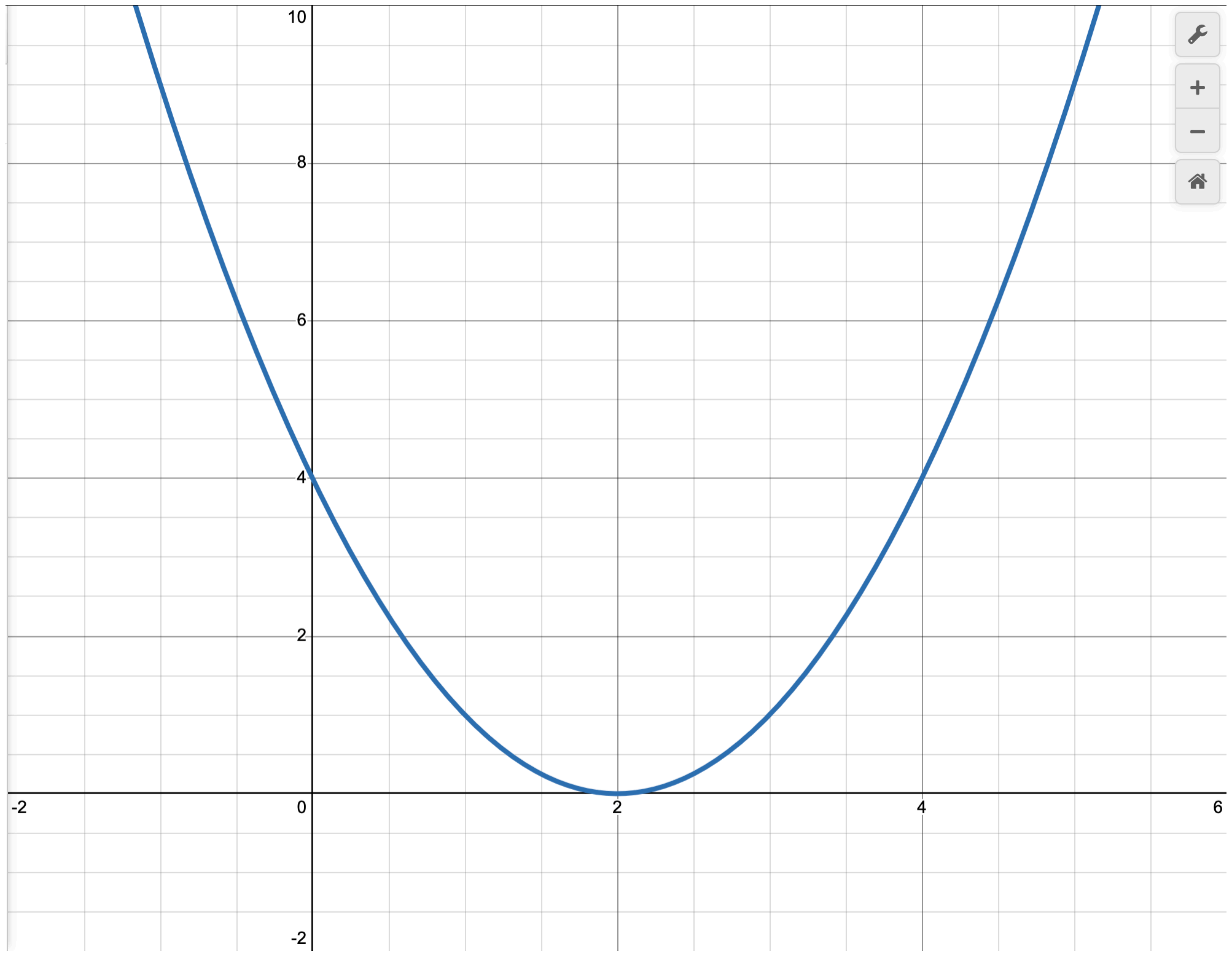

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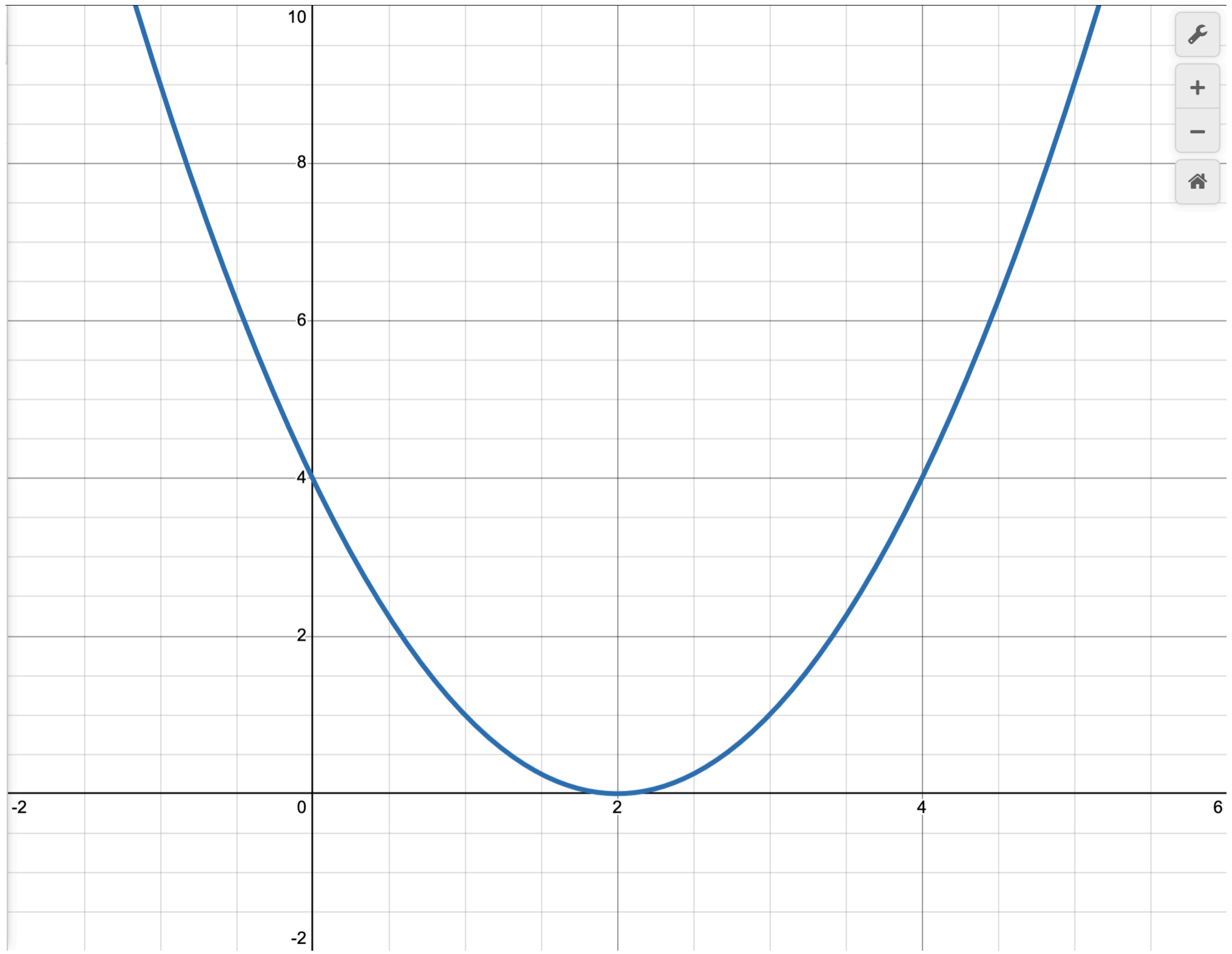

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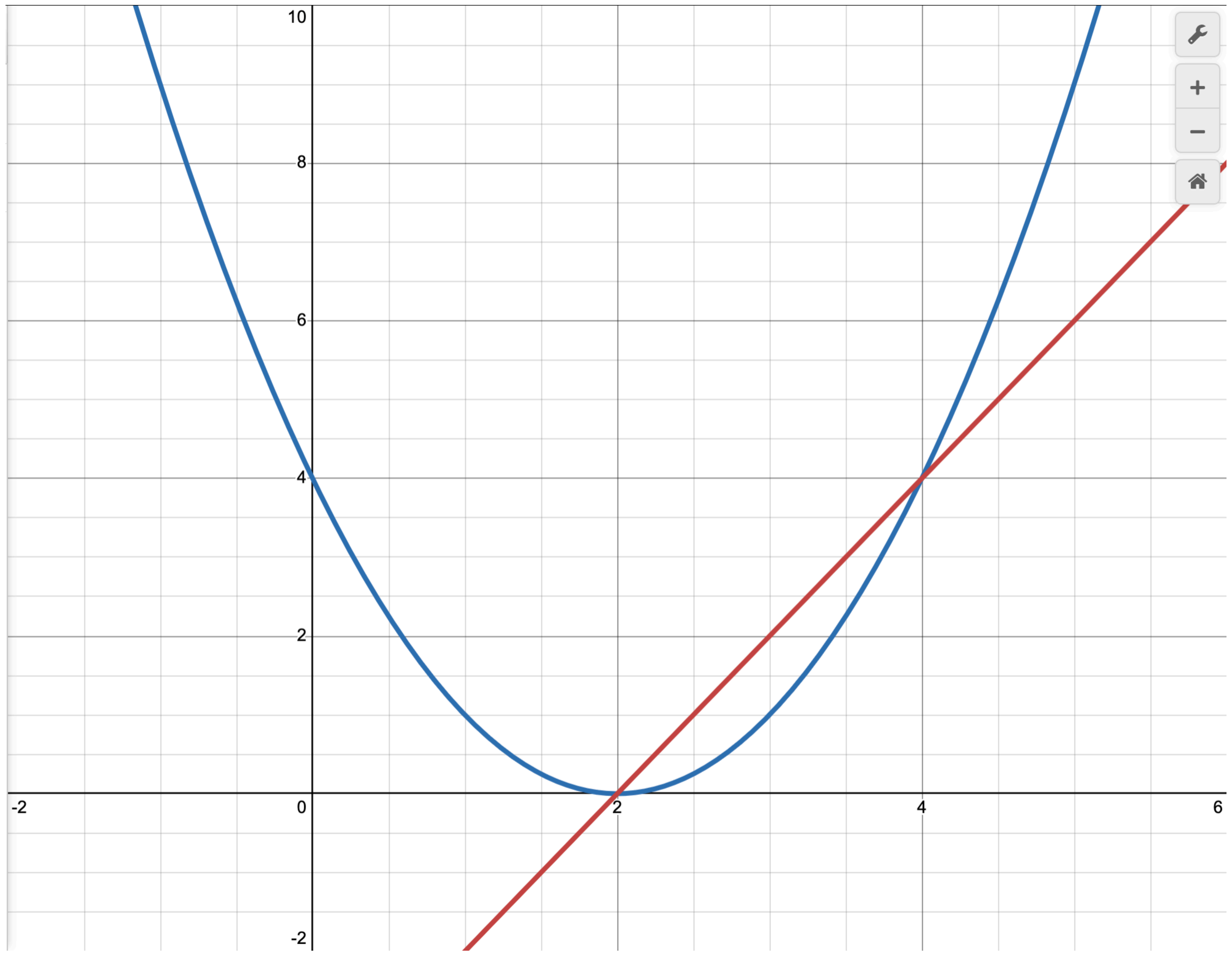

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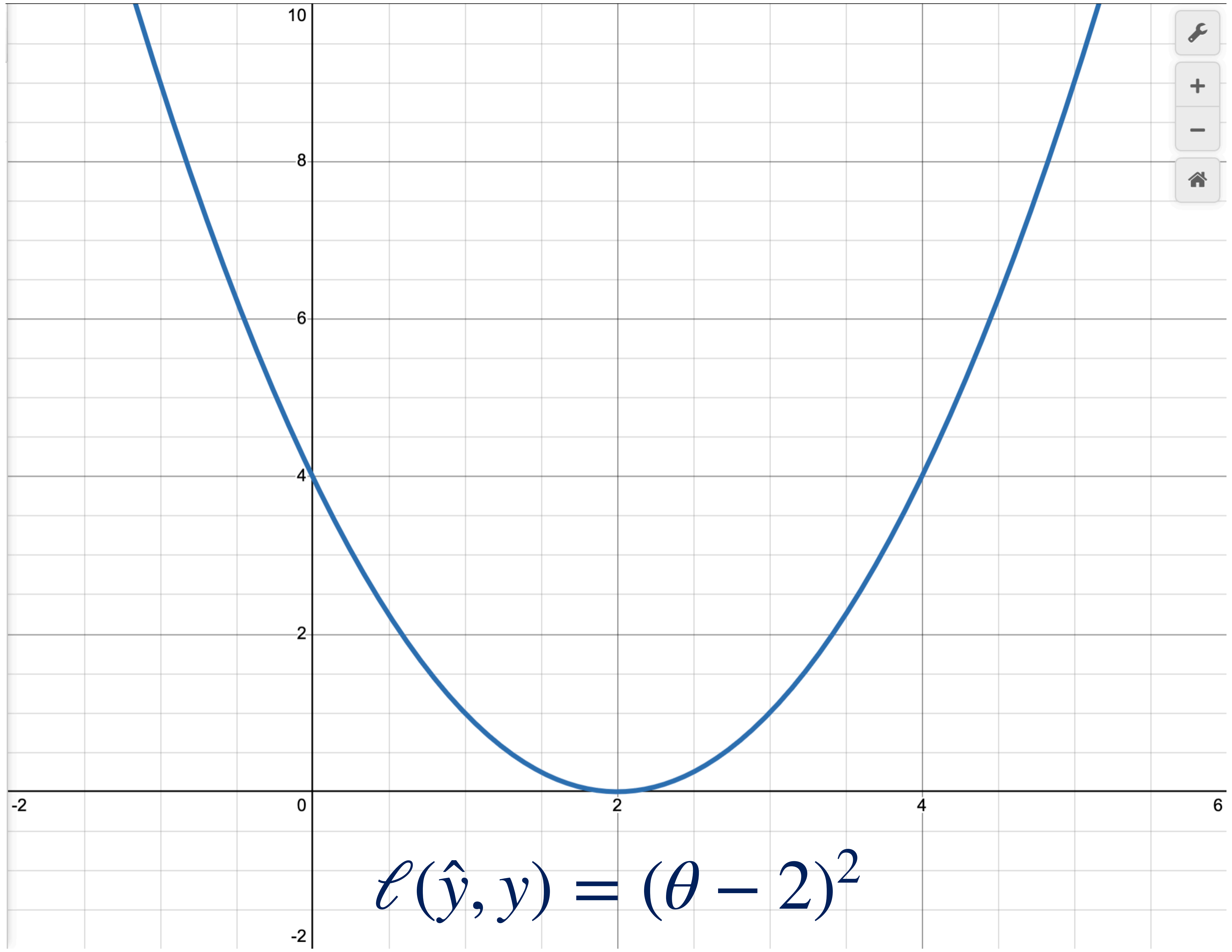

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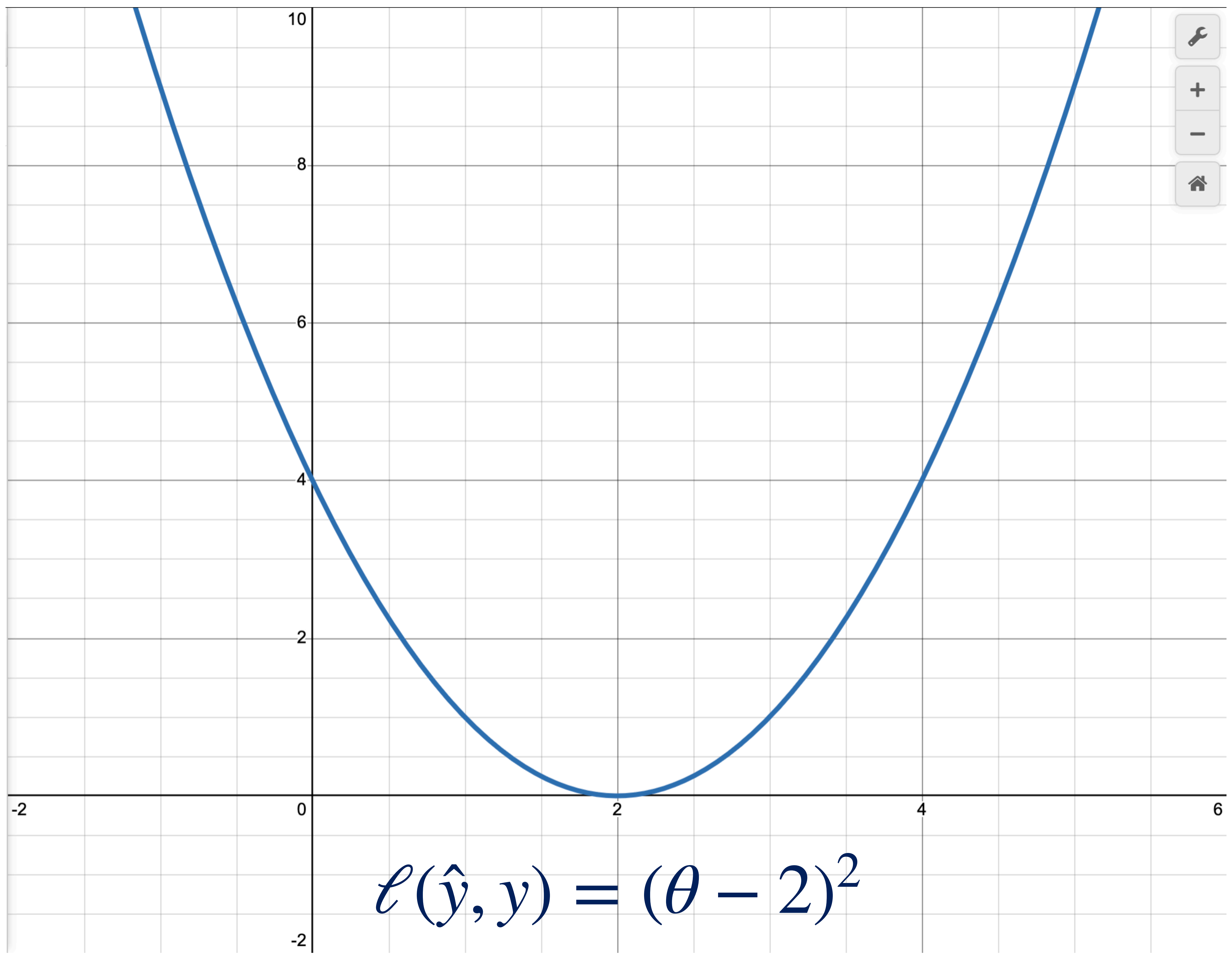

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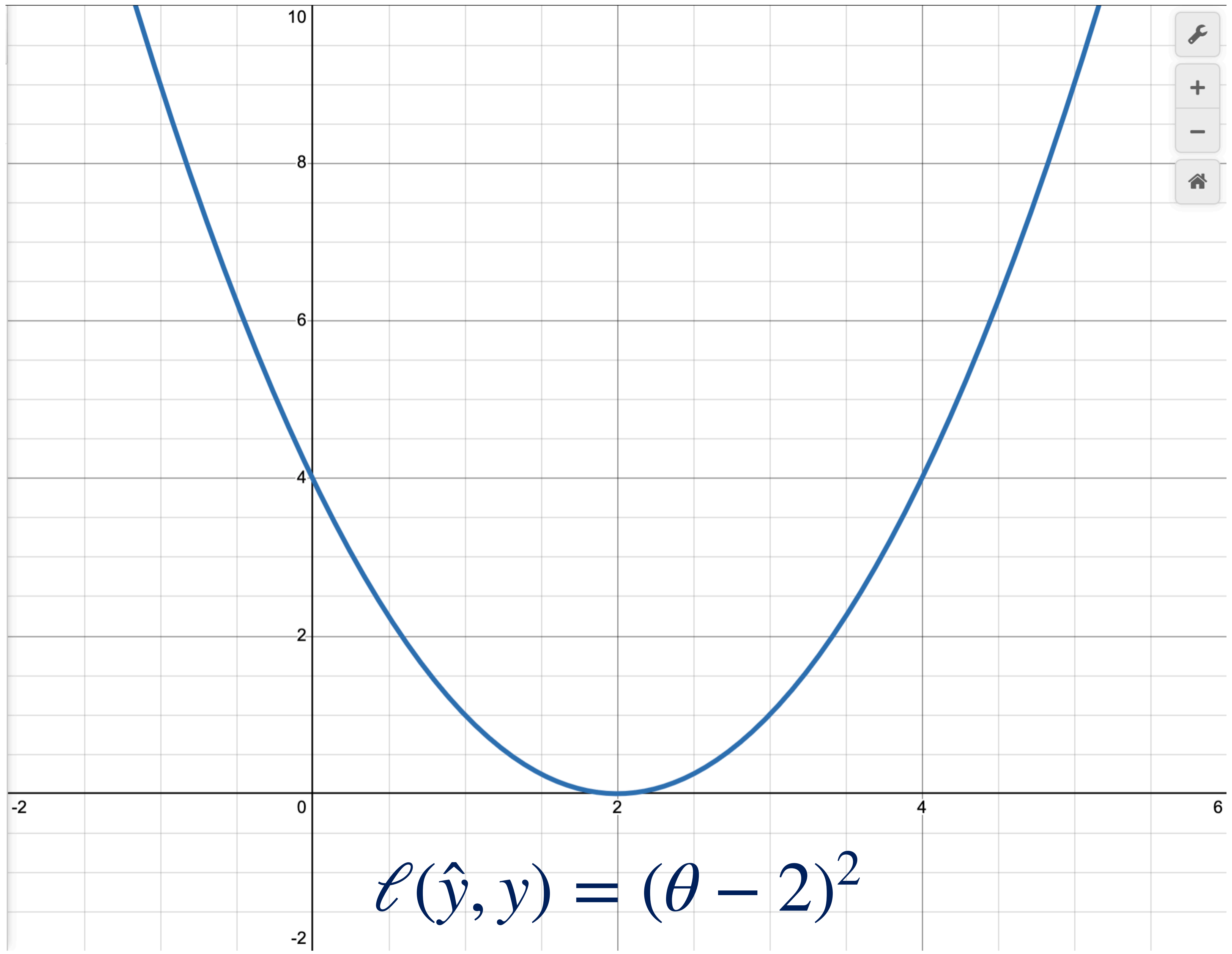

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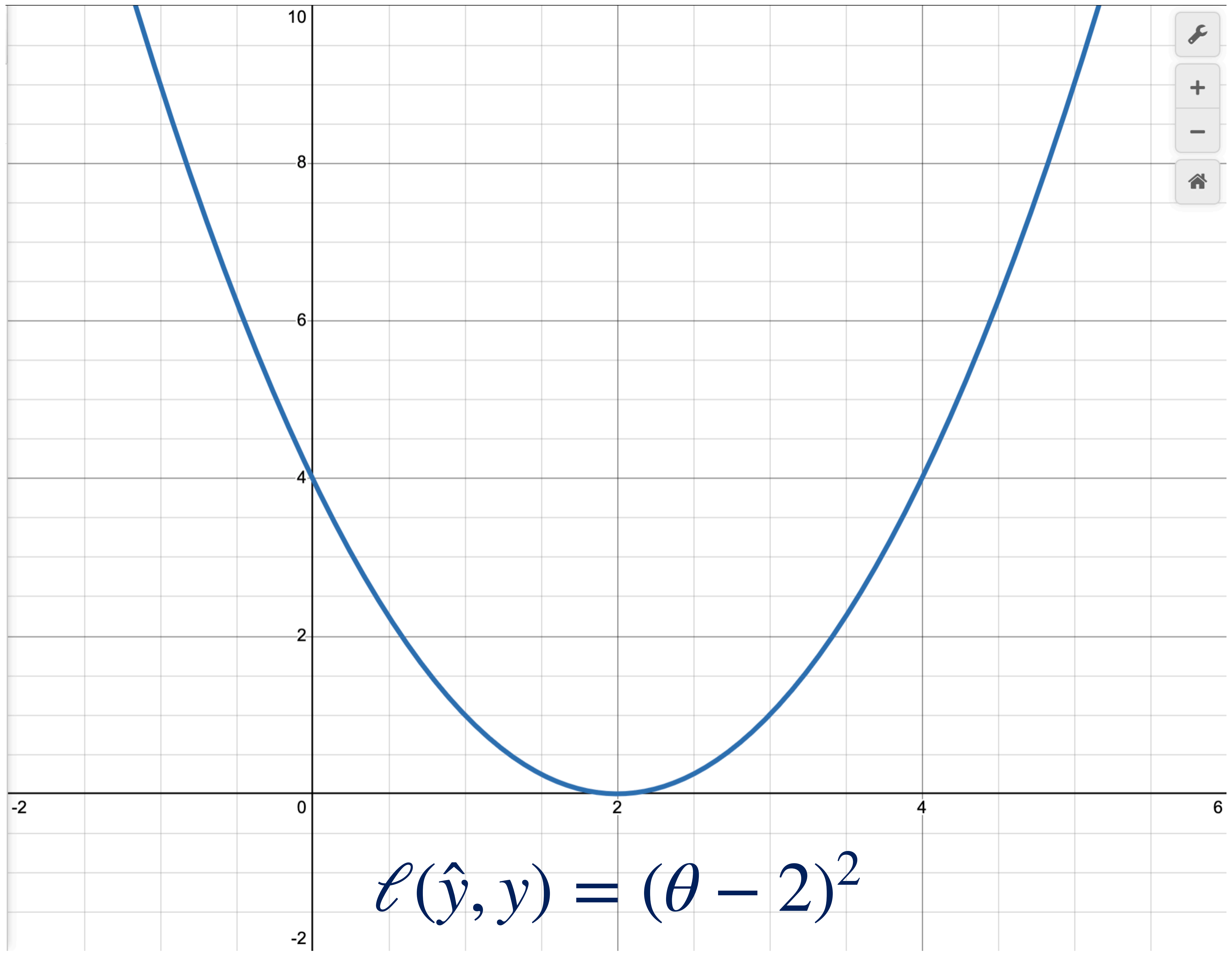
- For this example **ONLY**, solving for one datapoint solves the whole problem



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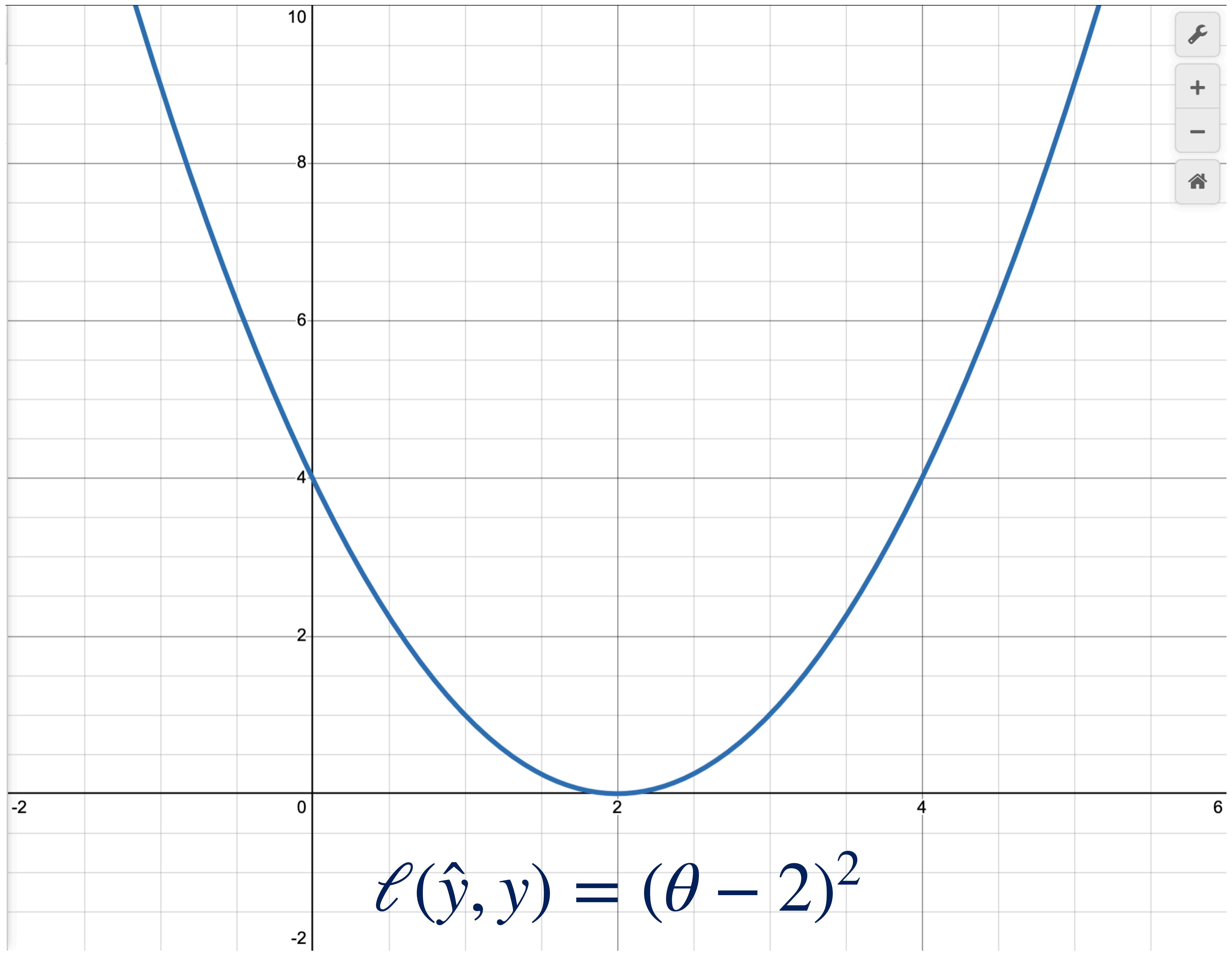

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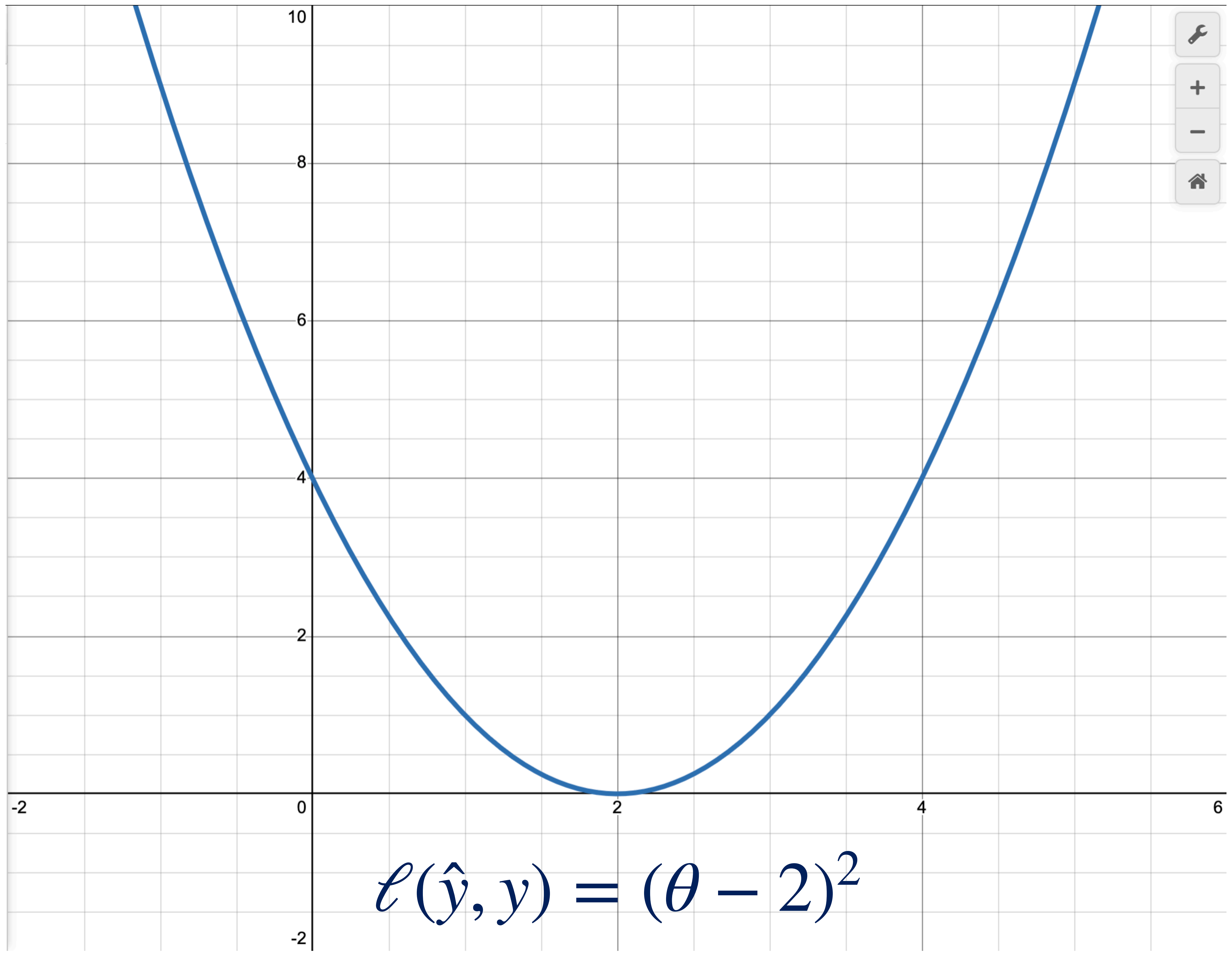

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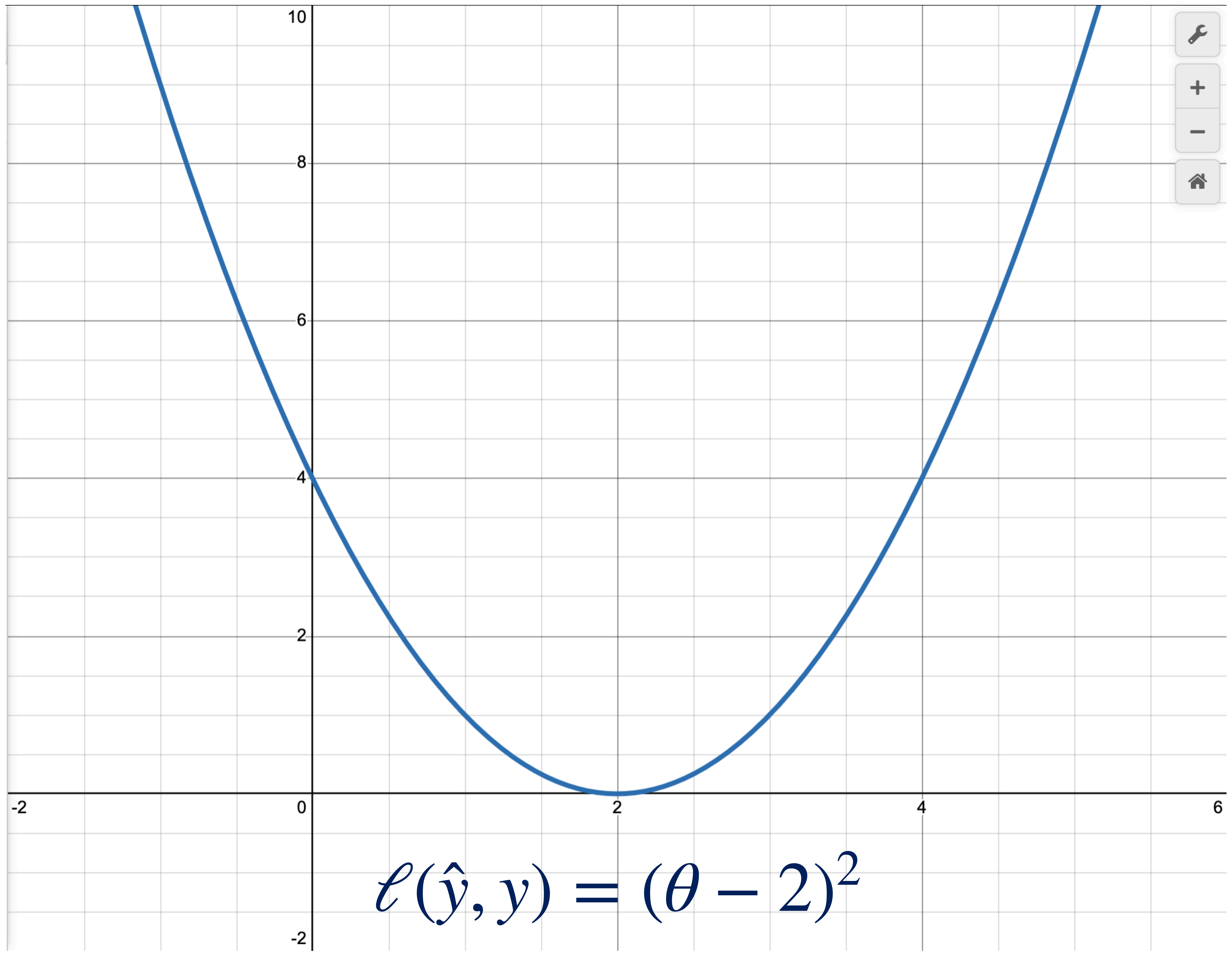

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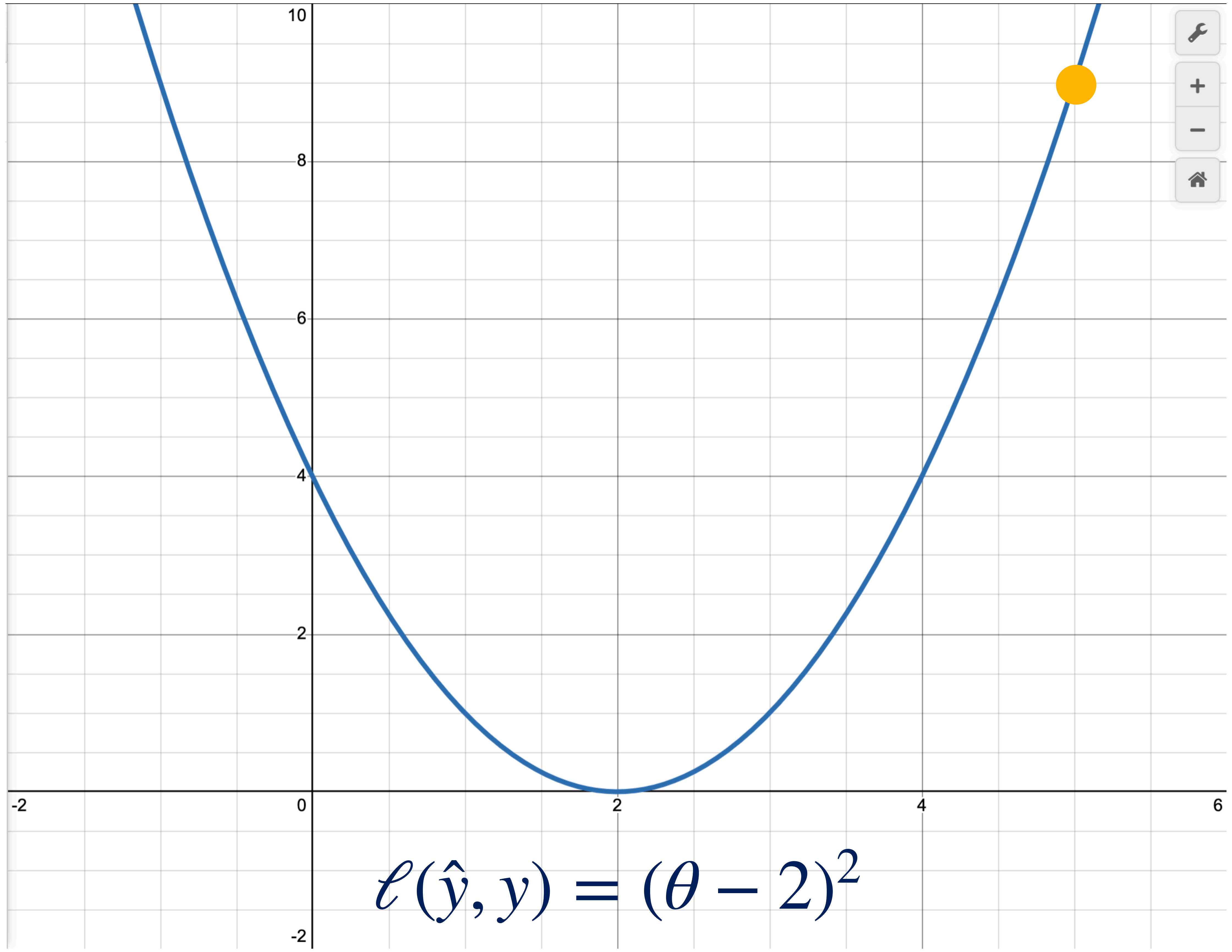

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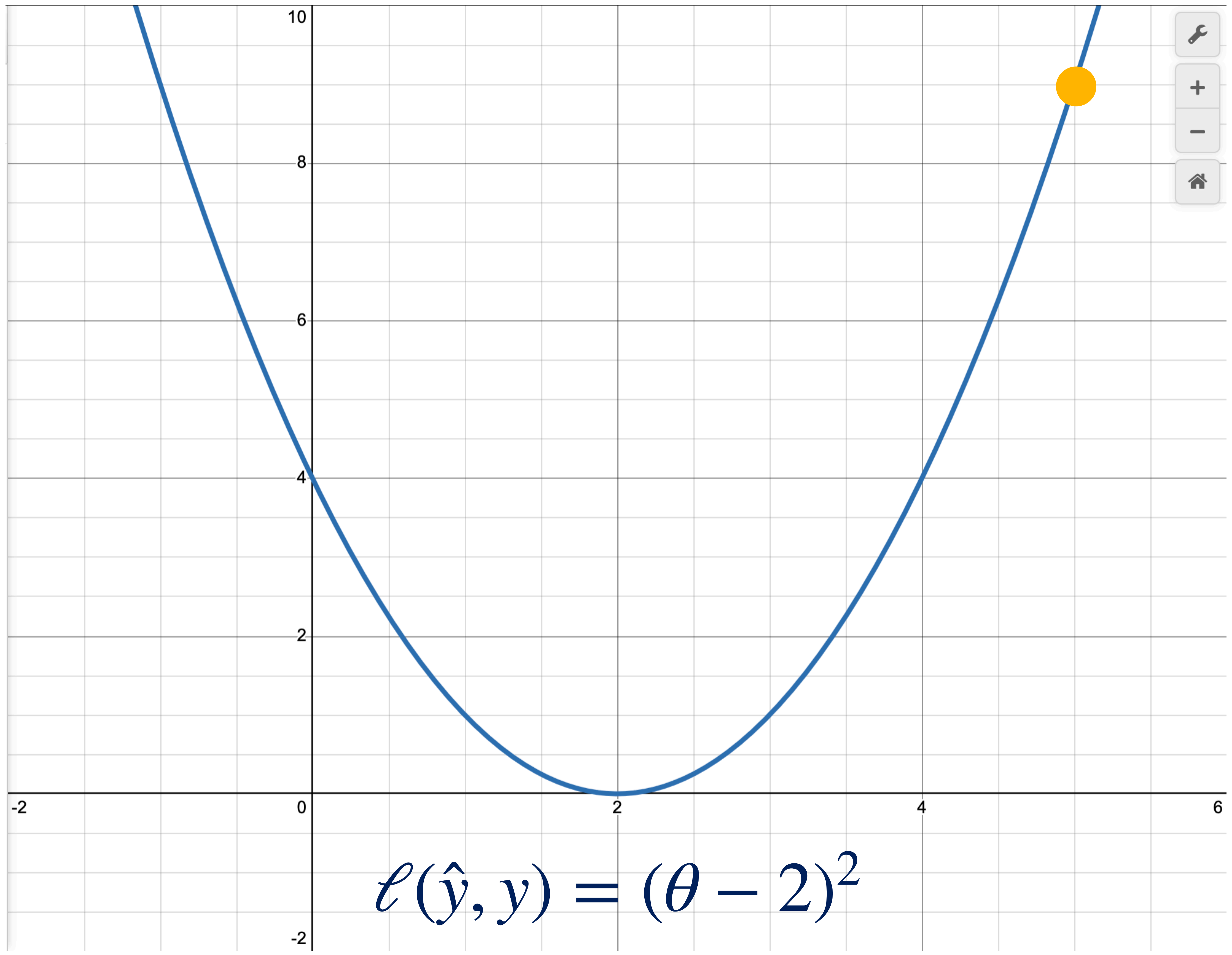

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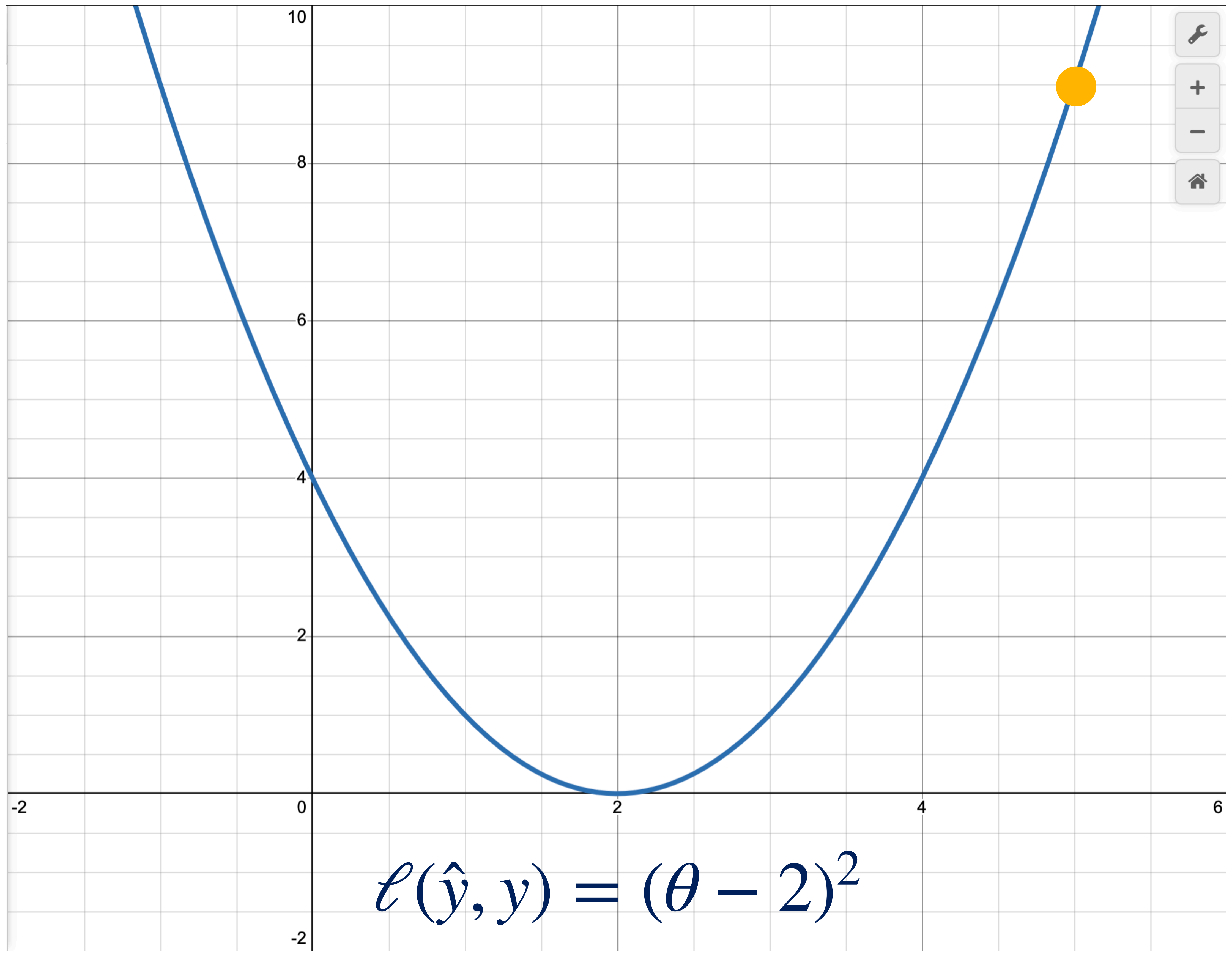

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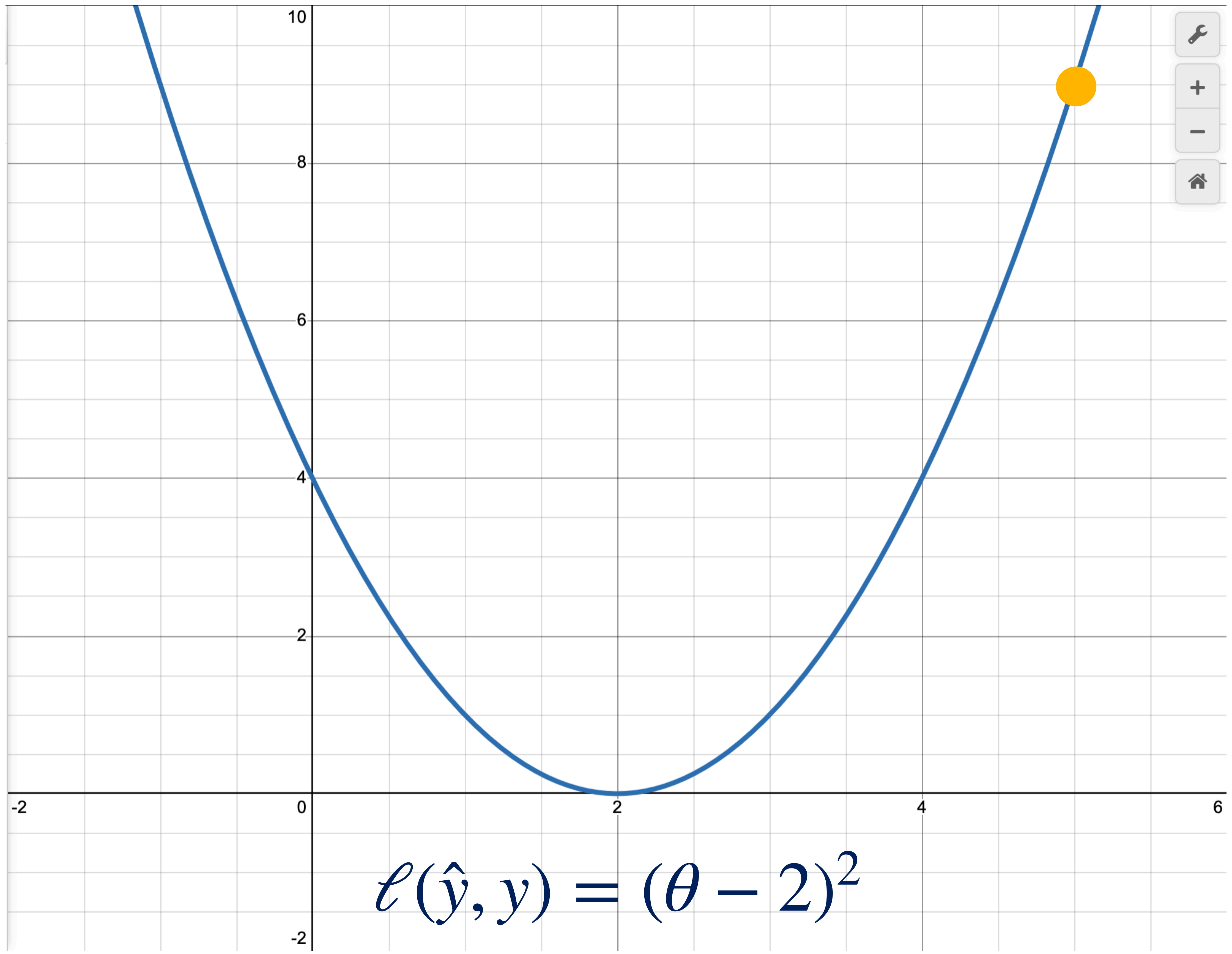

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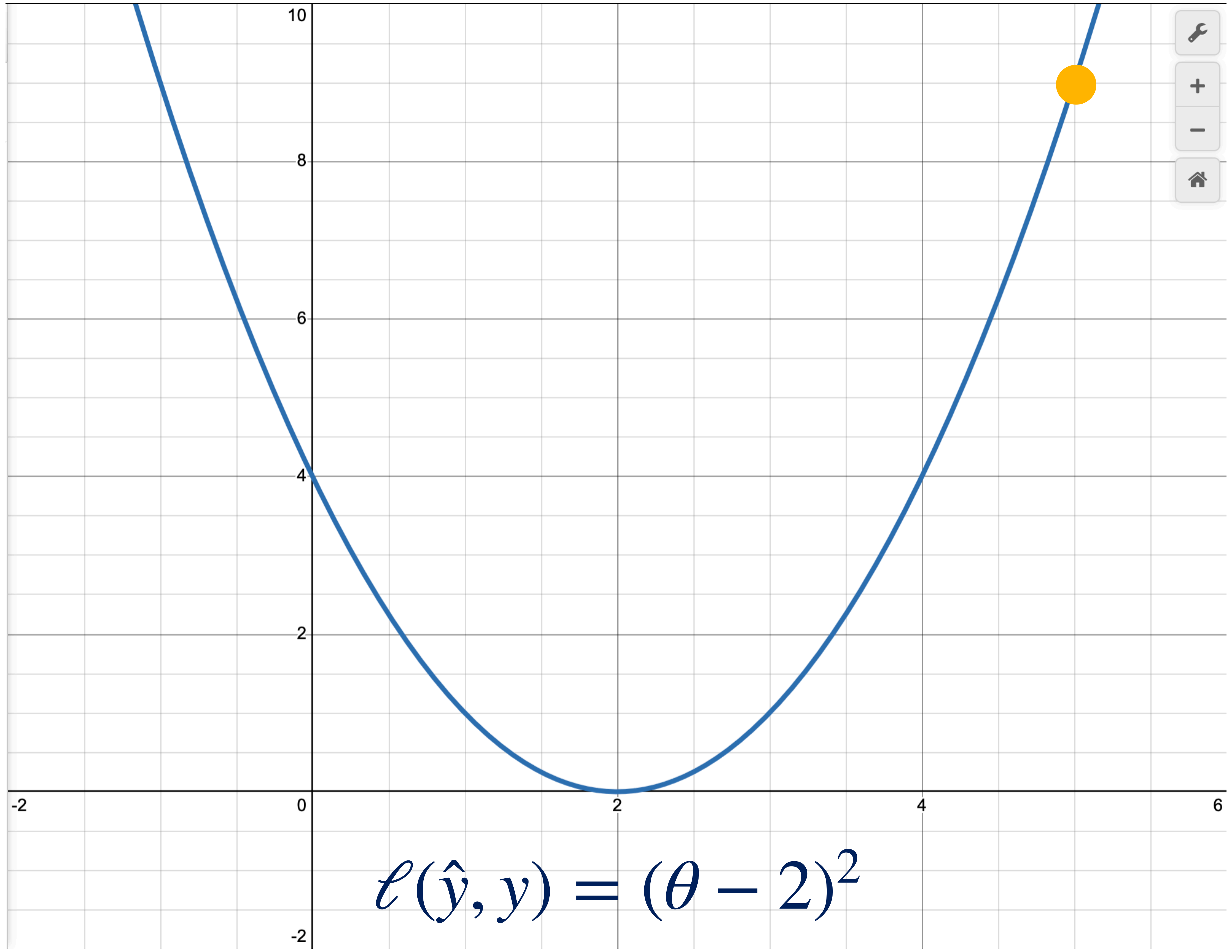
  - Calculate the **slope at the current point**
  - Adjust θ by the **negative of the slope** (because we want to **minimize** the function)

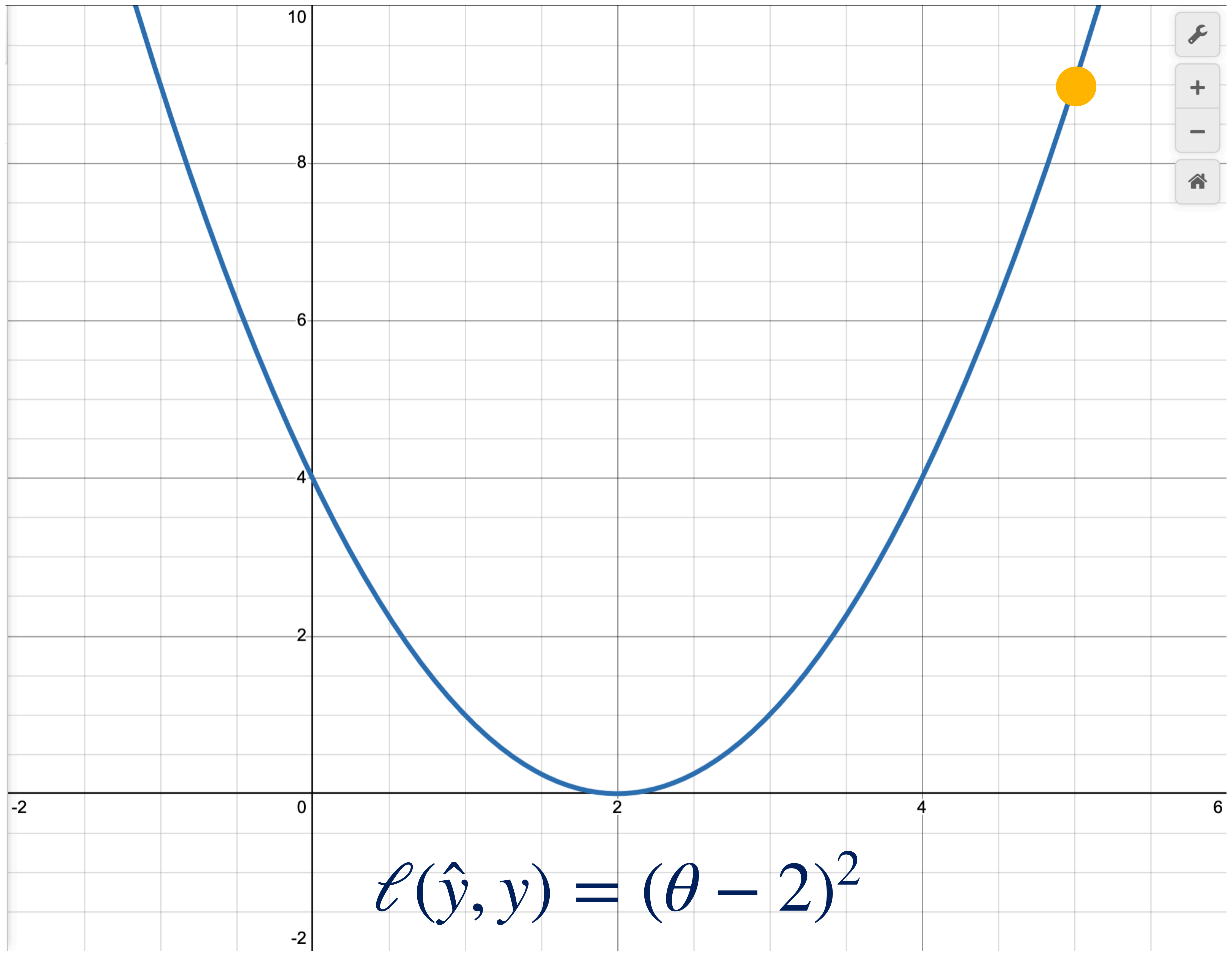


$$\ell(\hat{y}, y) = (\theta - 2)^2$$

$$\ell'(\hat{y}, y) = 2\theta - 4$$

Gradient Descent (first try)

- At each step of the algorithm:
  - Calculate the **slope at the current point**
  - Adjust θ by the **negative of the slope** (because we want to **minimize** the function)
- First step:



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- At each step of the algorithm:
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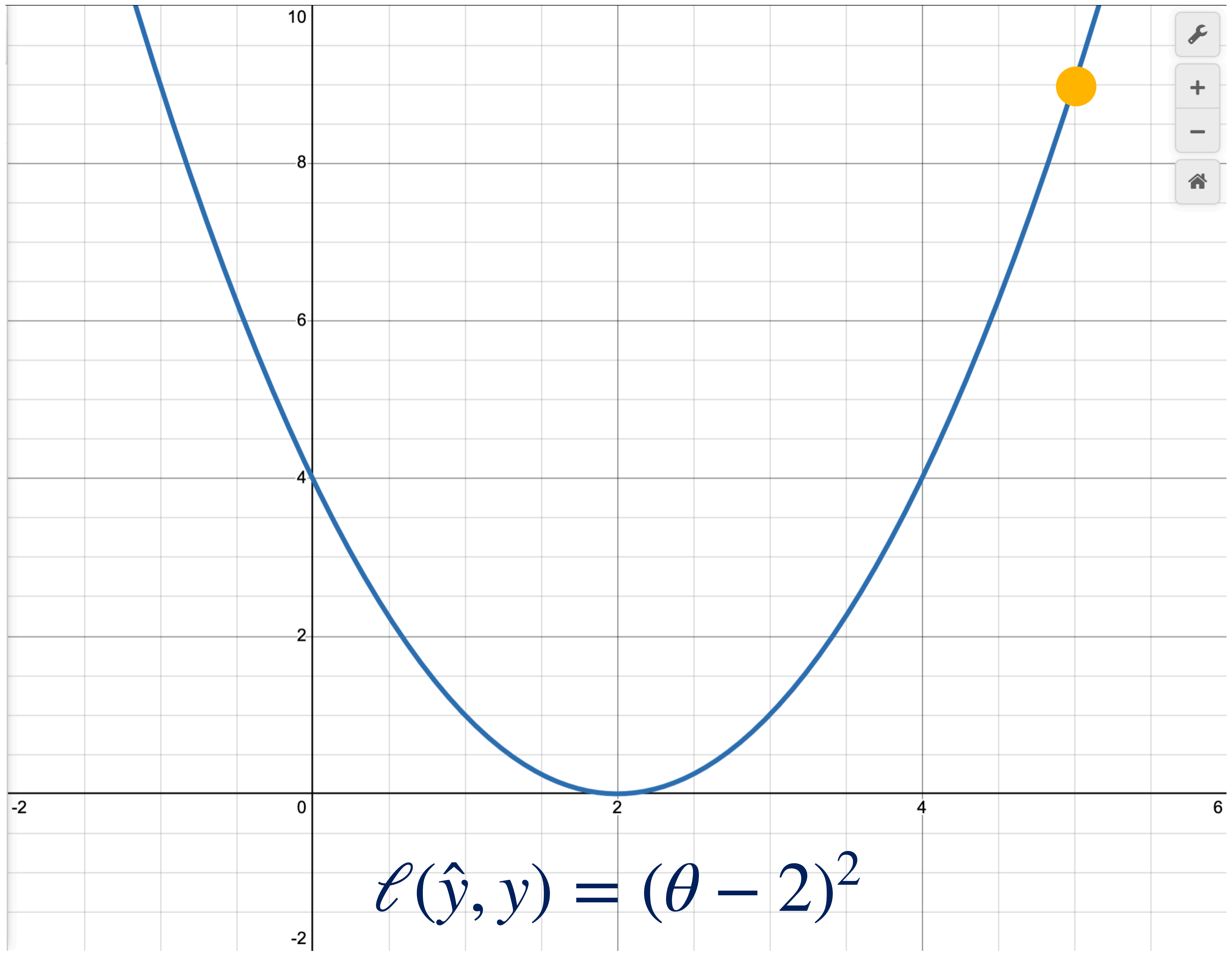
- First step:
  - Plug in θ to derivative:



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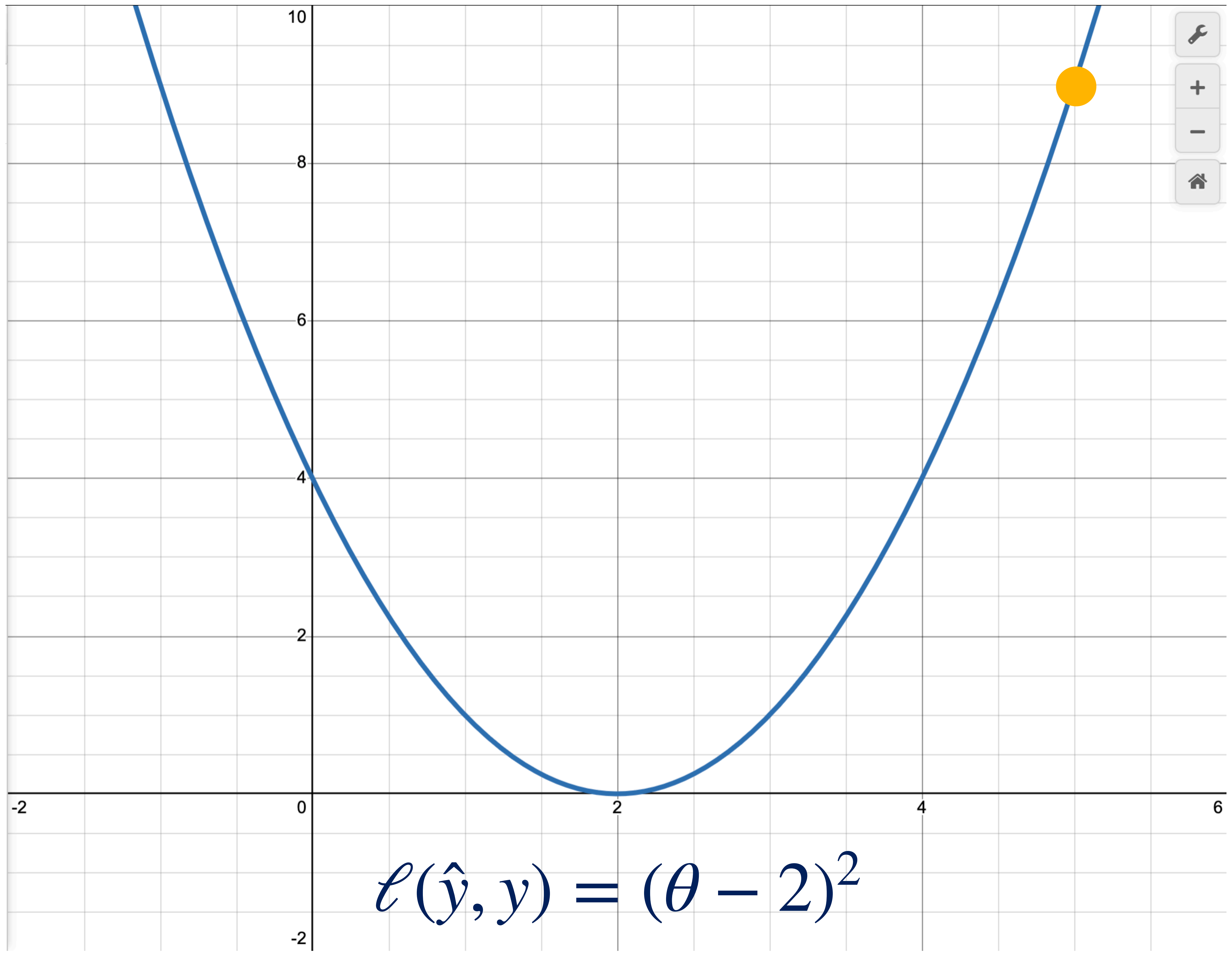

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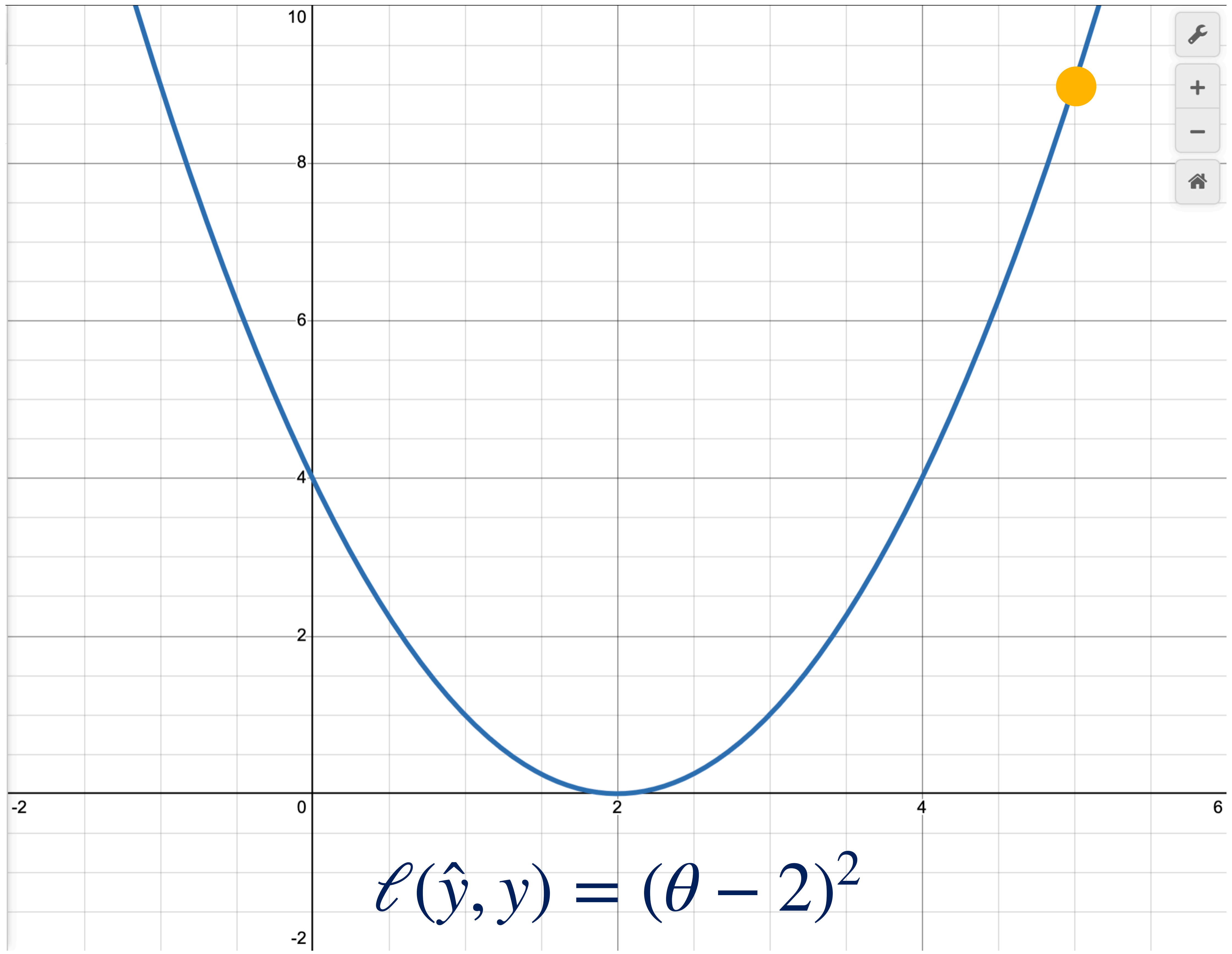
  - Update θ:



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- First step:
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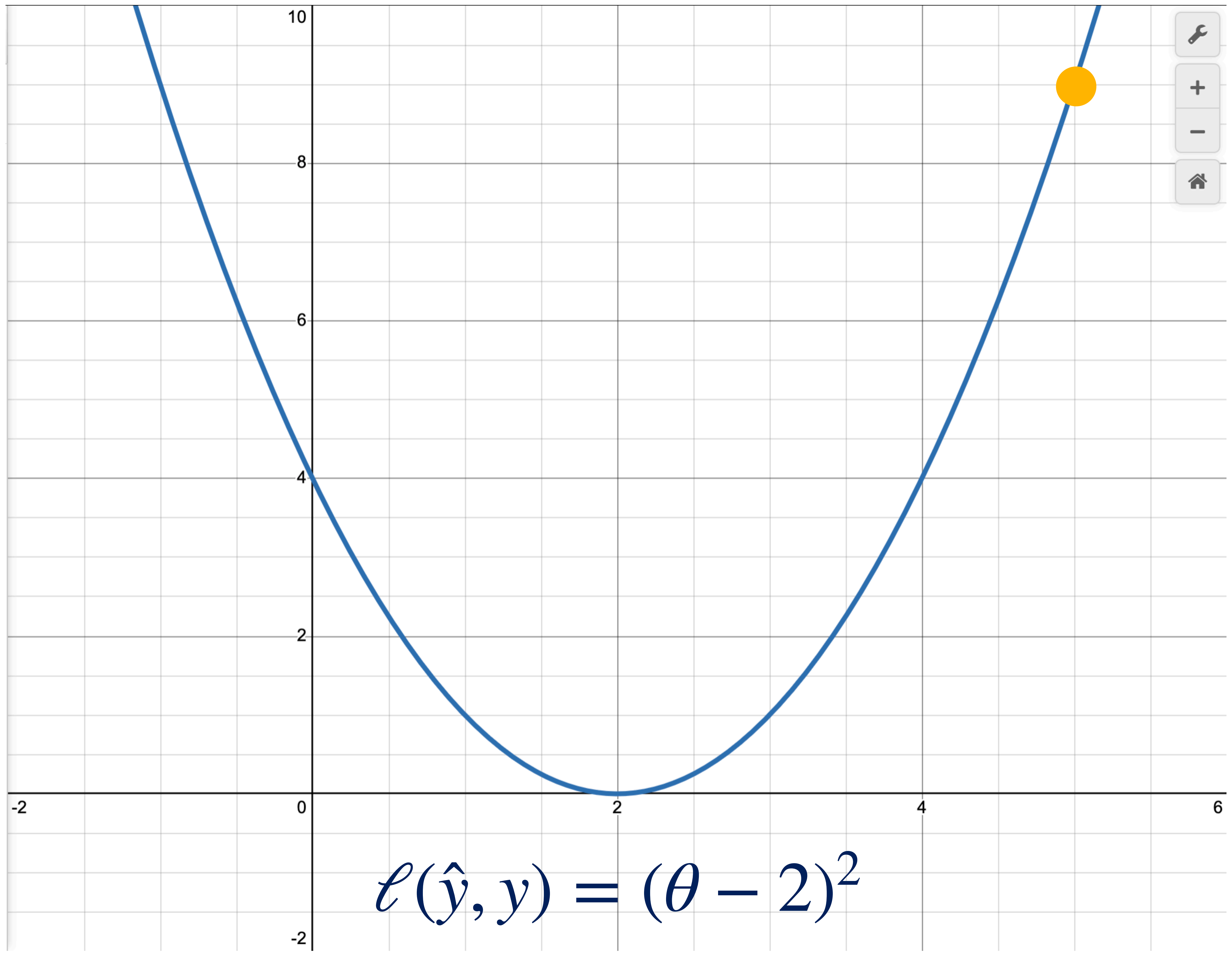
  - Update θ:
    - $\theta_1 = \theta_0 - 6 = -1$



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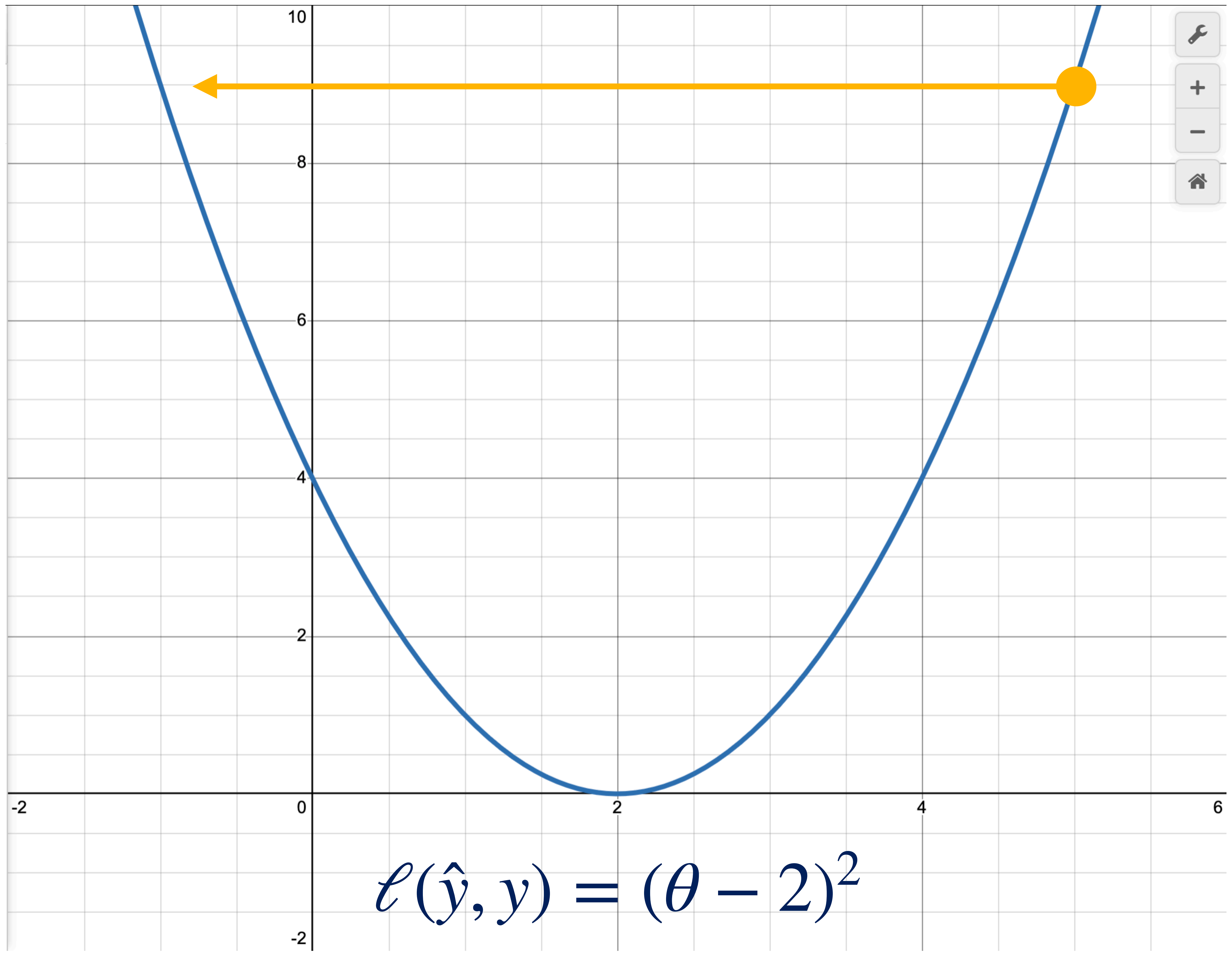

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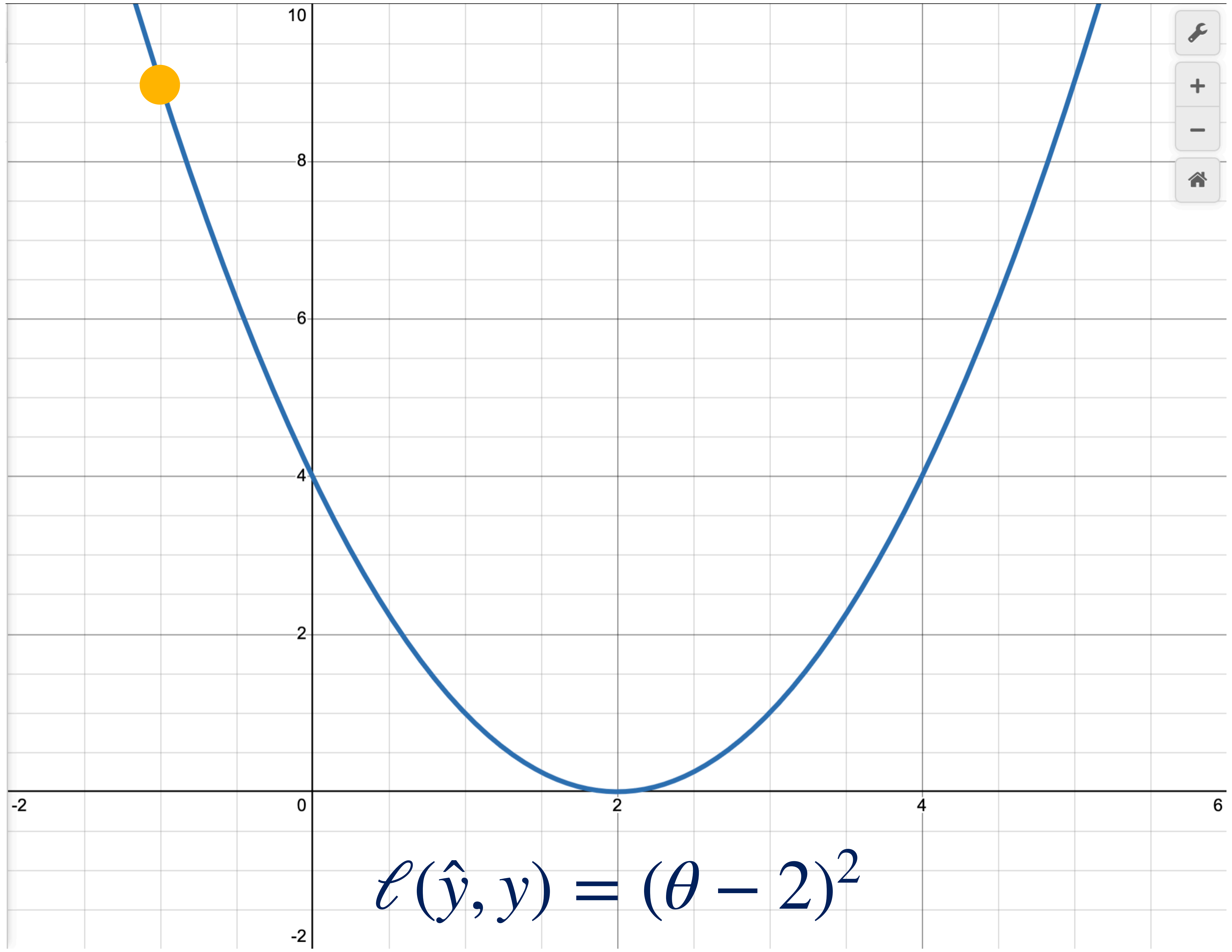
Gradient Descent (first try)



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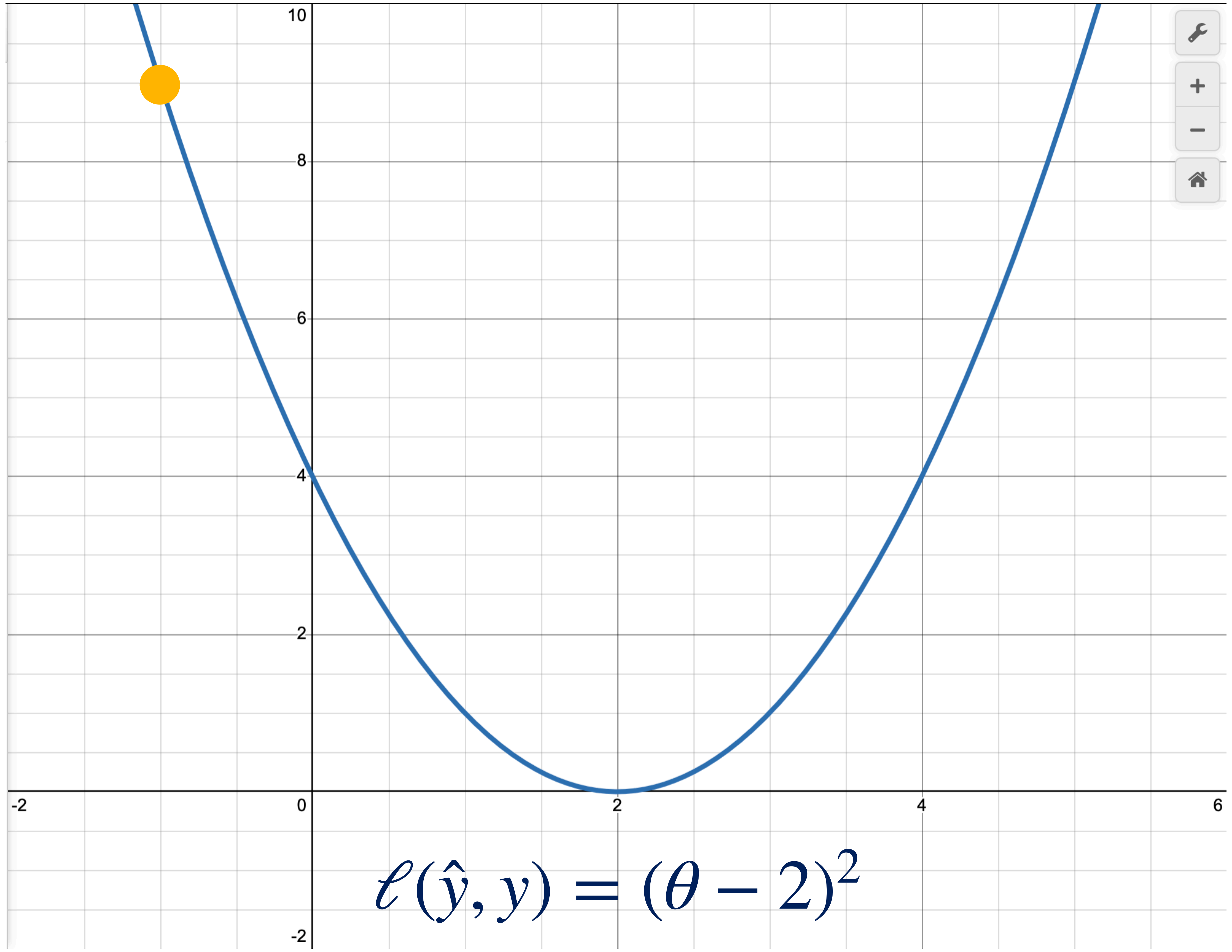
Gradient Descent (first try)

- Second step:



$$\ell(\hat{y}, y) = (\theta - 2)^2$$

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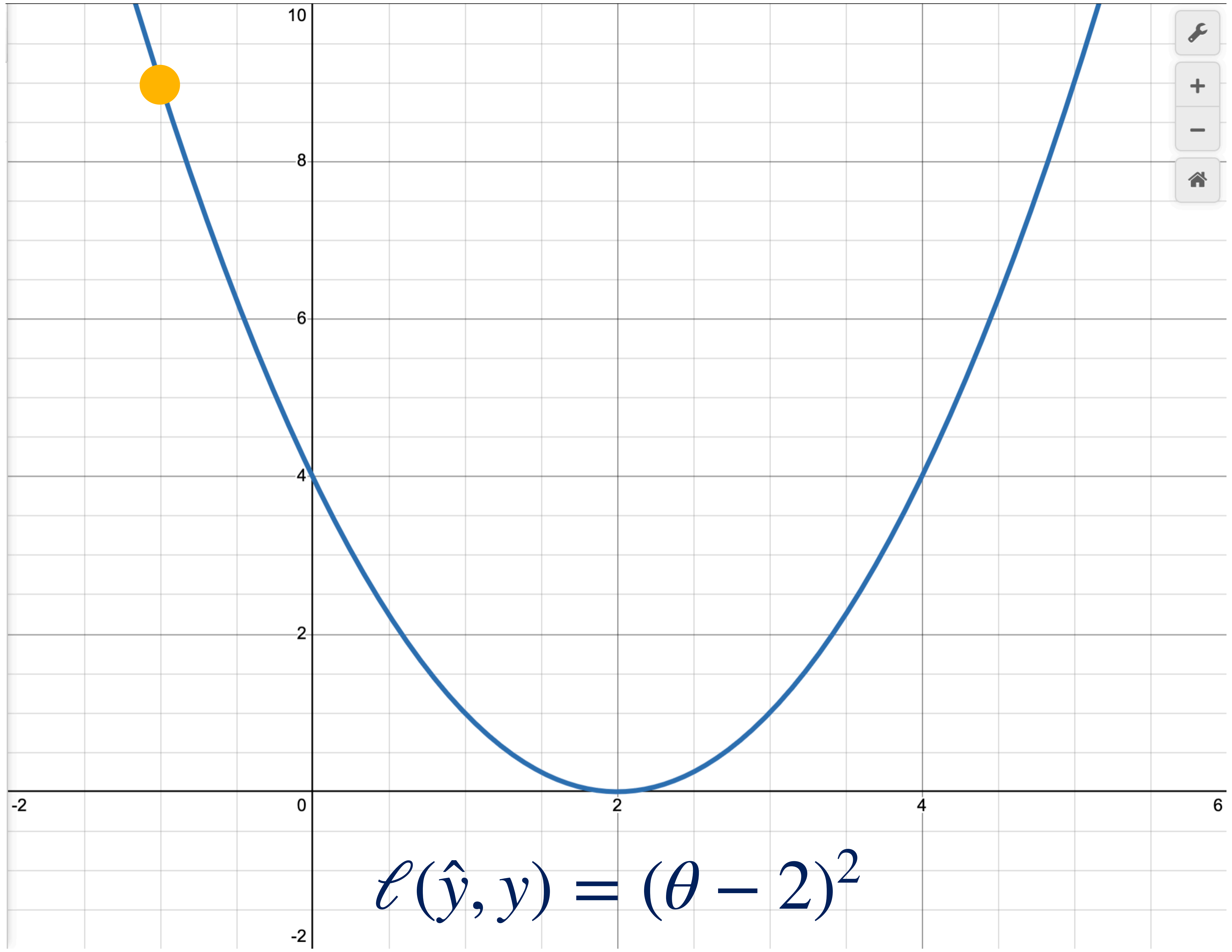
Gradient Descent (first try)

- Second step:
  - Plug in new θ_1:



$$\ell(\hat{y}, y) = (\theta - 2)^2$$

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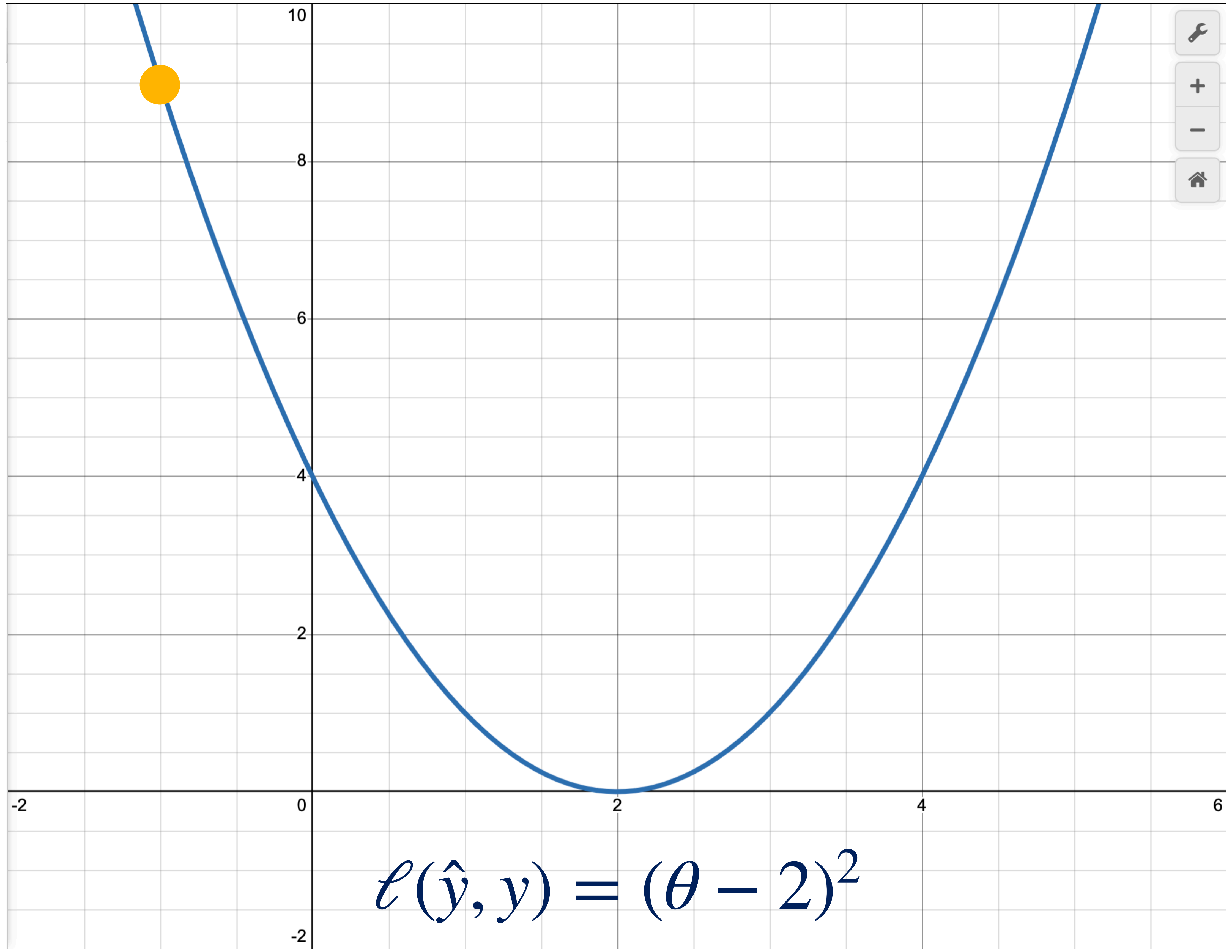
Gradient Descent (first try)

- Second step:
  - Plug in new θ_1:
    - $2 \cdot (-1) - 4 = -6$



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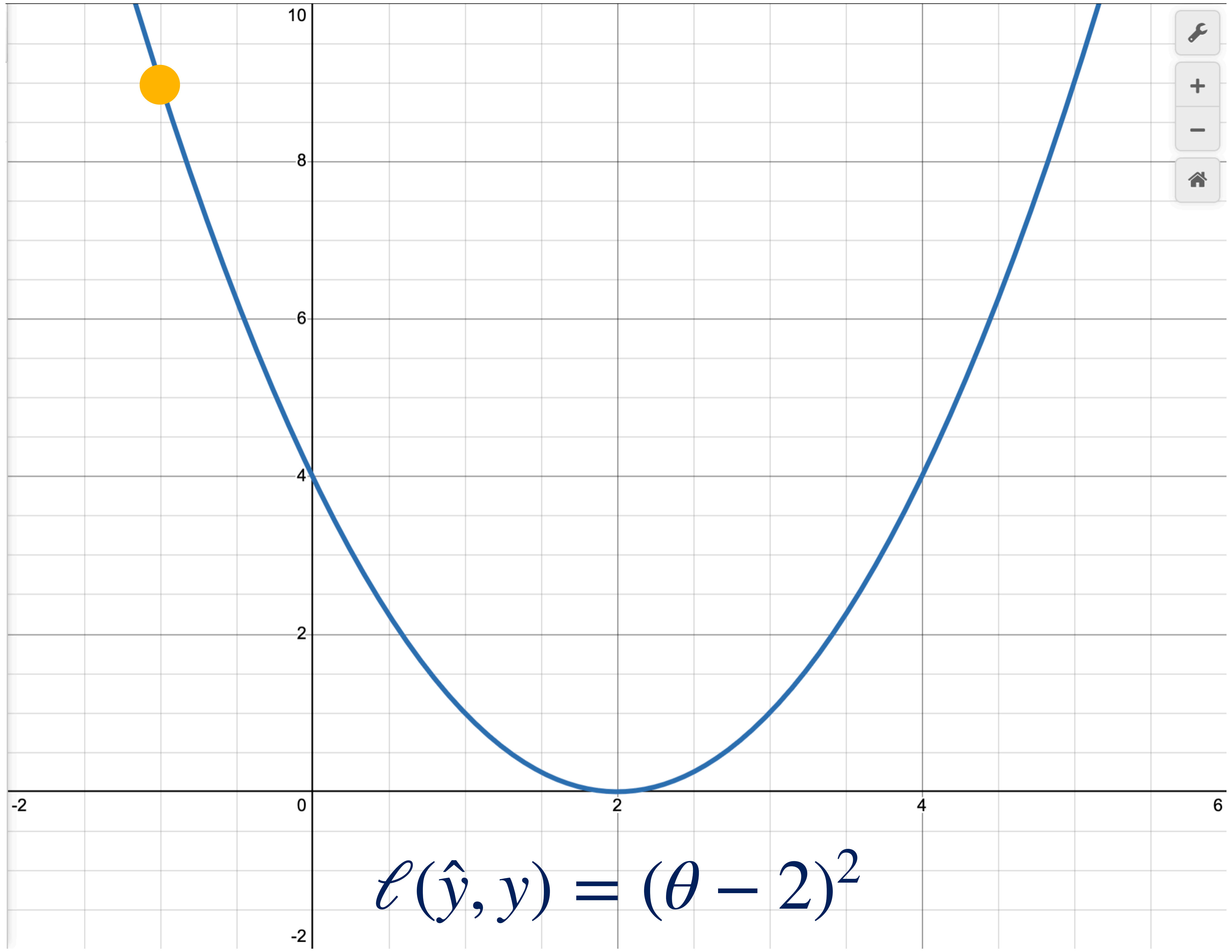
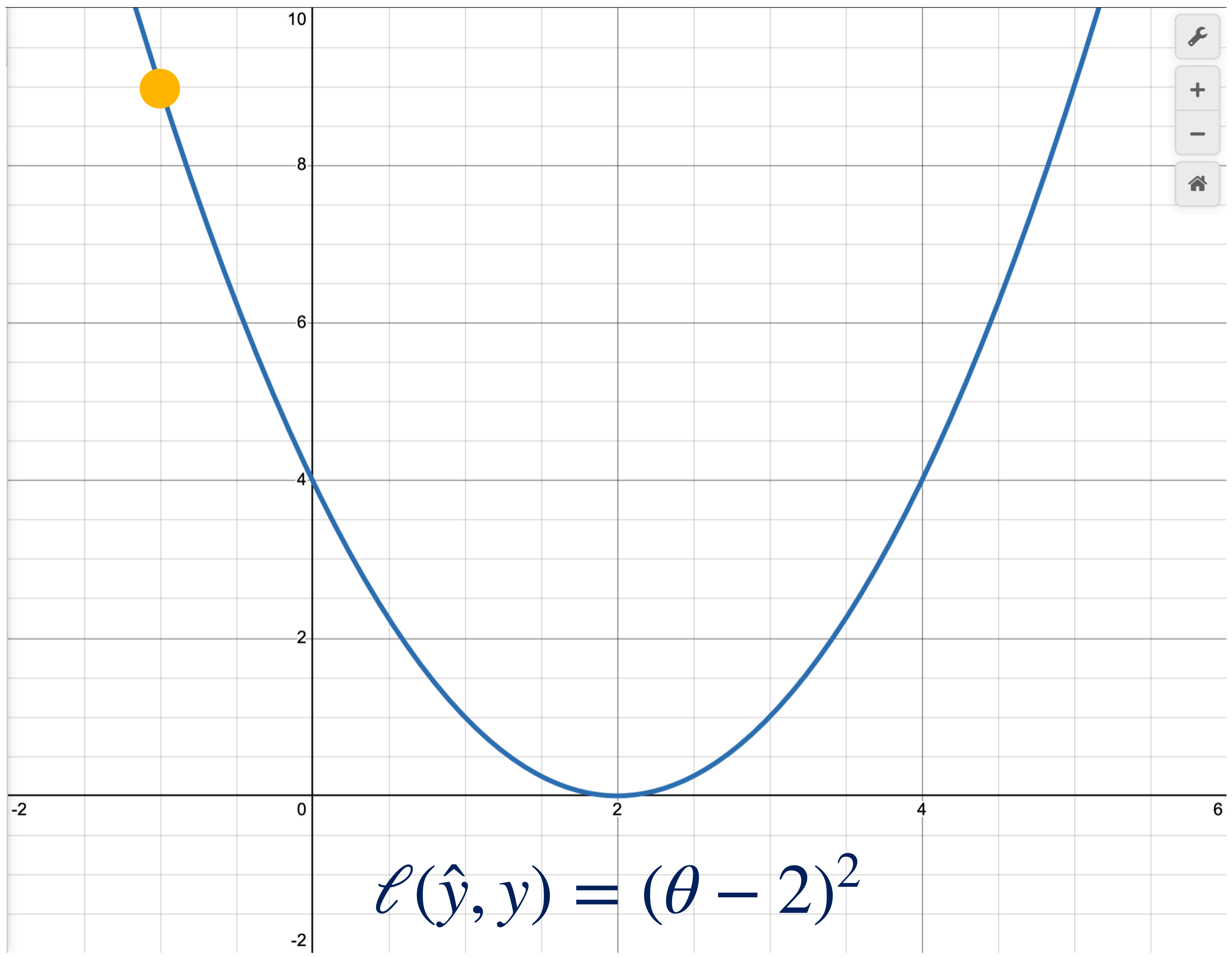


$\ell(\hat{y}, y) = (\theta - 2)^2$

$\ell'(\hat{y}, y) = 2\theta - 4$

Gradient Descent (first try)

- Second step:
  - Plug in new θ_1:
    - $2 \cdot (-1) - 4 = -6$
  - Update θ:
    - $\theta_2 = \theta_1 - (-6) = 5$



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Gradient Descent (first try)

- Second step:
  - Plug in new θ_1:
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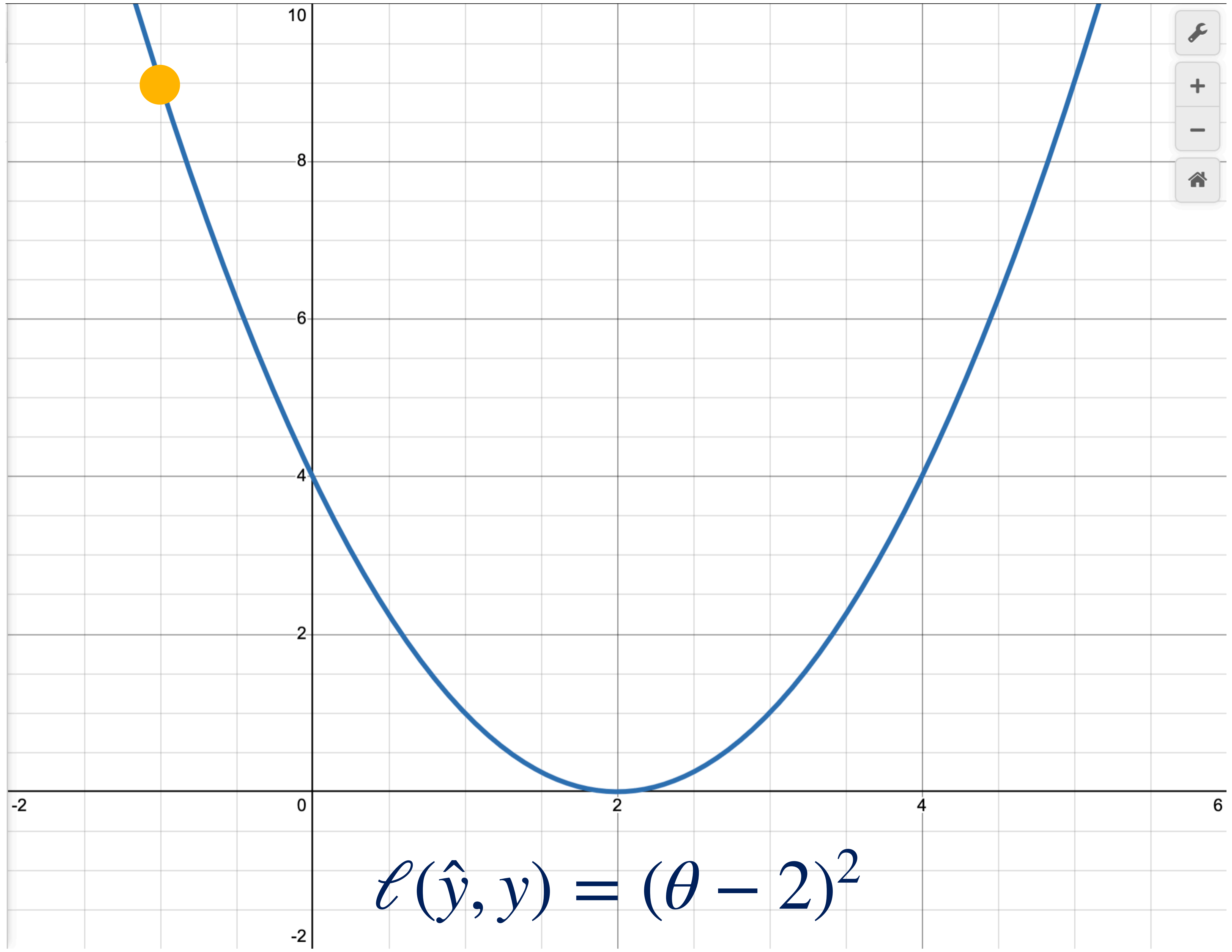
  - Update θ:
    - $\theta_2 = \theta_1 - (-6) = 5$
- Whoops! We're **back where we started!**



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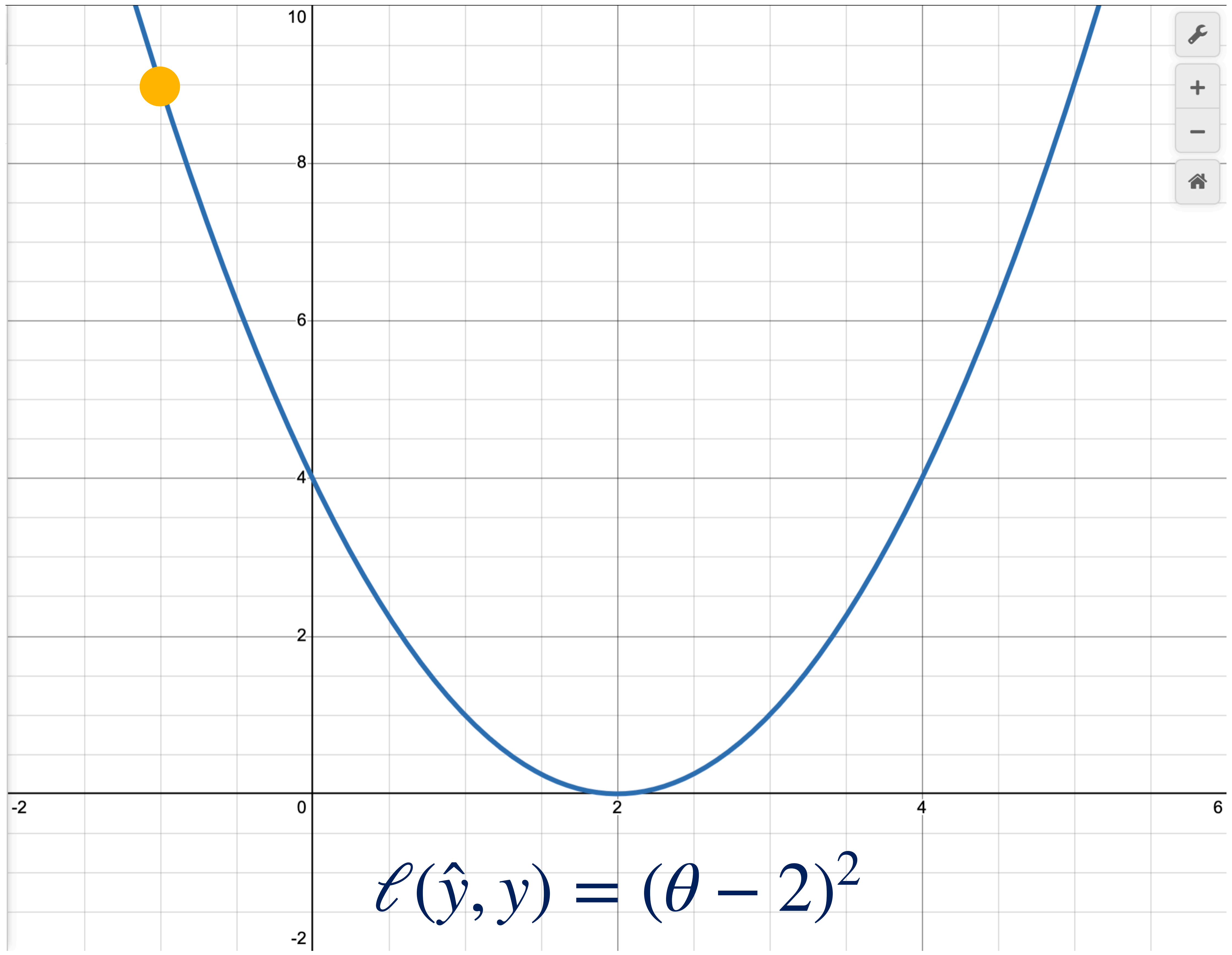
- Whoops! We're **back where we started!**
  - This process would **"bounce back and forth"** forever!



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Gradient Descent (first try)

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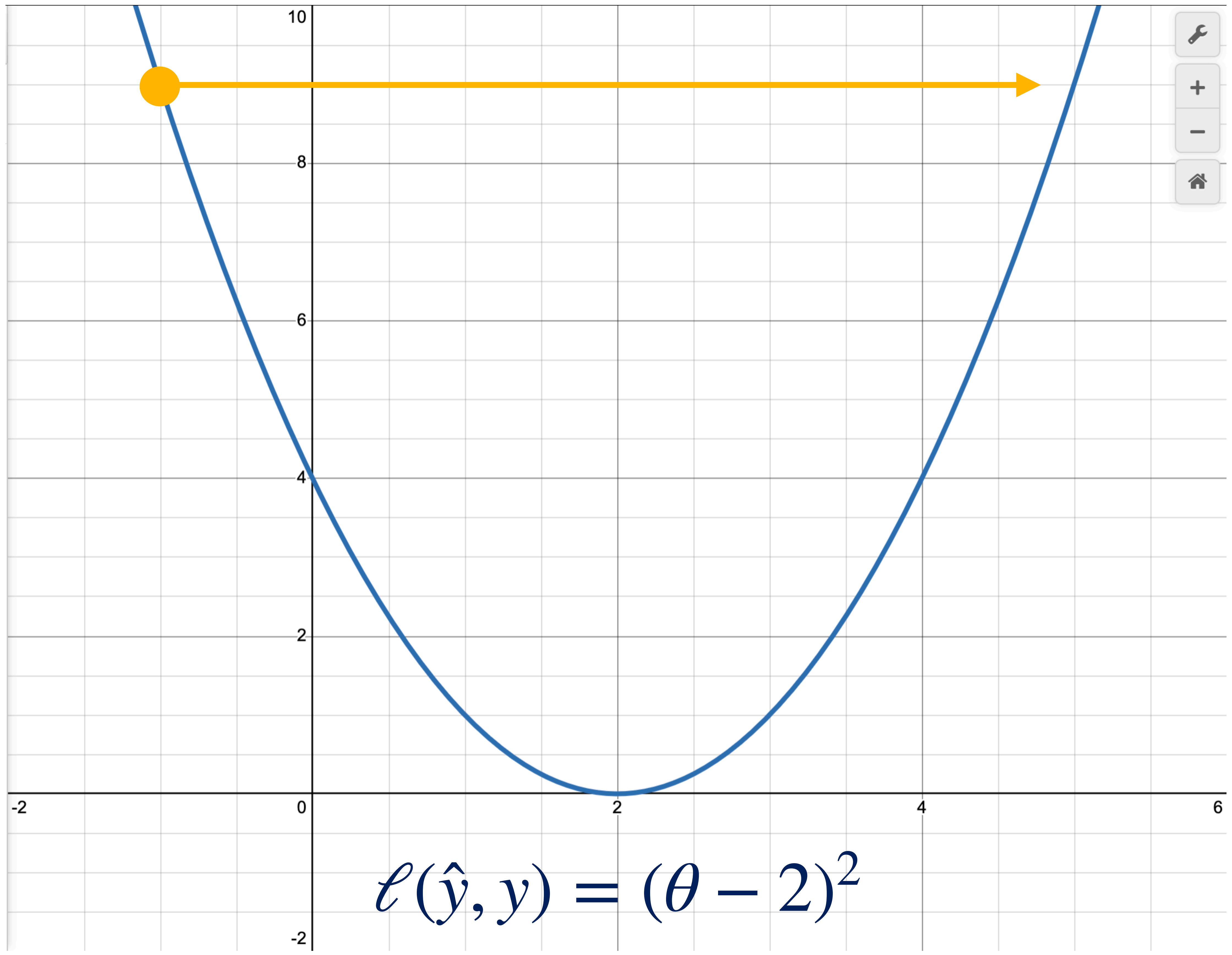

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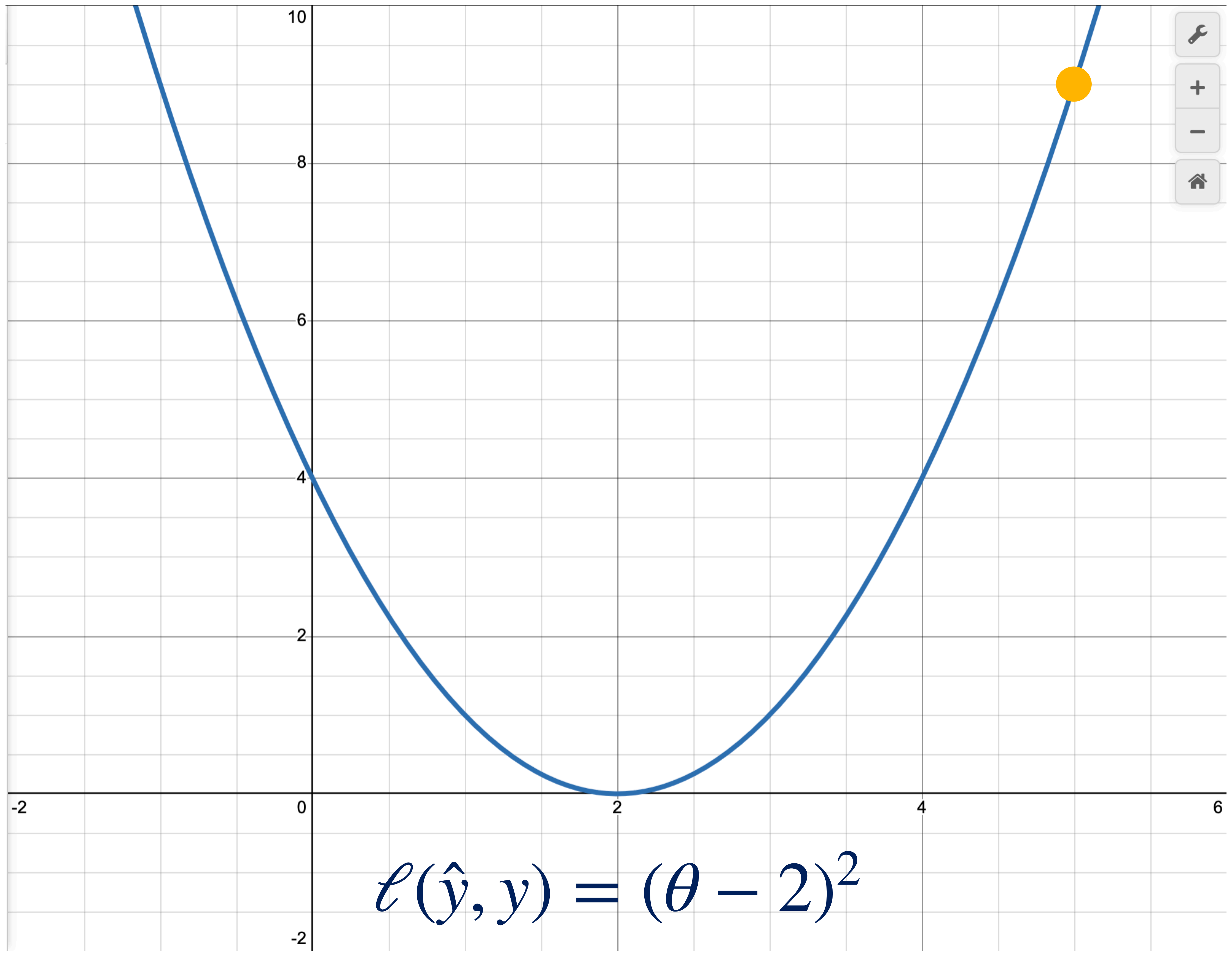
Gradient Descent (second try)



$$\ell(\hat{y}, y) = (\theta - 2)^2$$

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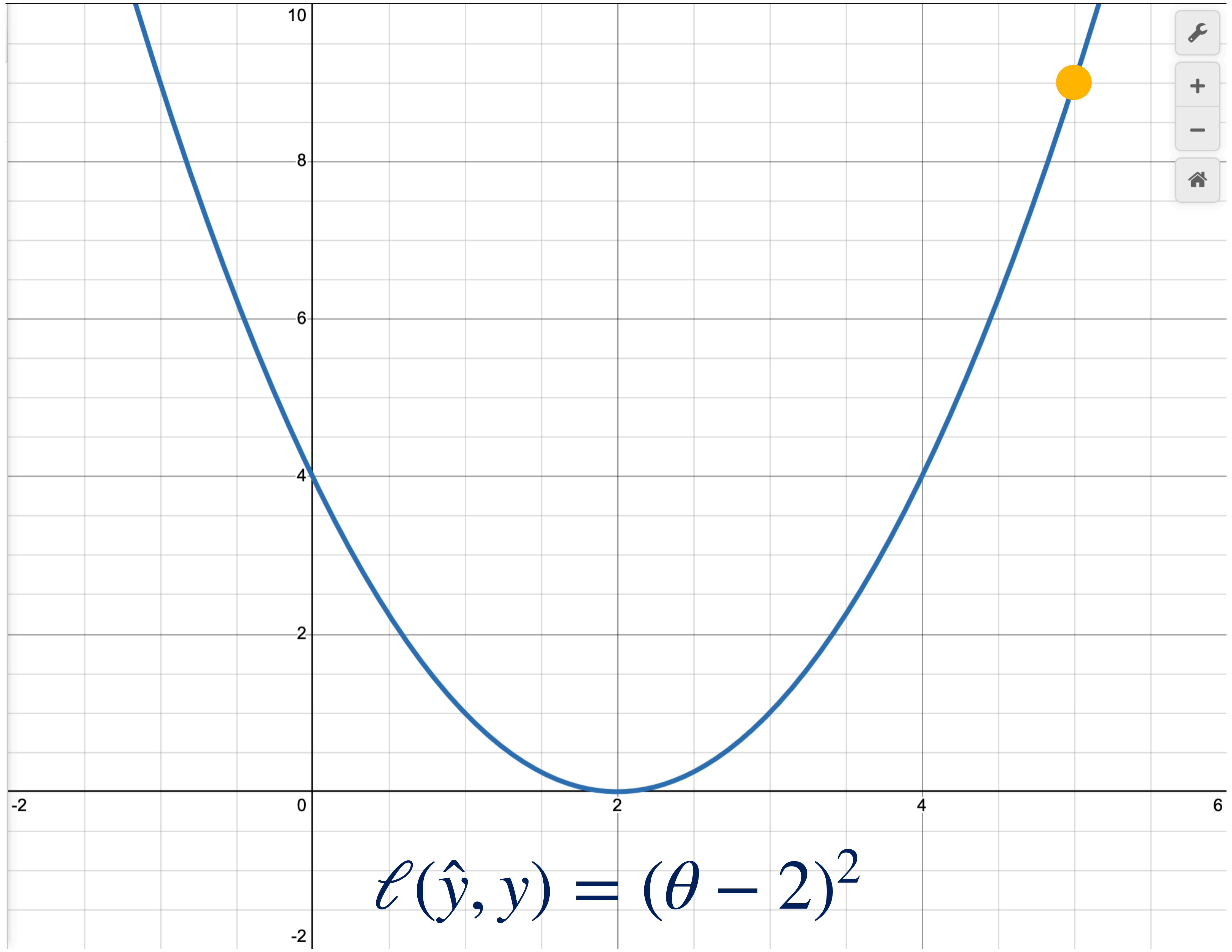
Gradient Descent (second try)

- What if we adjusted θ by a **fraction** of the slope? (Say, 0.75)



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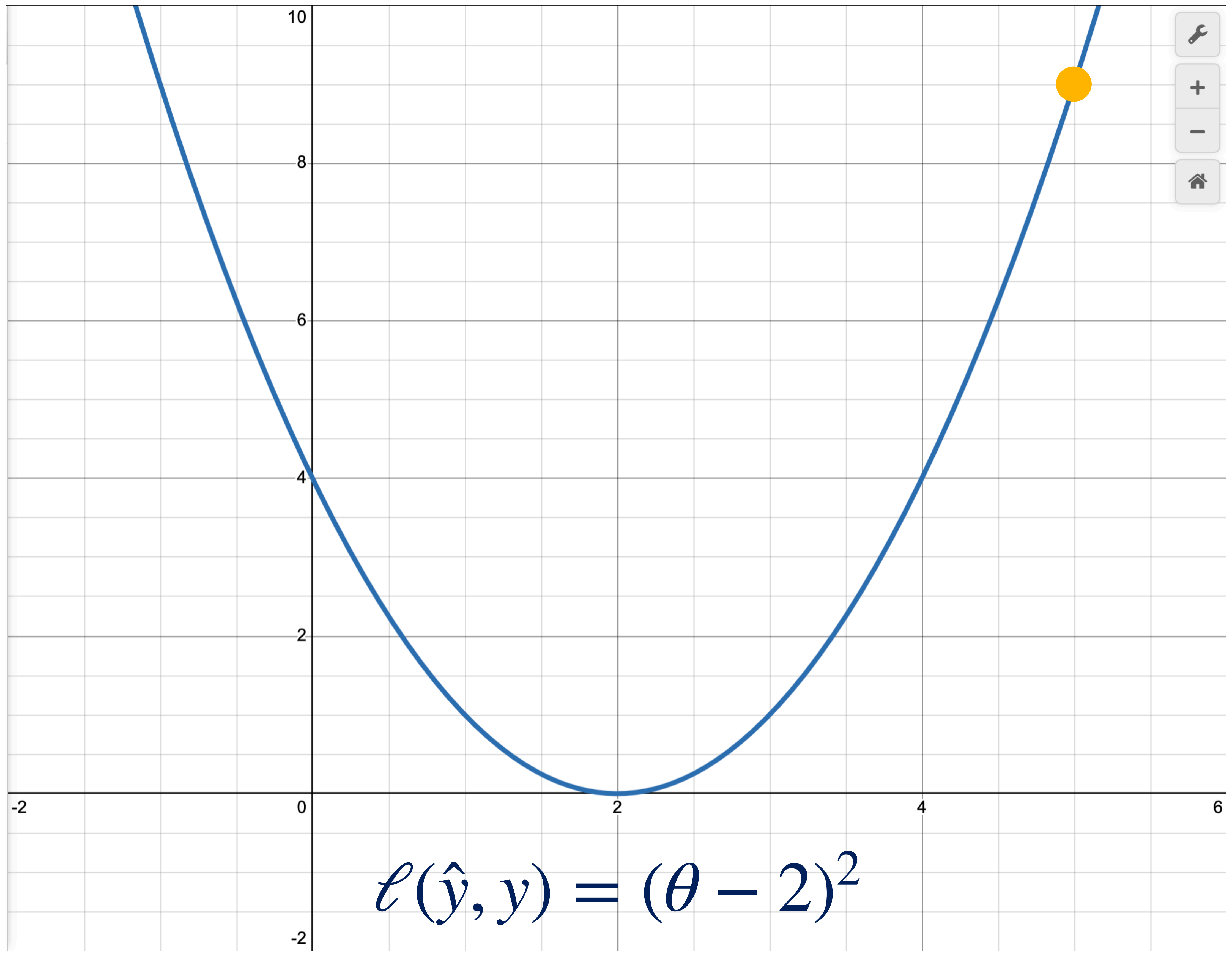
Gradient Descent (second try)

- What if we adjusted θ by a **fraction** of the slope? (Say, 0.75)
- Let's start again:



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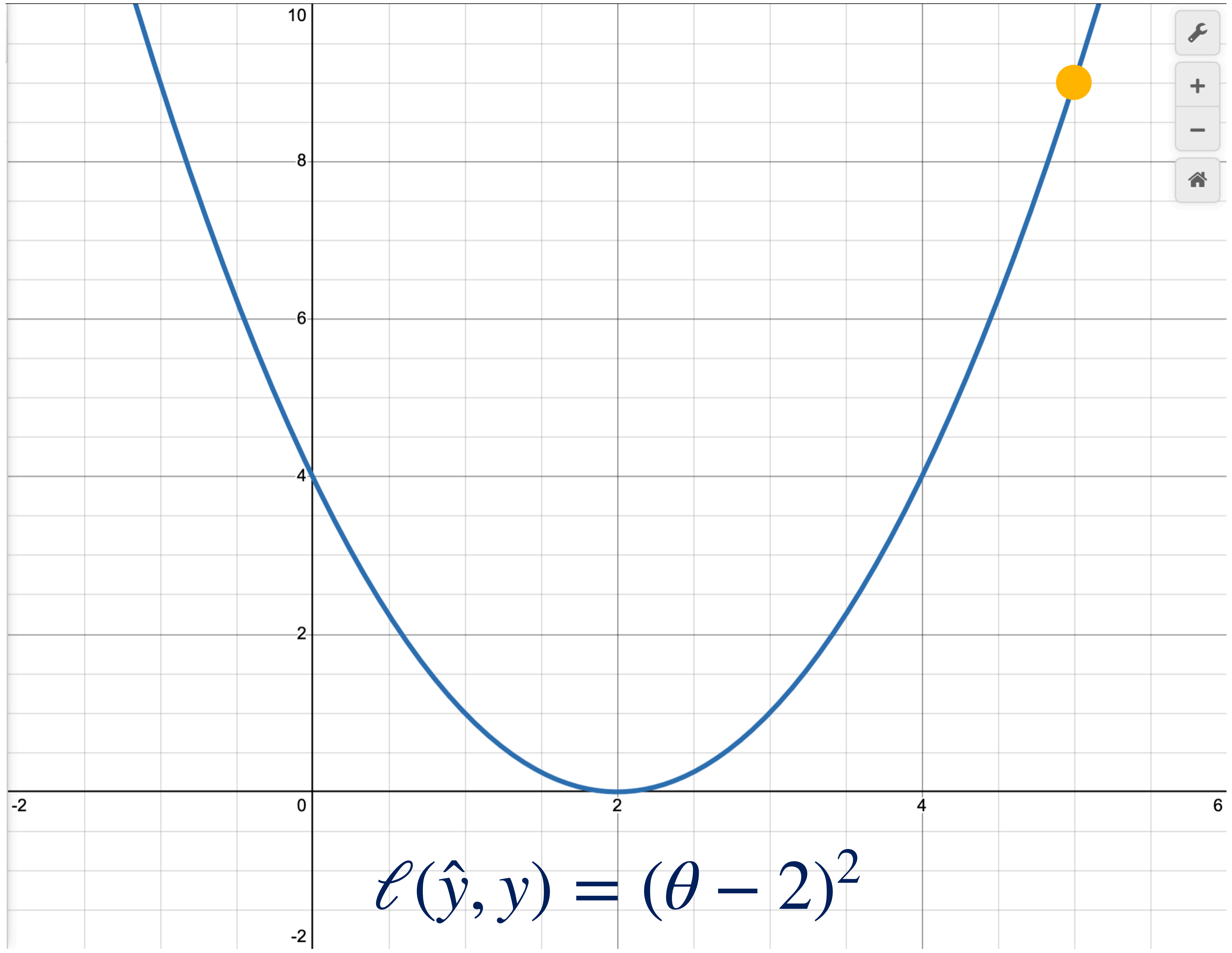
Gradient Descent (second try)

- What if we adjusted θ by a **fraction** of the slope? (Say, 0.75)
- Let's start again:
  - Plug in θ_0 to derivative:



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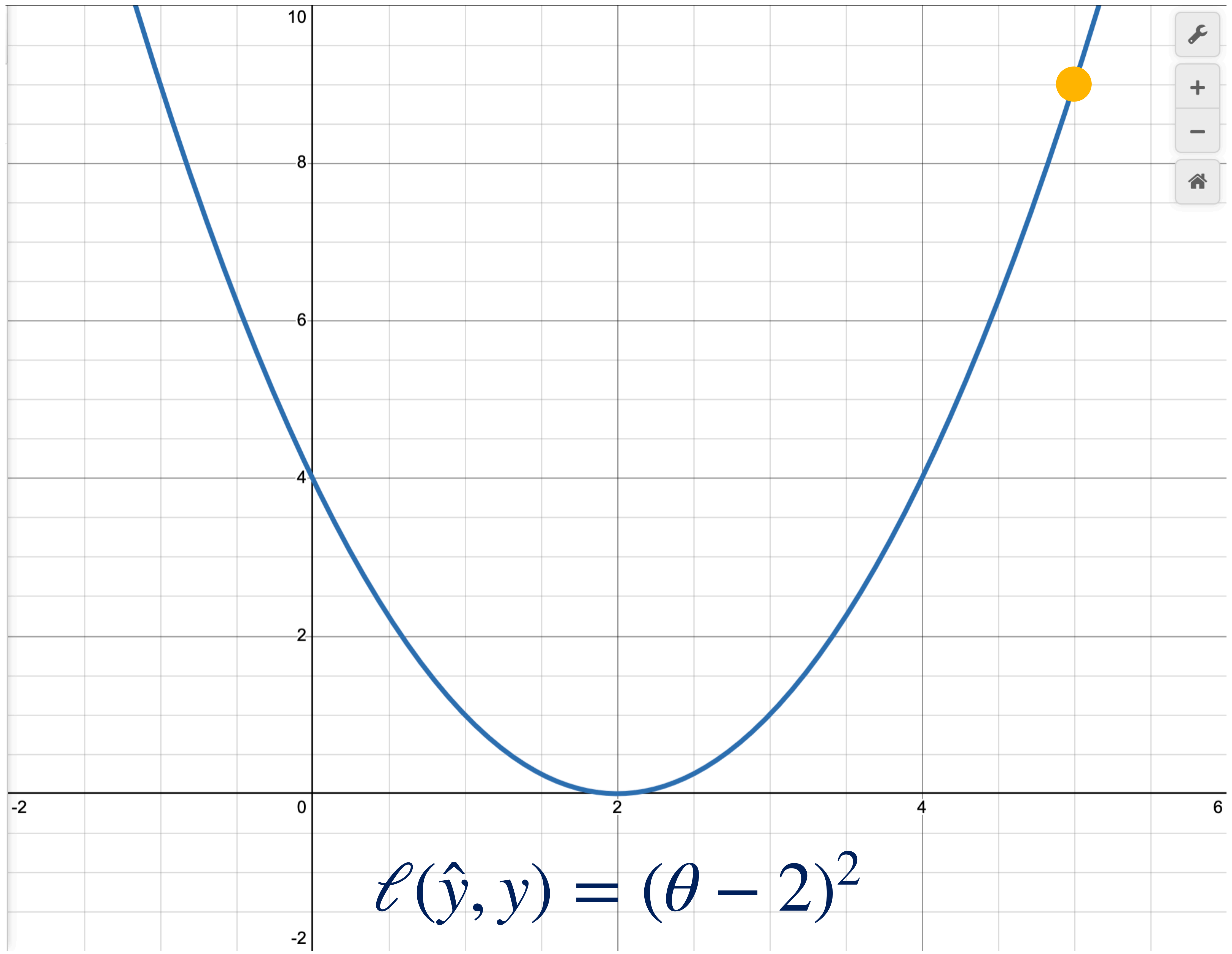

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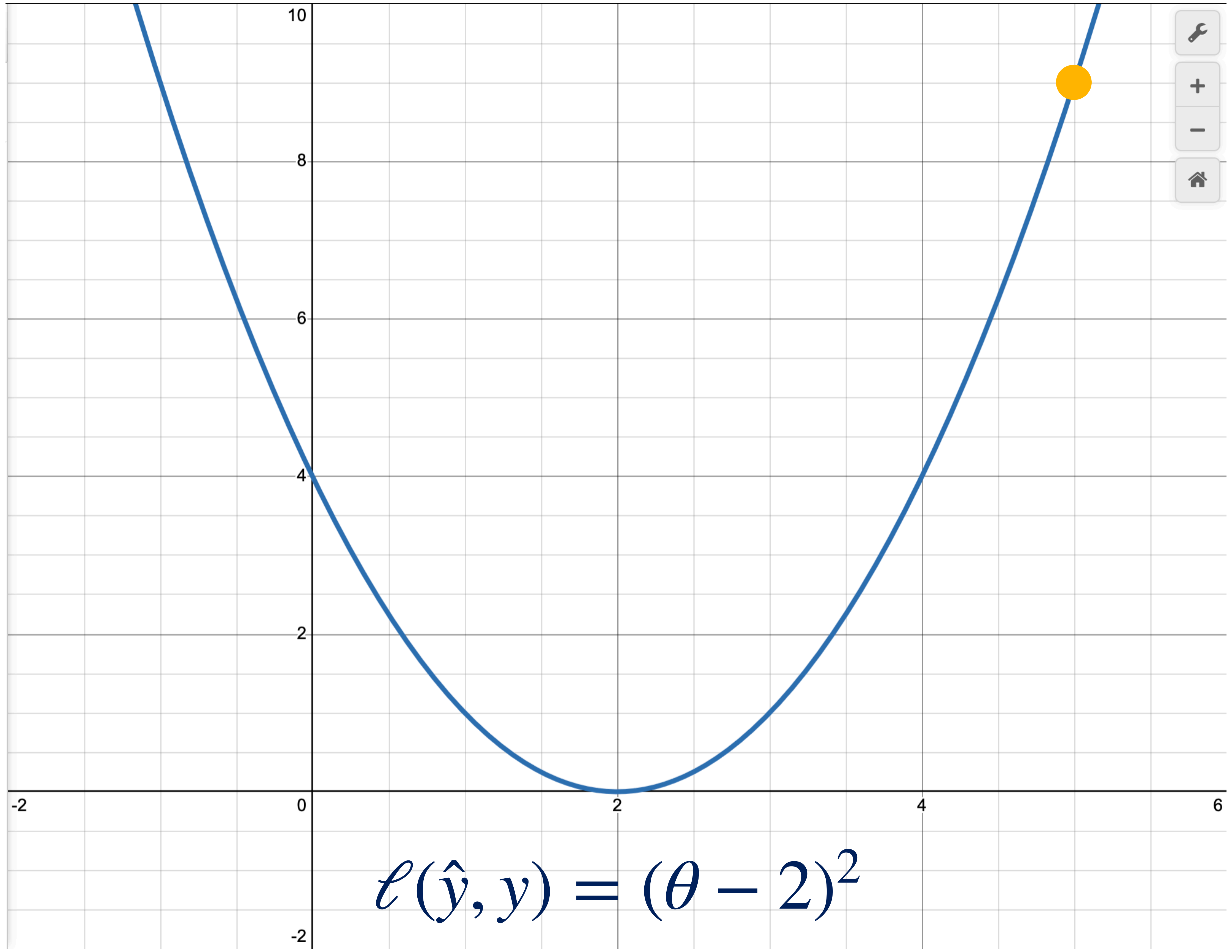

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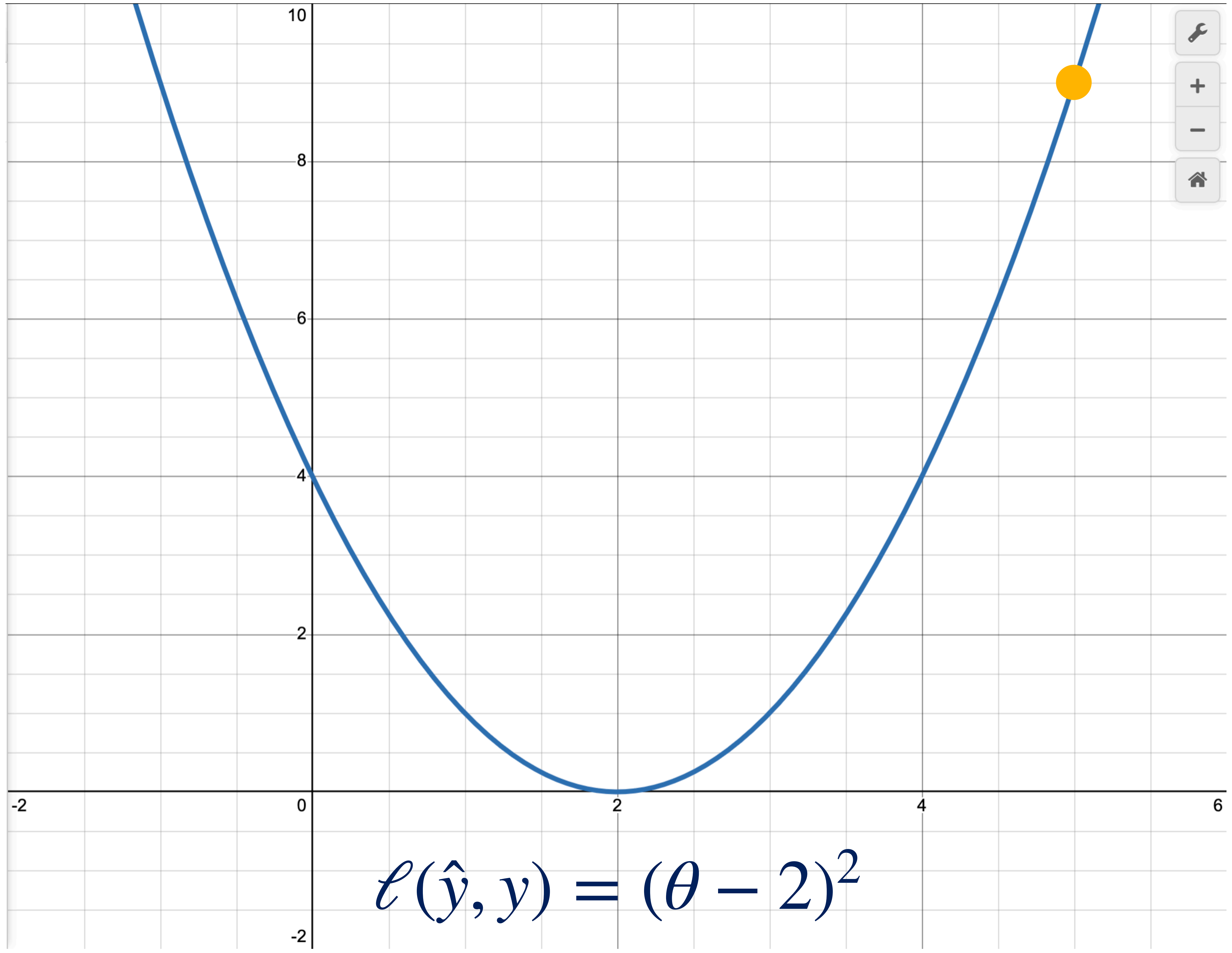
- Let's start again:
  - Plug in θ_0 to derivative:
    - $2 \cdot 5 - 4 = 6$
  - Update θ:
    - $\theta_1 = \theta_0 - 0.75 \cdot 6 = 0.5$



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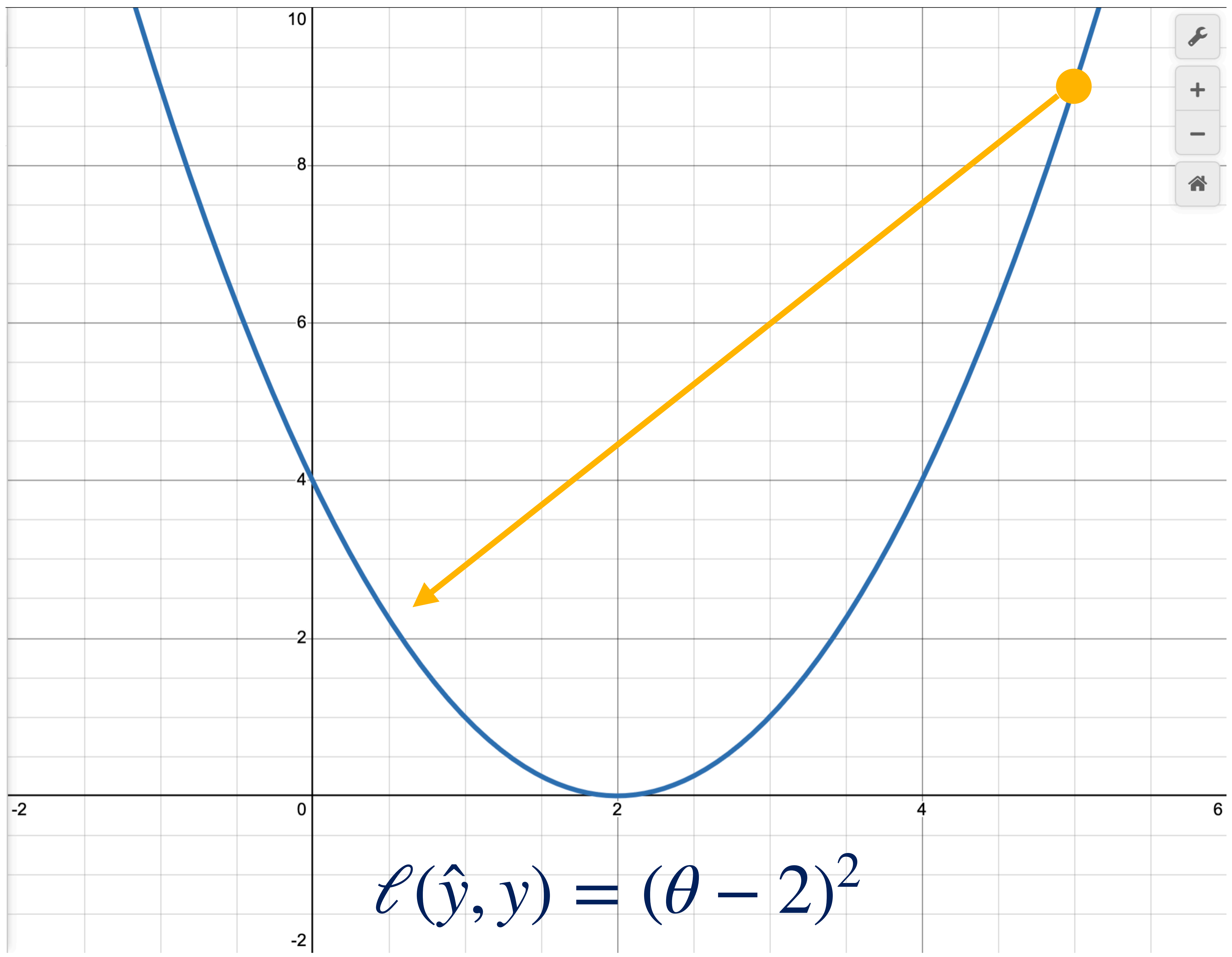

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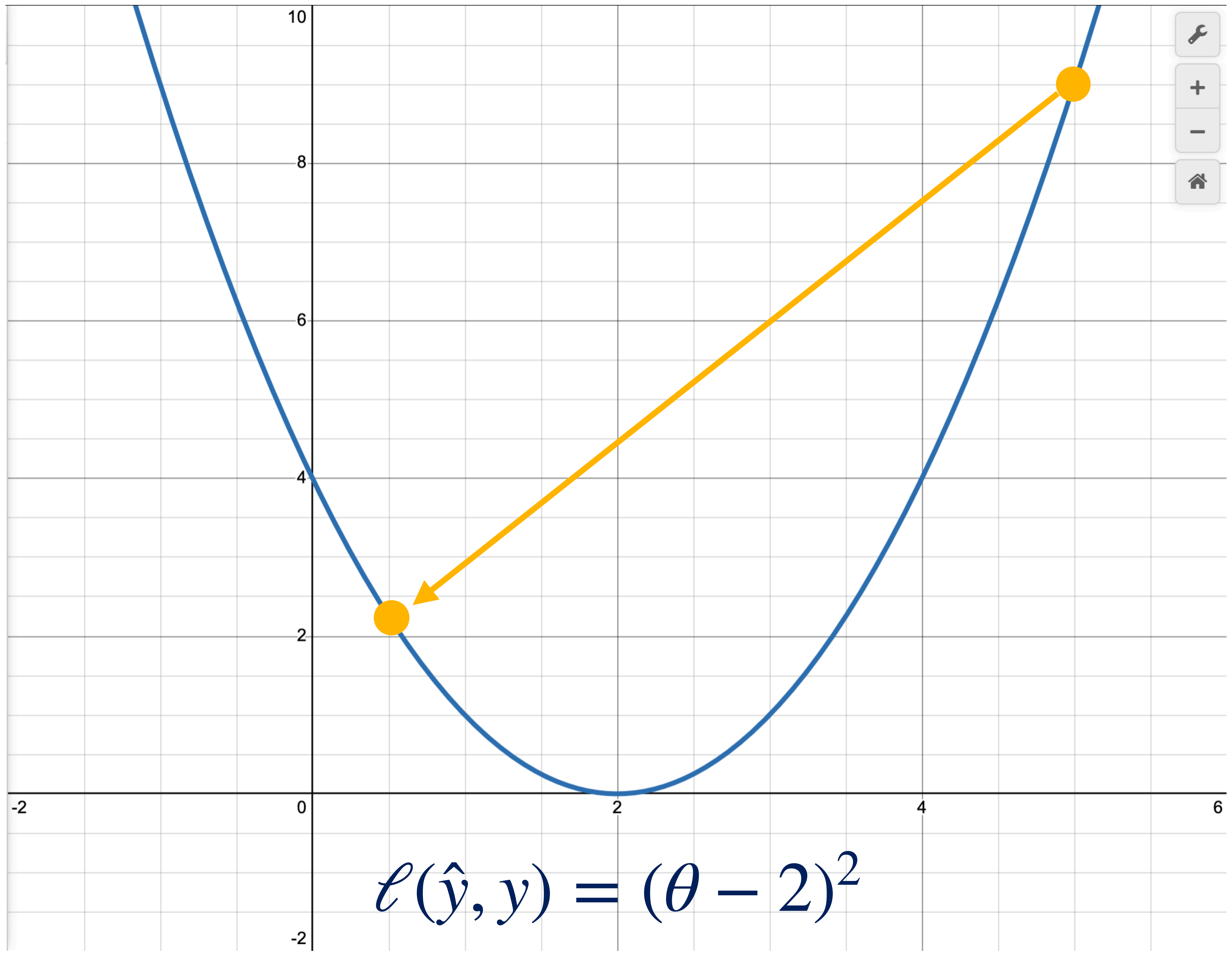
Gradient Descent (second try)



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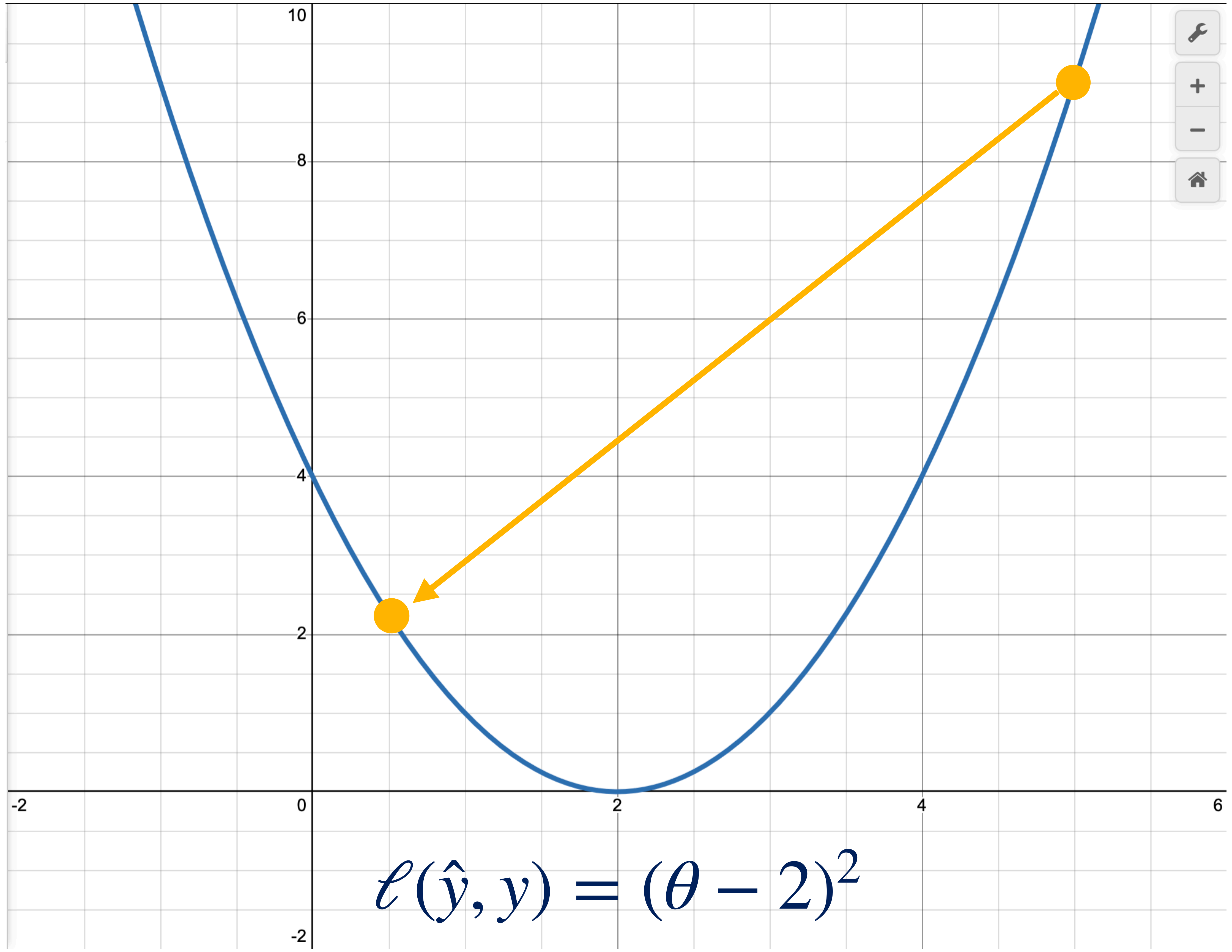
Gradient Descent (second try)

- Second step:



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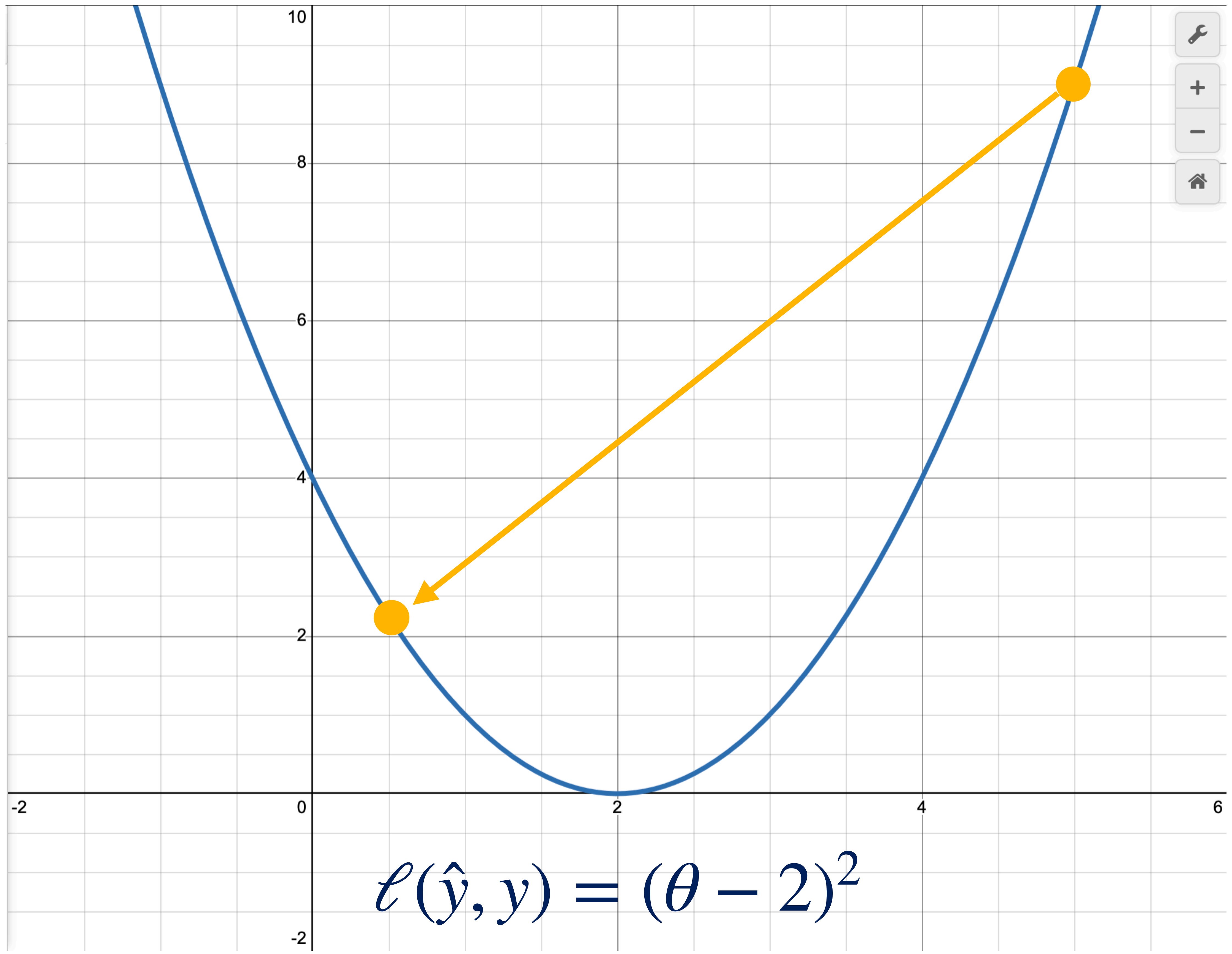
  - Plug in θ_1 to derivative:



$$\ell(\hat{y}, y) = (\theta - 2)^2$$

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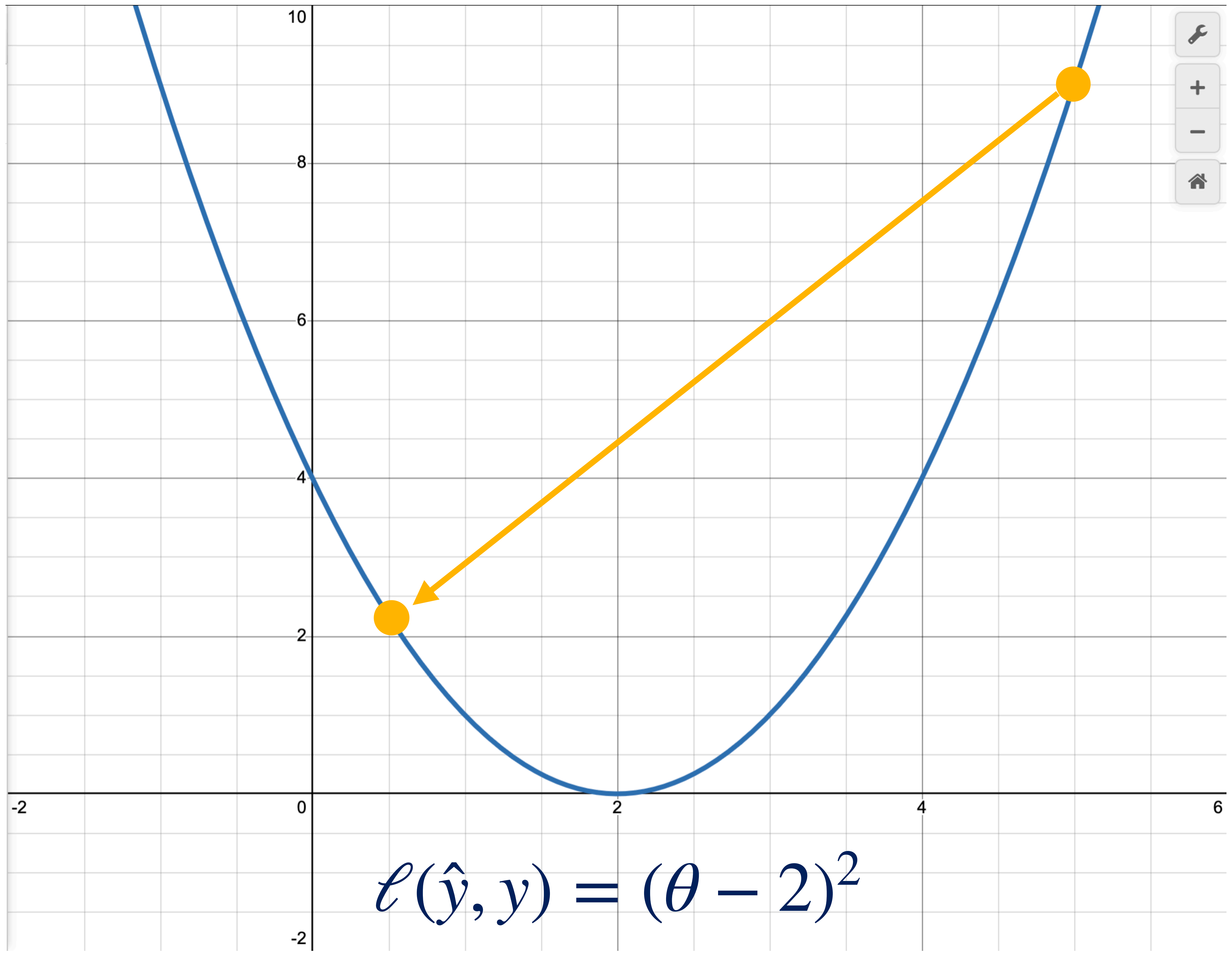
Gradient Descent (second try)

- Second step:
  - Plug in θ_1 to derivative:
    - $2 \cdot 0.5 - 4 = -3$



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- Second step:
  - Plug in θ_1 to derivative:
    - $2 \cdot 0.5 - 4 = -3$
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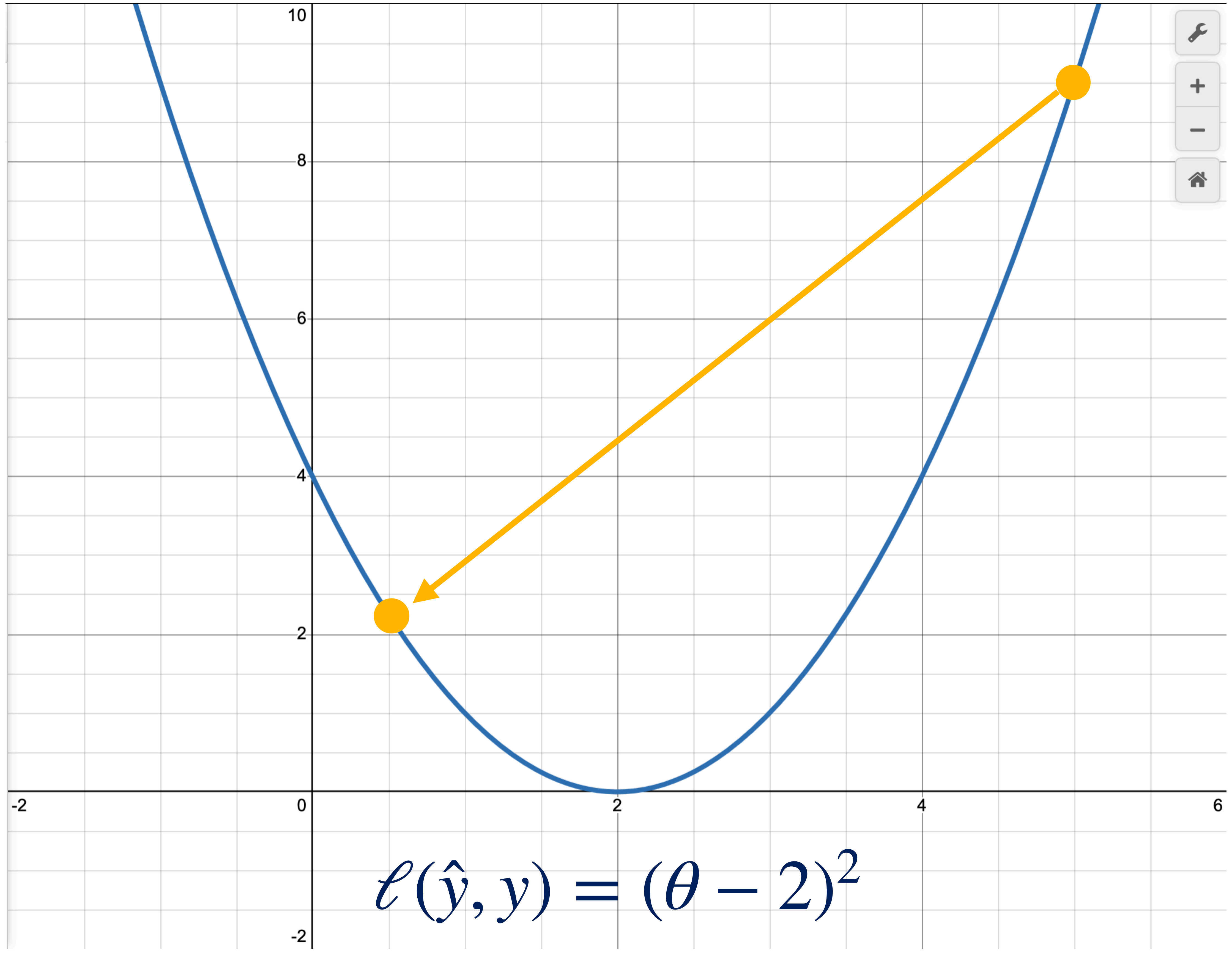

$$\ell(\hat{y}, y) = (\theta - 2)^2$$

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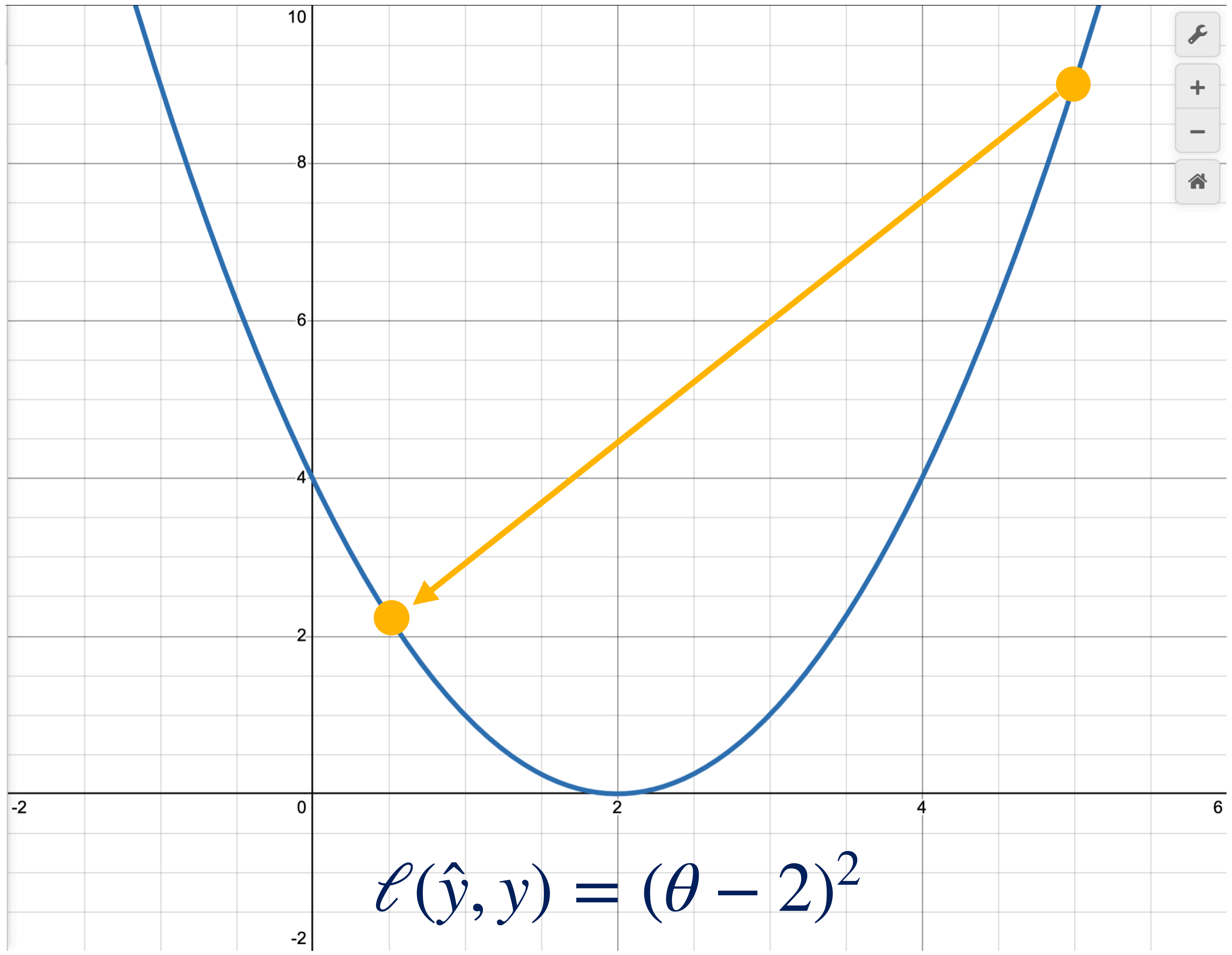
Gradient Descent (second try)

- Second step:
  - Plug in θ_1 to derivative:
    - $2 \cdot 0.5 - 4 = -3$
  - Update θ:
    - $\theta_2 = \theta_1 - 0.75 \cdot (-3) = 2.25$



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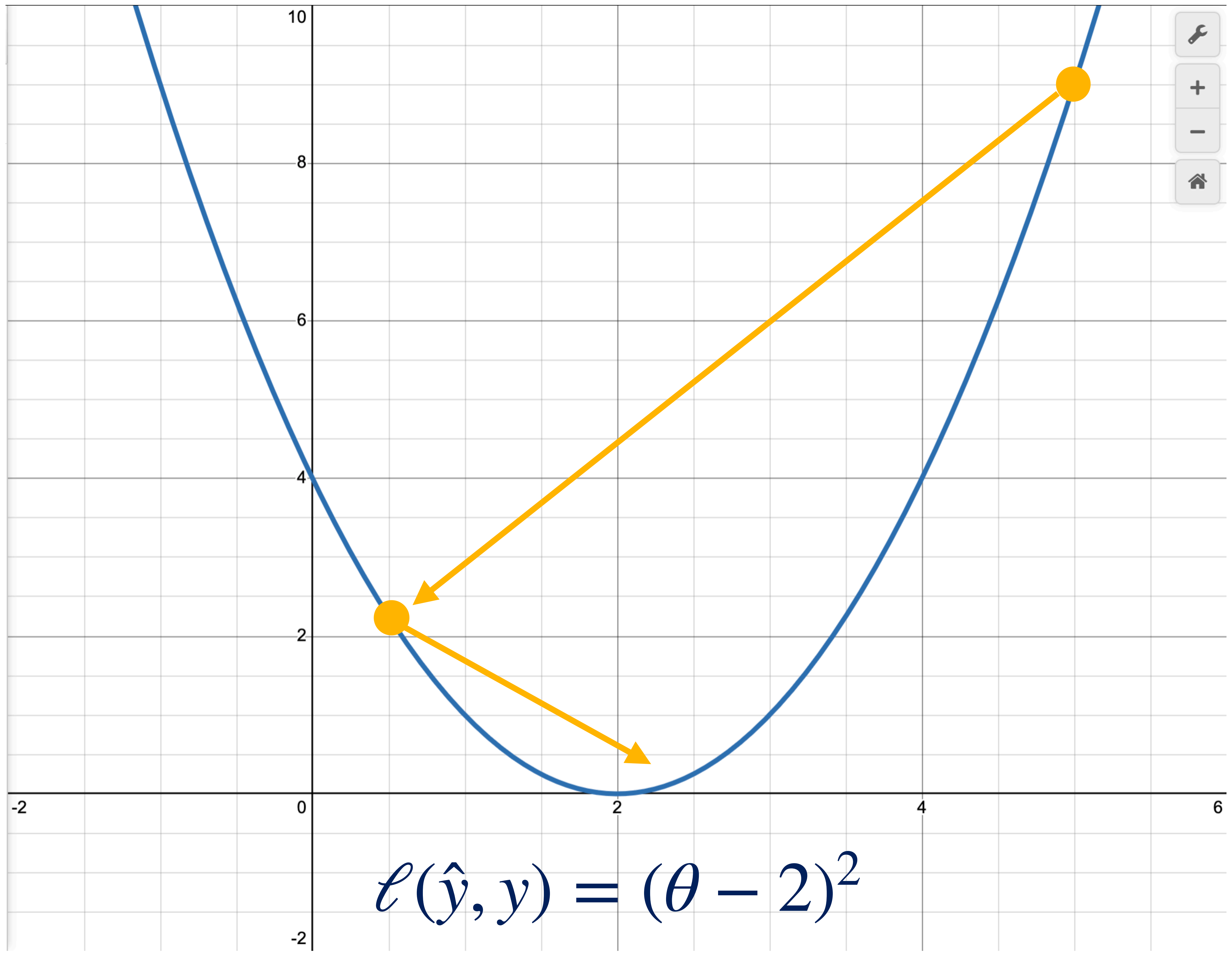

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Gradient Descent (second try)

- Second step:
  - Plug in θ_1 to derivative:
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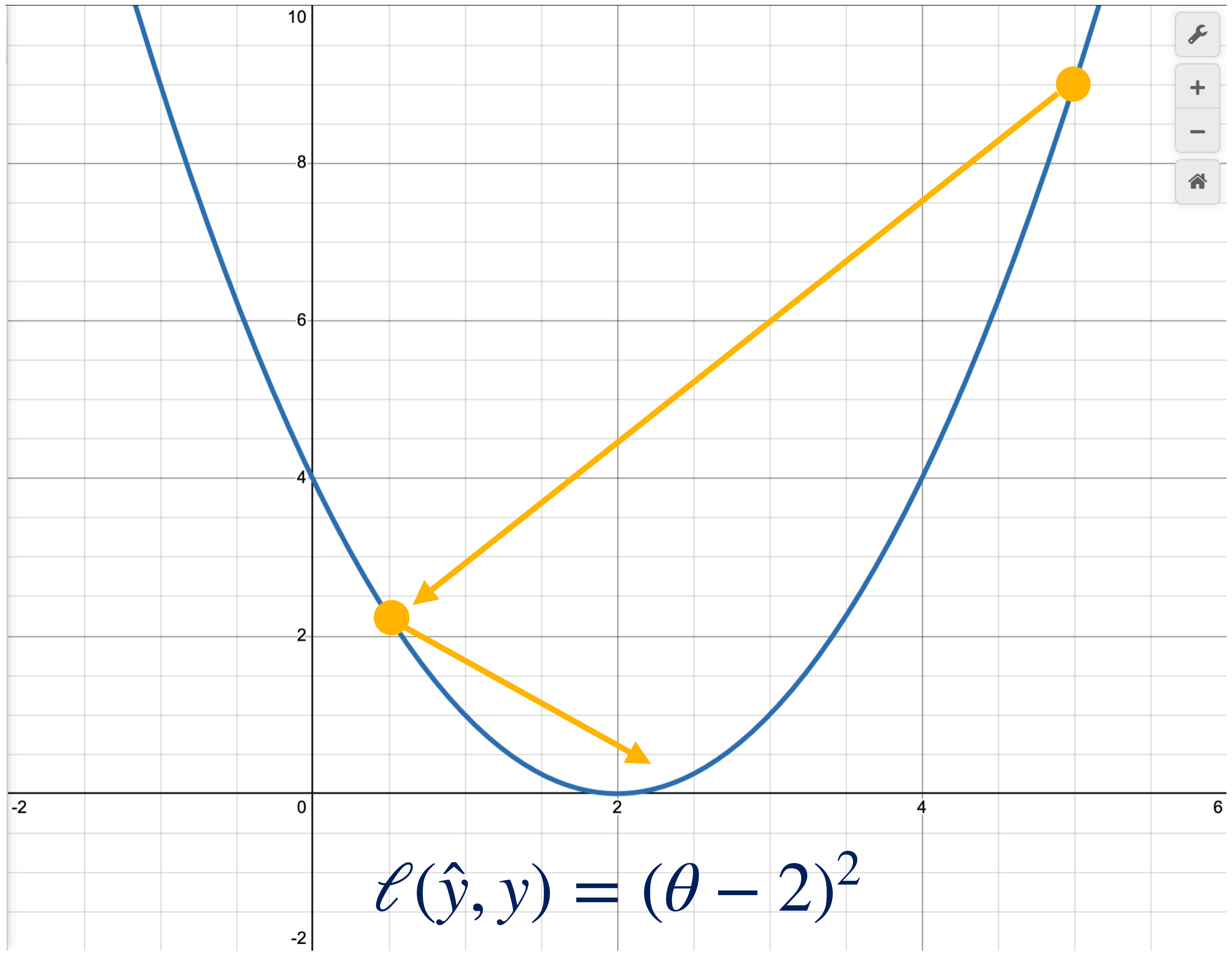
  - Update θ:
    - $\theta_2 = \theta_1 - 0.75 \cdot (-3) = 2.25$
- (This is looking better)



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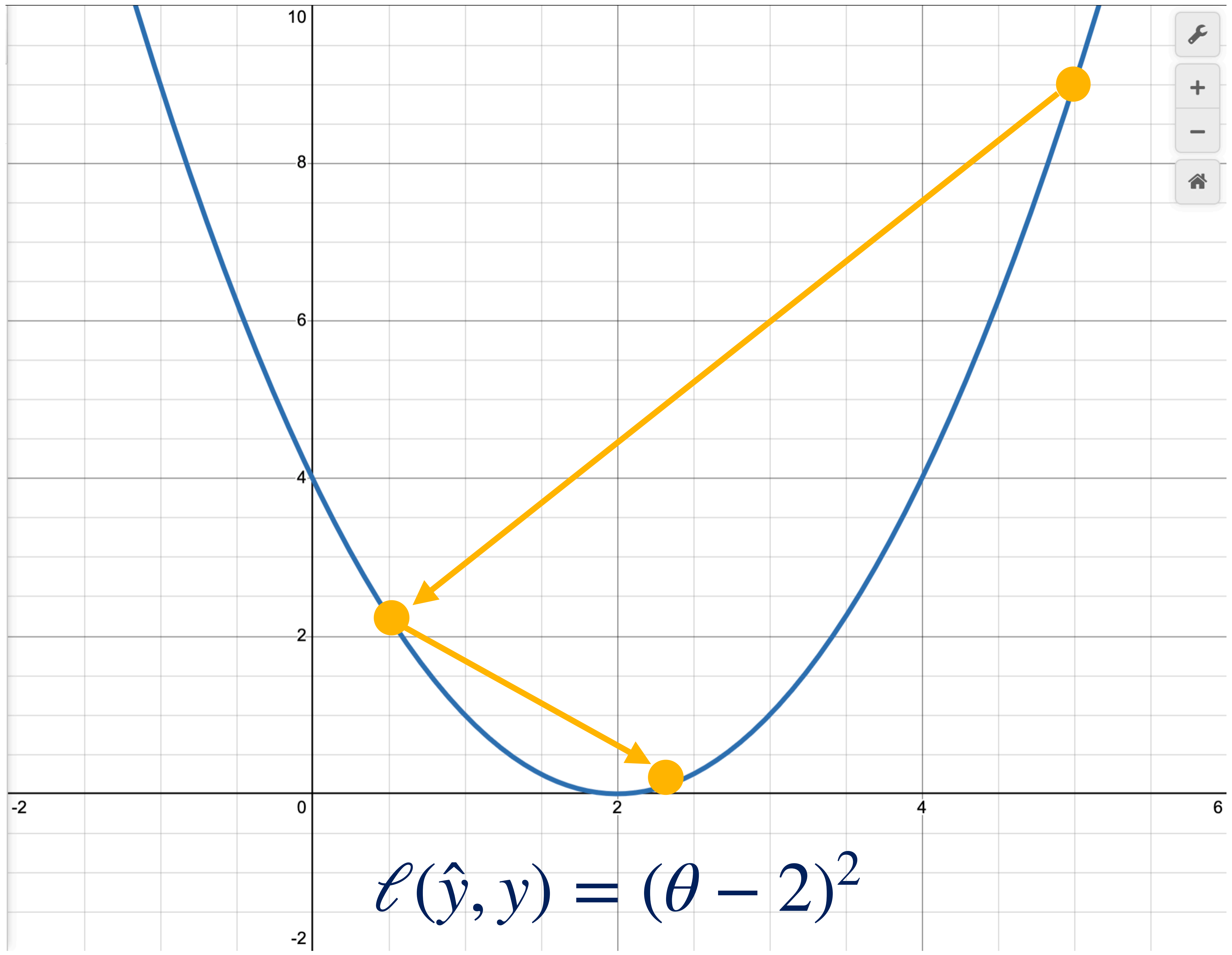
Gradient Descent (second try)



$$\ell(\hat{y}, y) = (\theta - 2)^2$$

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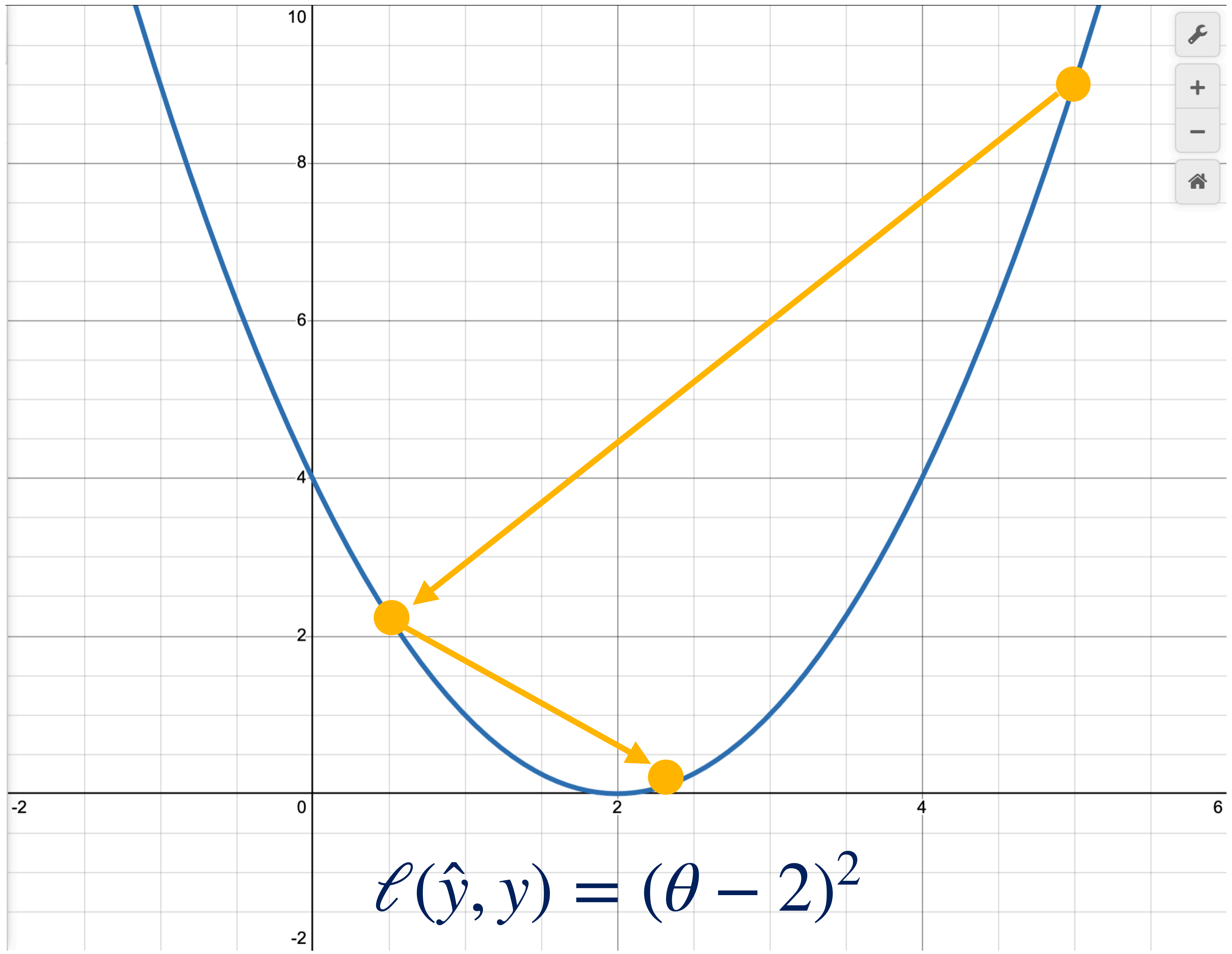
Gradient Descent (second try)

- Third step:
  - Plug in θ_2 to derivative:
    - $2 \cdot 2.25 - 4 = 0.5$
  - Update θ:
    - $\theta_3 = \theta_2 - 0.75 \cdot 0.5 = 1.875$



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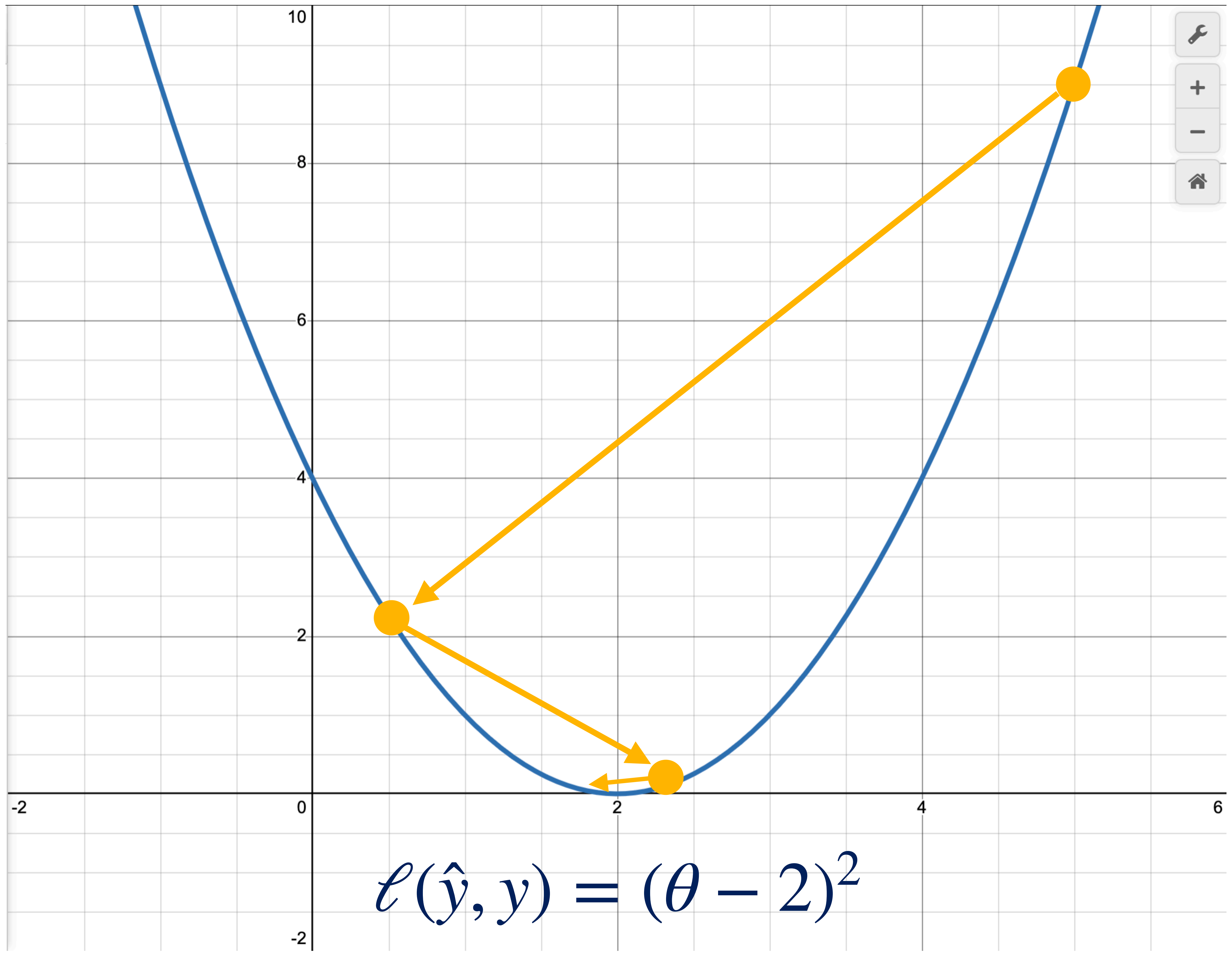

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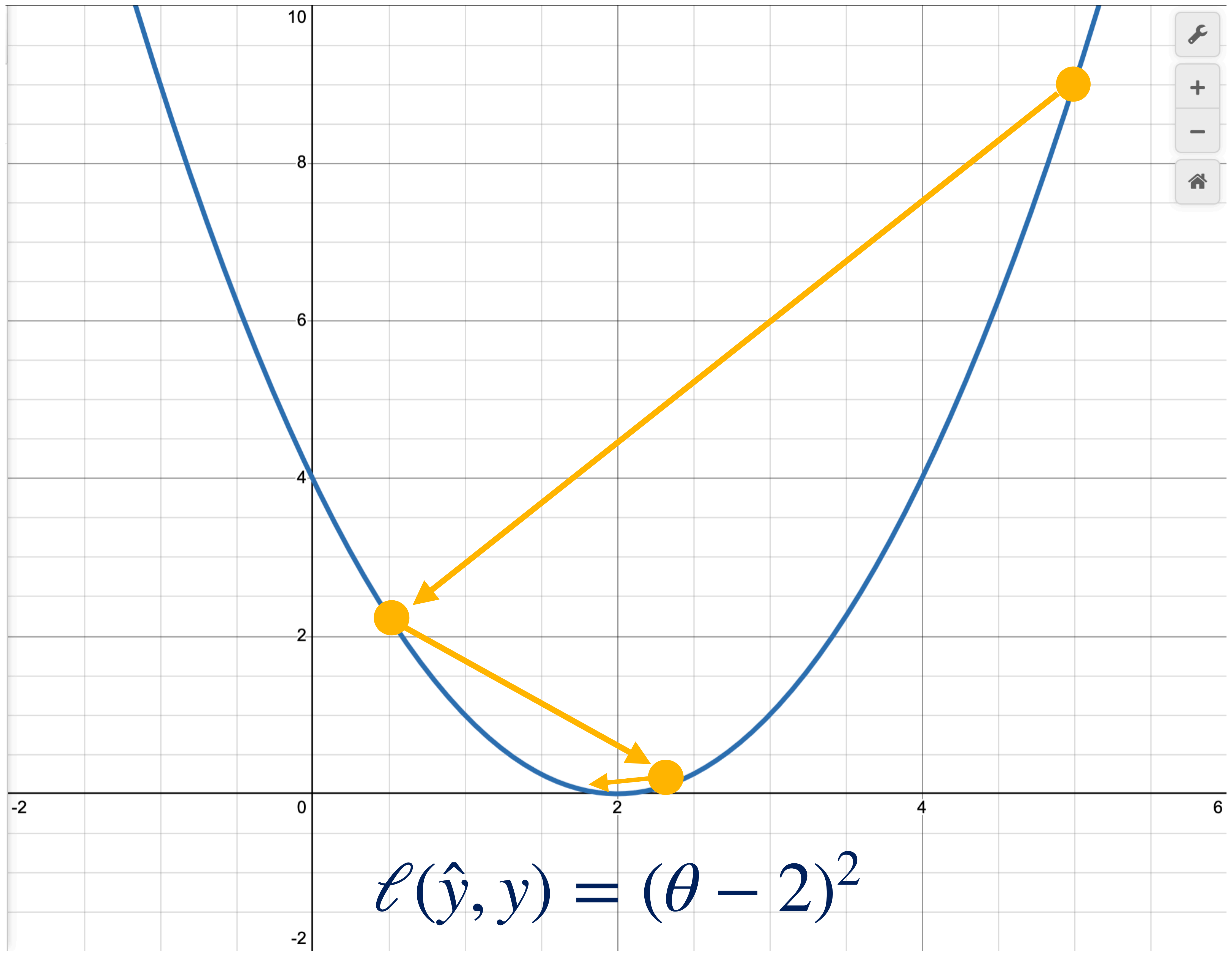
- **Loss gets lower with every step!**



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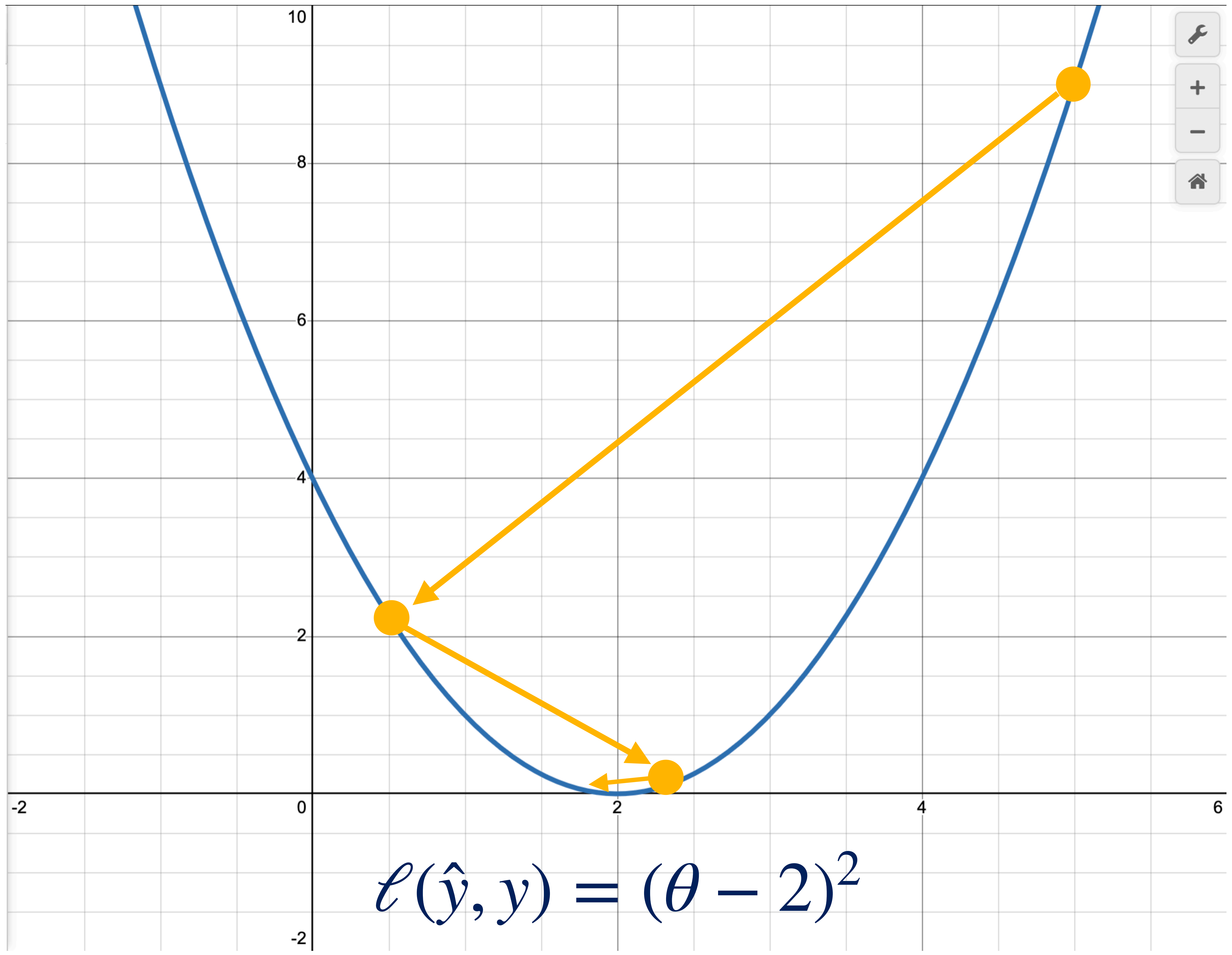
- **Loss gets lower with every step!**
- θ gets **arbitrarily close to the optimal value** with more steps



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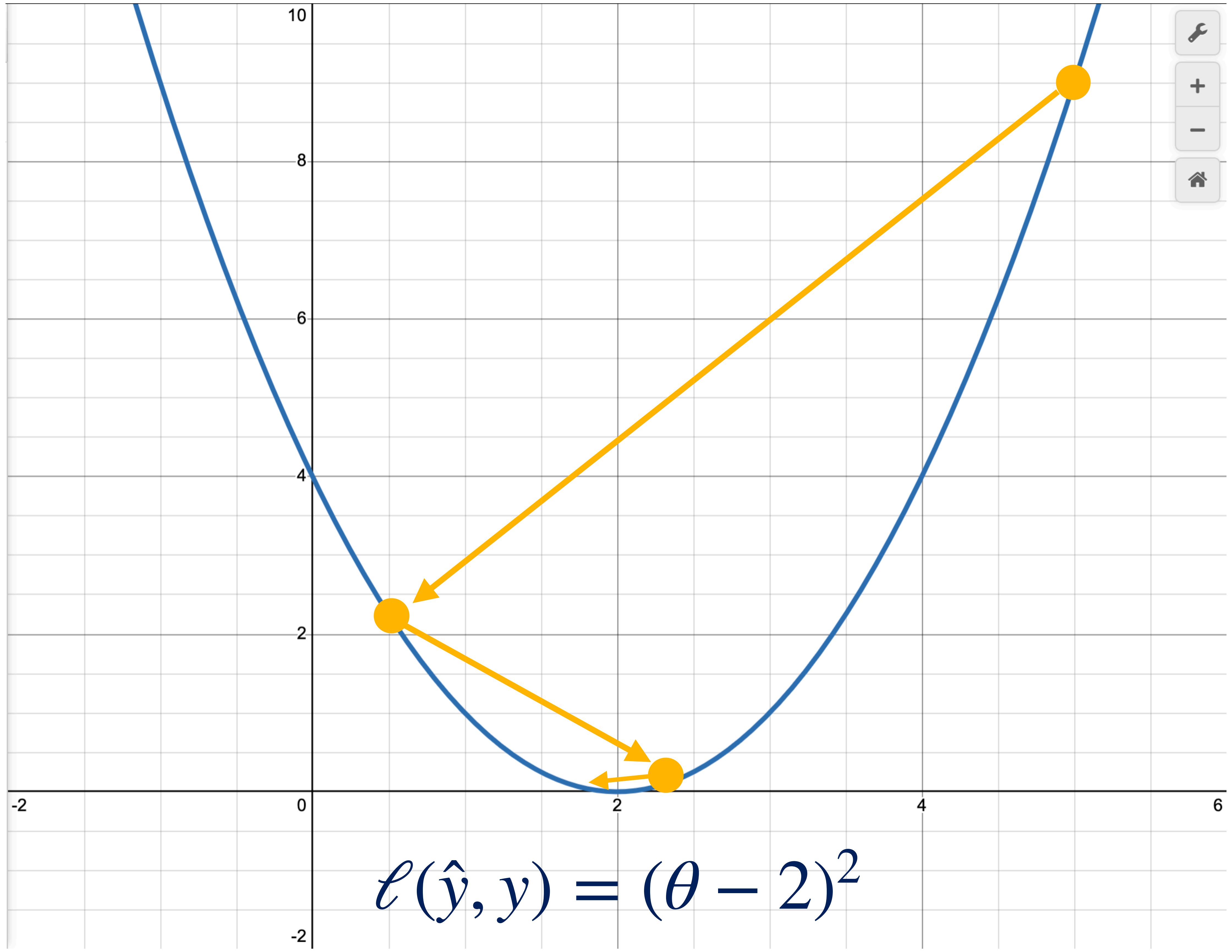
Learning Rate



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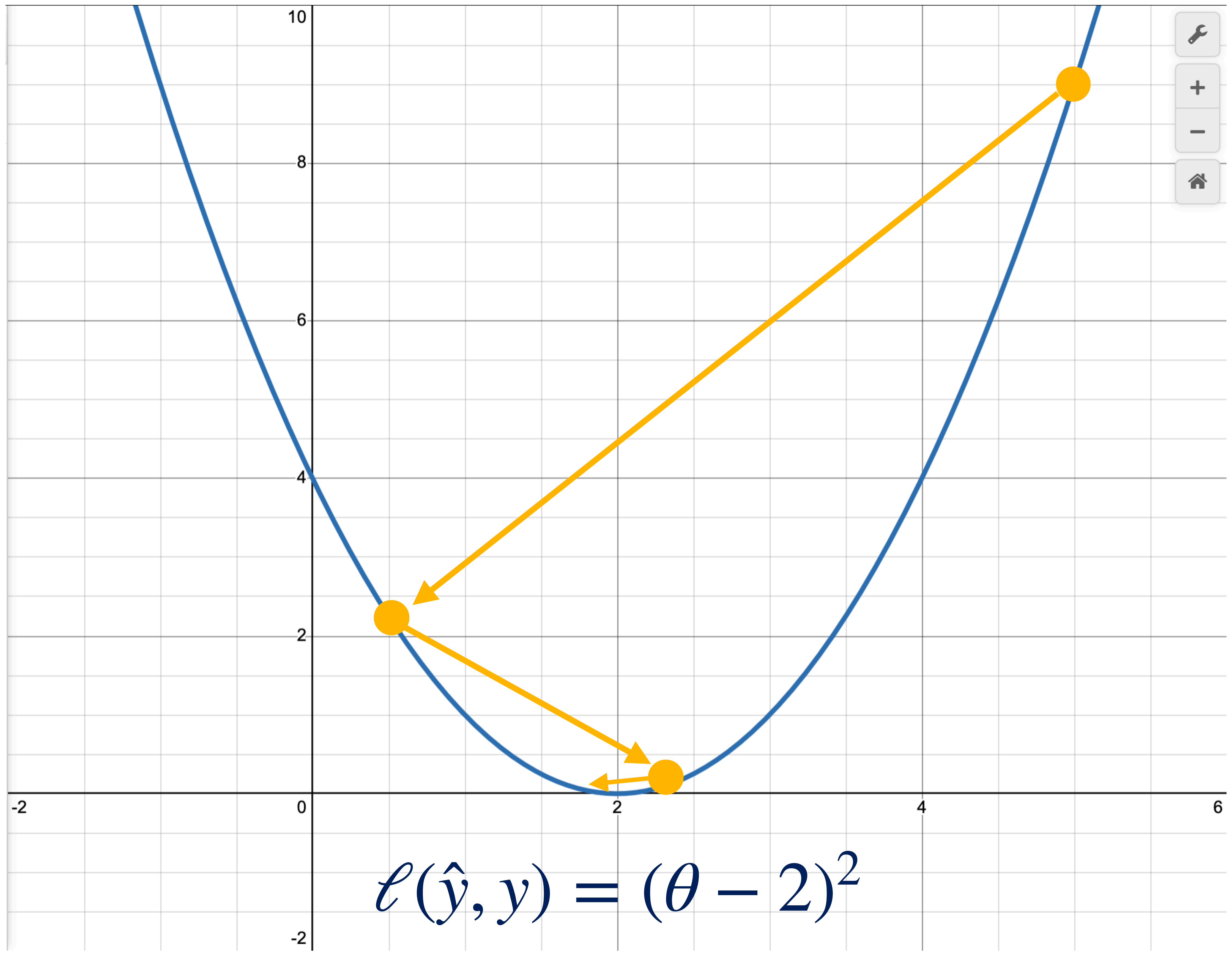
Learning Rate

- The fraction by which we multiplied our adjustment is called the **Learning Rate**



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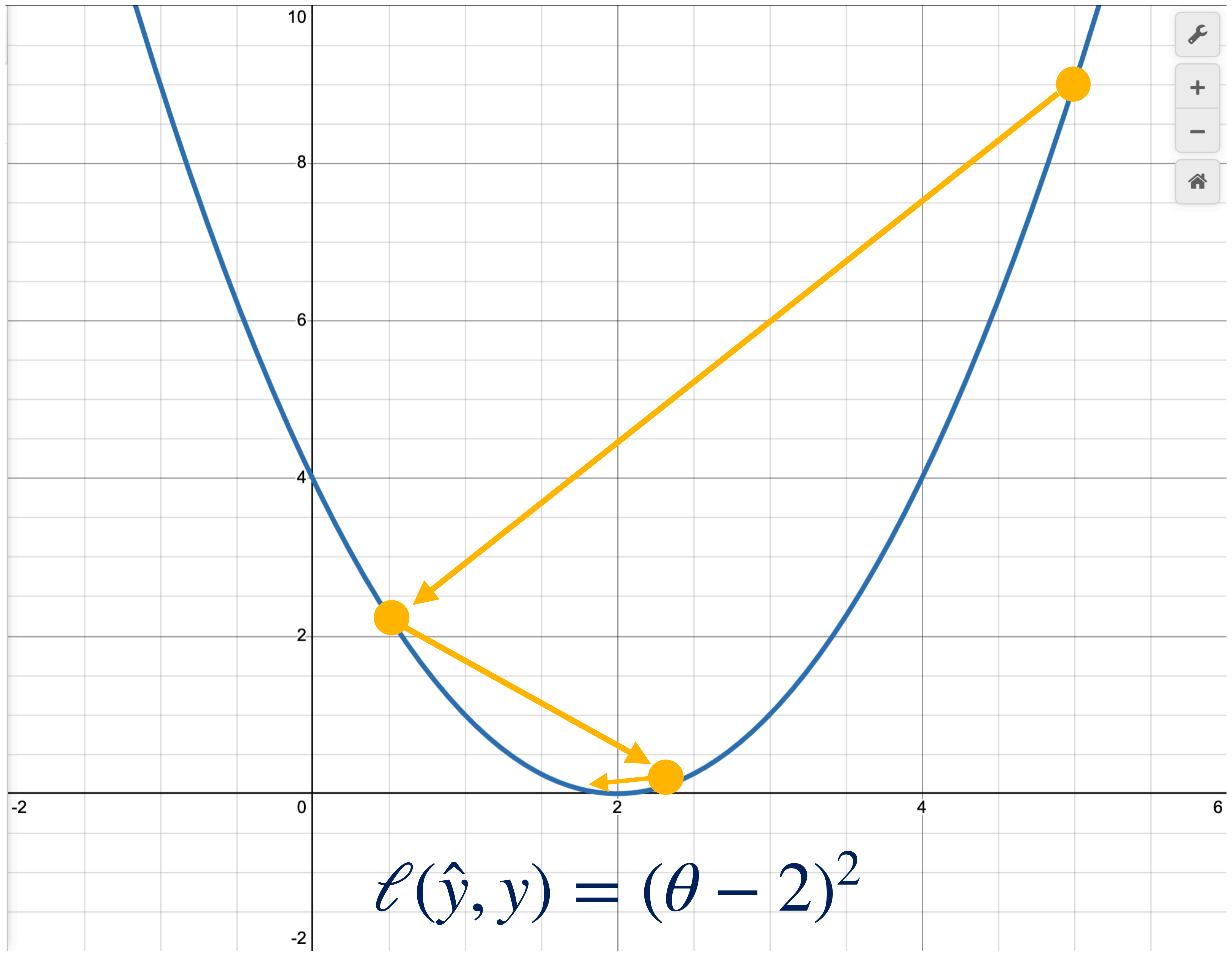
Learning Rate

- The fraction by which we multiplied our adjustment is called the **Learning Rate**
- In practice, LR is chosen by **trial and error** (called "tuning")



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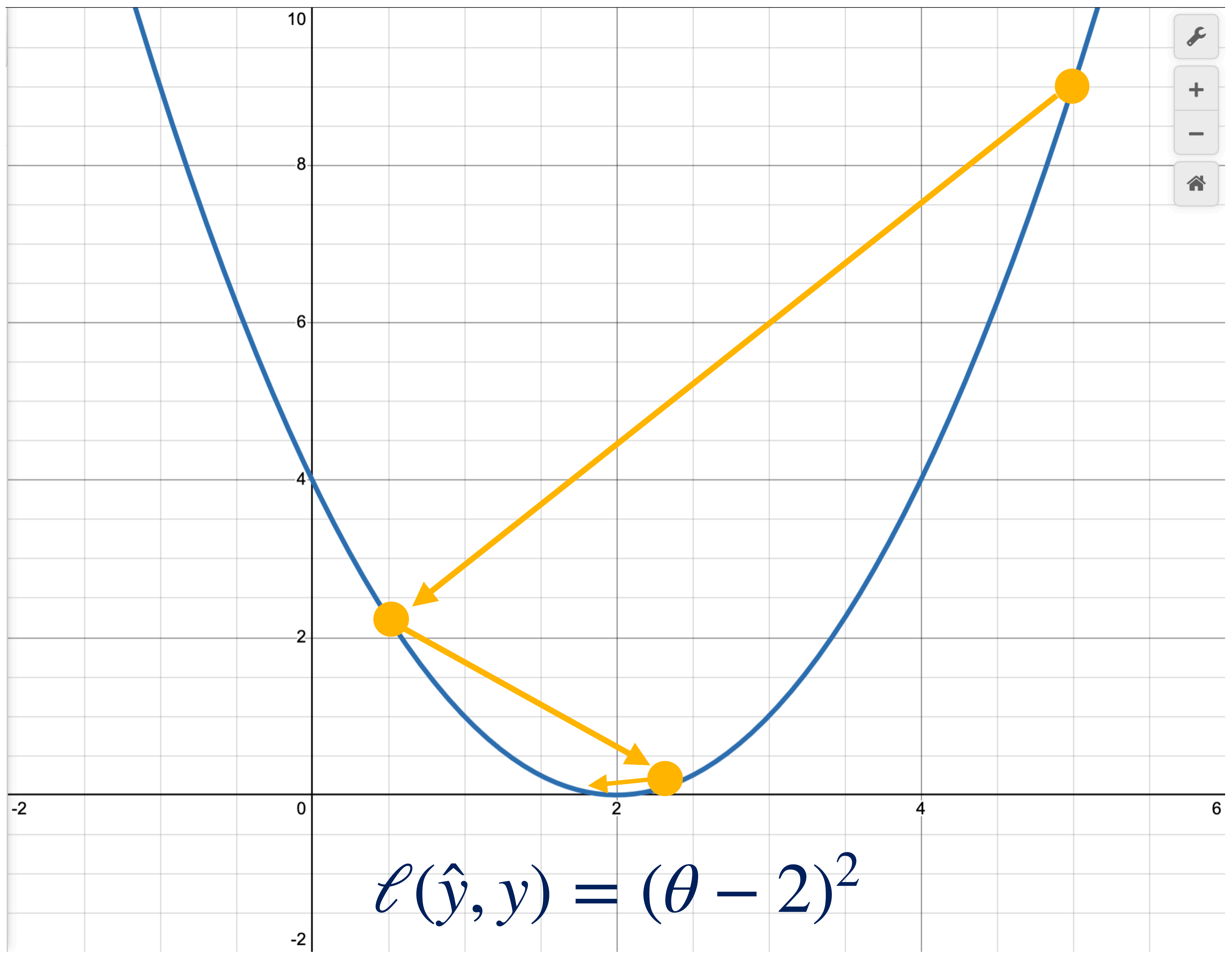
Learning Rate

- The fraction by which we multiplied our adjustment is called the **Learning Rate**
- In practice, LR is chosen by **trial and error** (called "tuning")
- Risks of different values
  - Too high → **"bouncing around"** and missing an optimum
  - Too low → taking **many steps** to reach the optimum



$$\ell(\hat{y}, y) = (\theta - 2)^2$$

$$\ell'(\hat{y}, y) = 2\theta - 4$$

Noisy Secret Number Game

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- Unlike the previous example, we need to **define the loss function** over the **entire dataset**

Global loss function

$$\mathscr{L}(f(X,\theta),Y)=\frac{1}{N}\sum_{i=1}^{N}\ell(f(x_i,\theta),y_i)$$

loss over **all datapoints**

average

loss for a **single datapoint**

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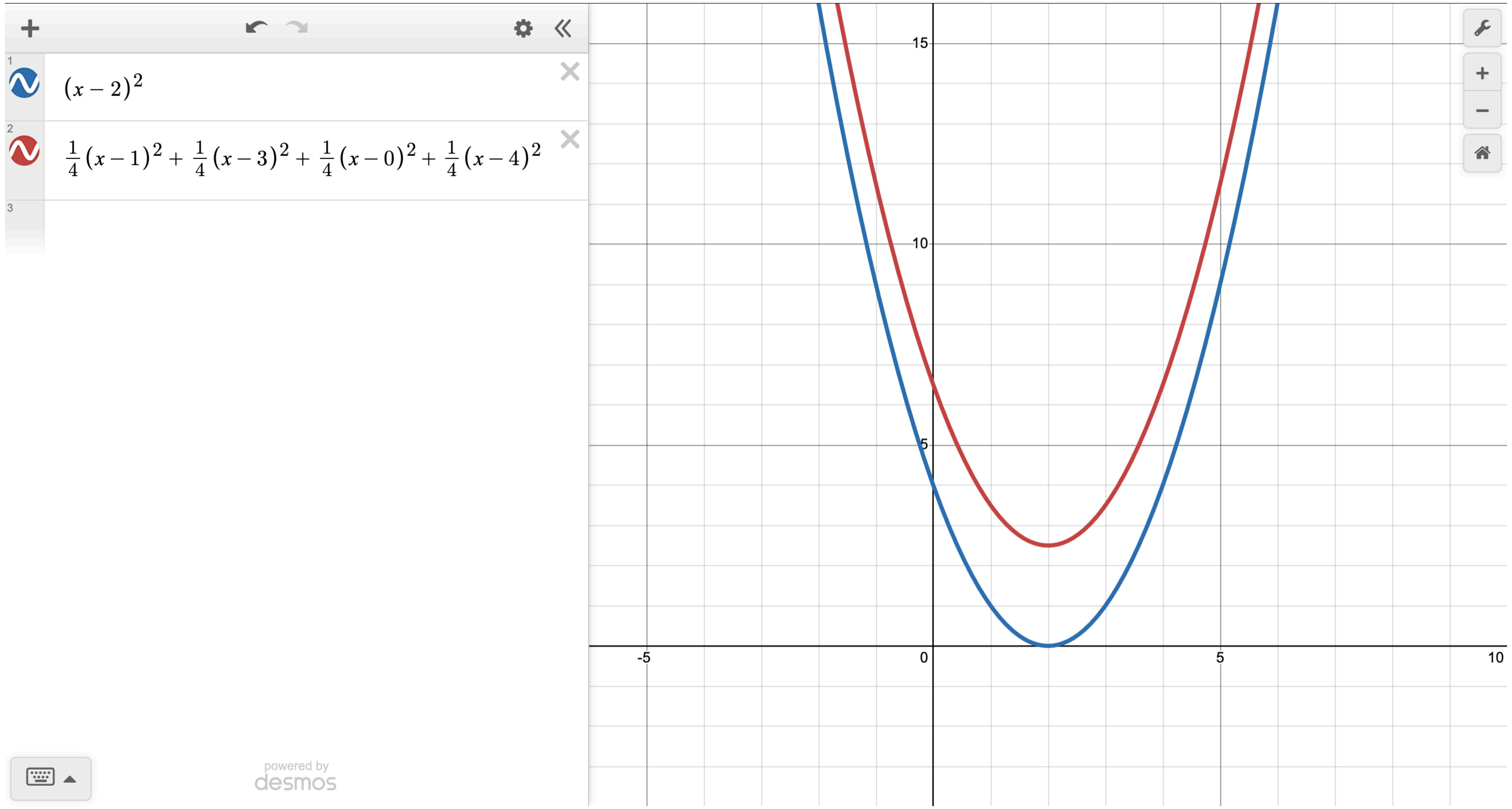
- **Any guesses** what the optimal value of θ is for our **noisy dataset?**
  - $D = \{(2, 3), (3, 6), (5, 5), (8, 12)\}$
  - Hint, look at the **difference between each pair**

Batch Gradient Descent

- The optimal θ is still 2! (The **average** input/output difference)
- Note that the **optimal loss is > 0** (i.e. there is some error left over!)



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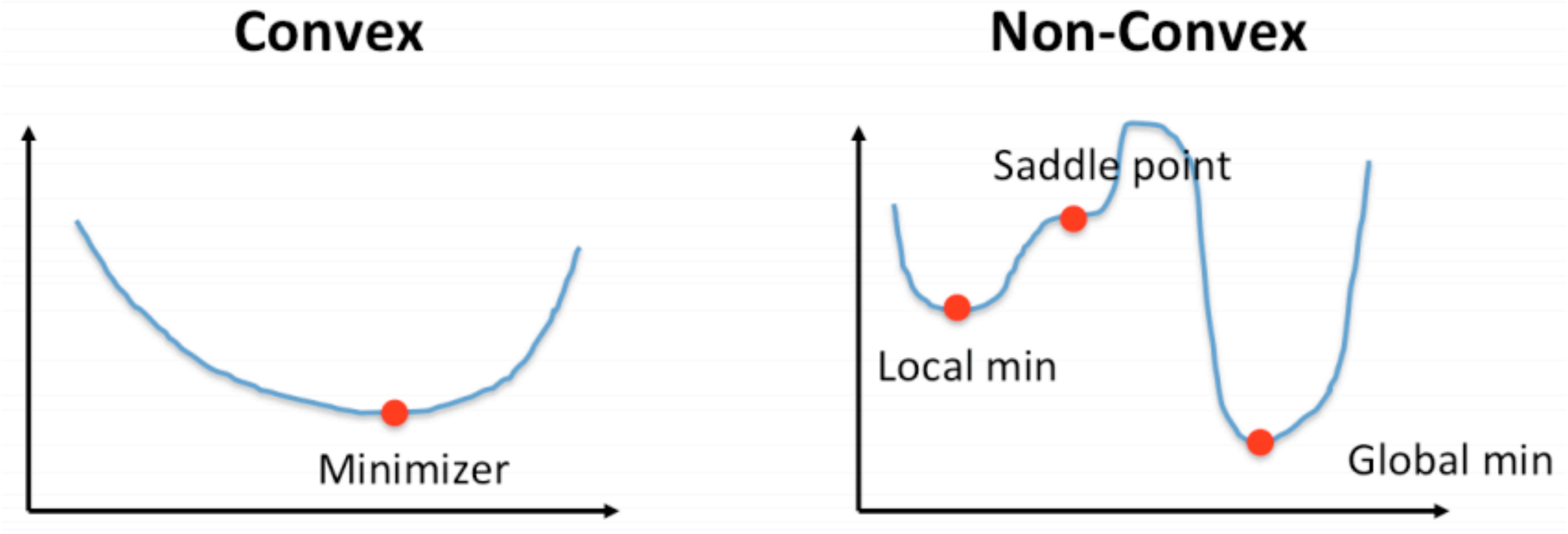
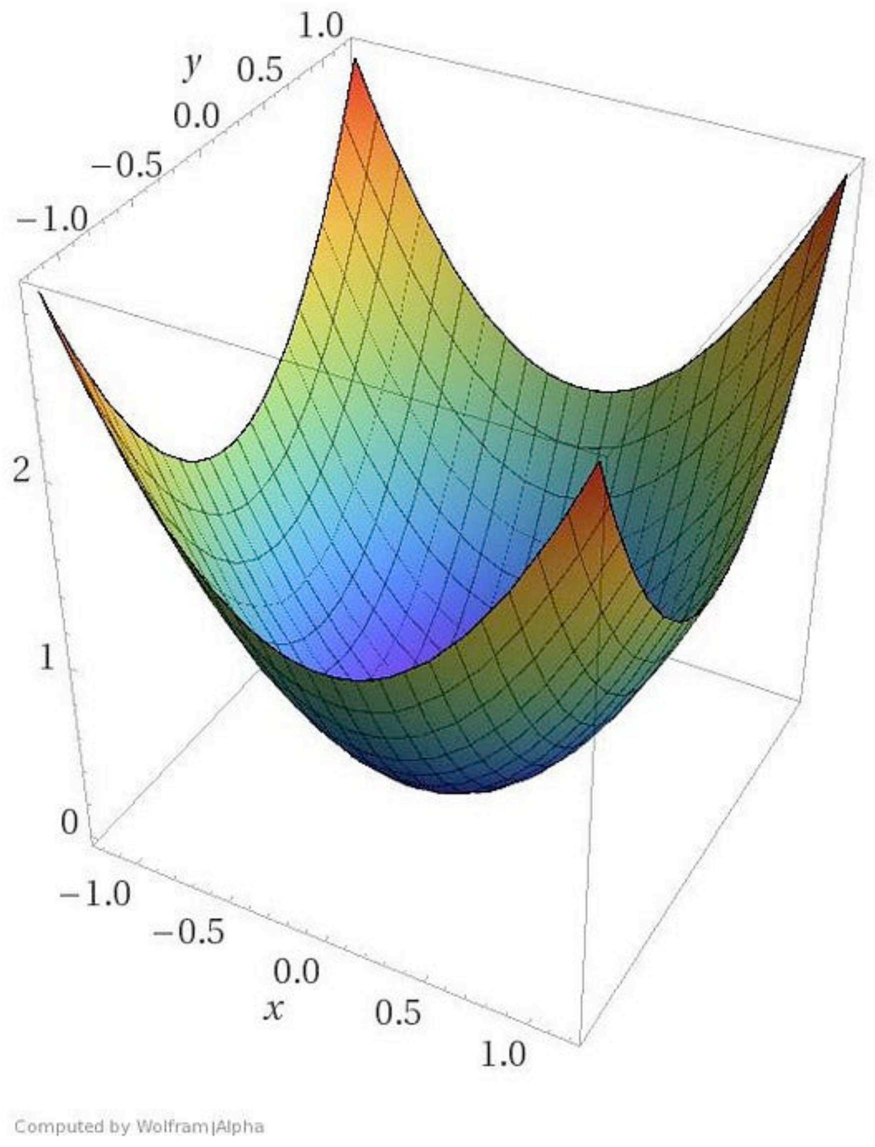
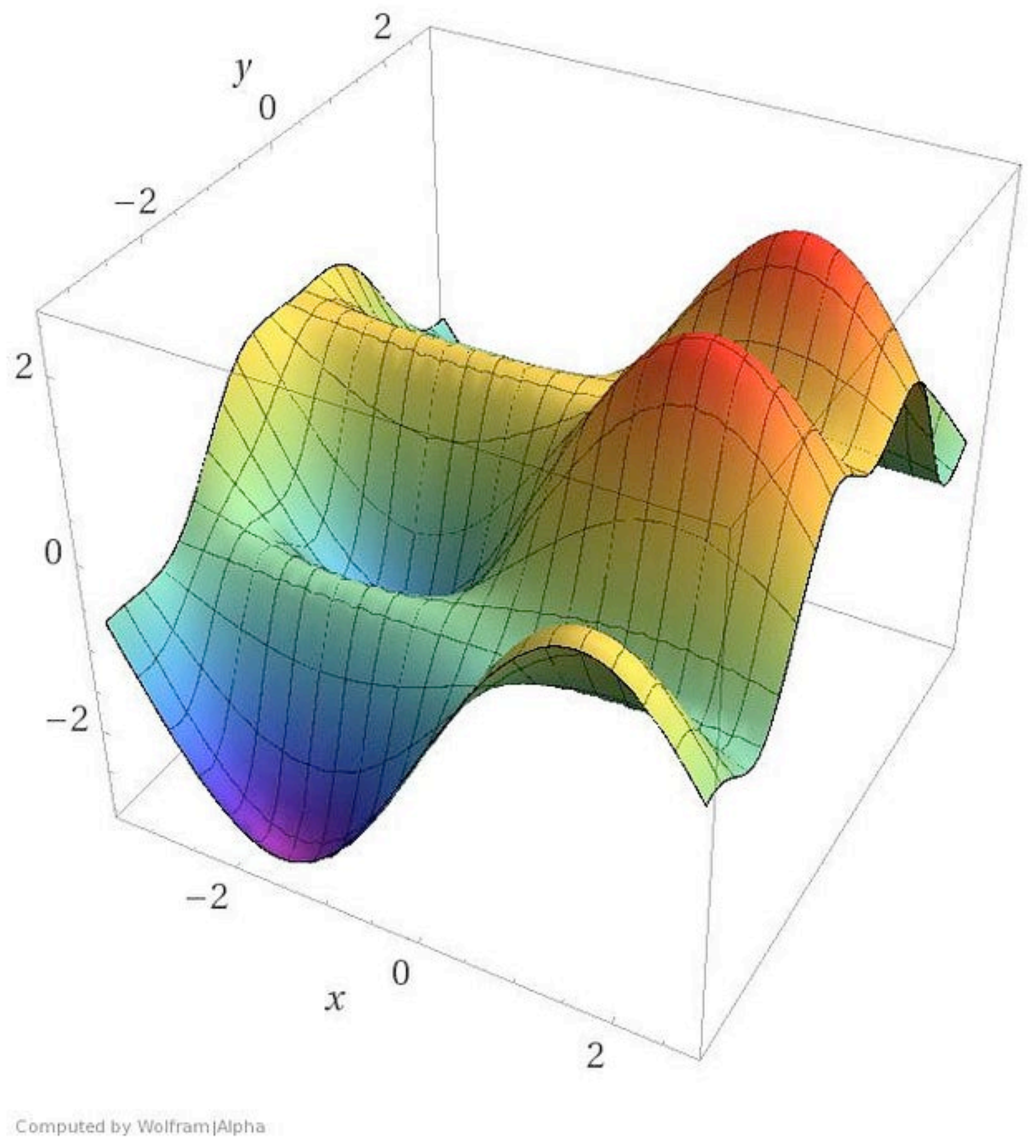
  - You can have **as many parameters as you want!**

Convex vs. Non-convex

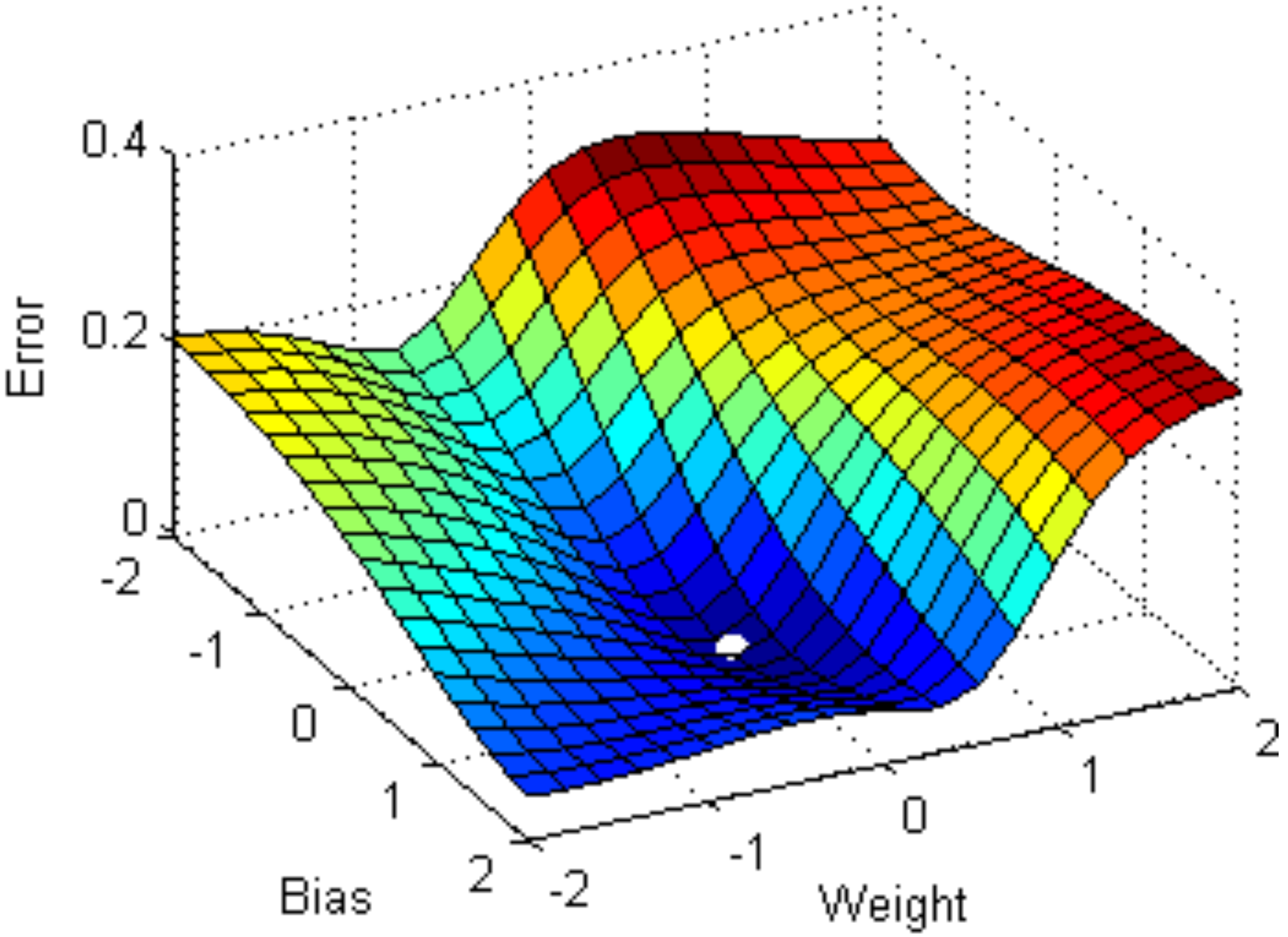






Multi-variable Functions / Gradients

Gradient Descent: Intuition



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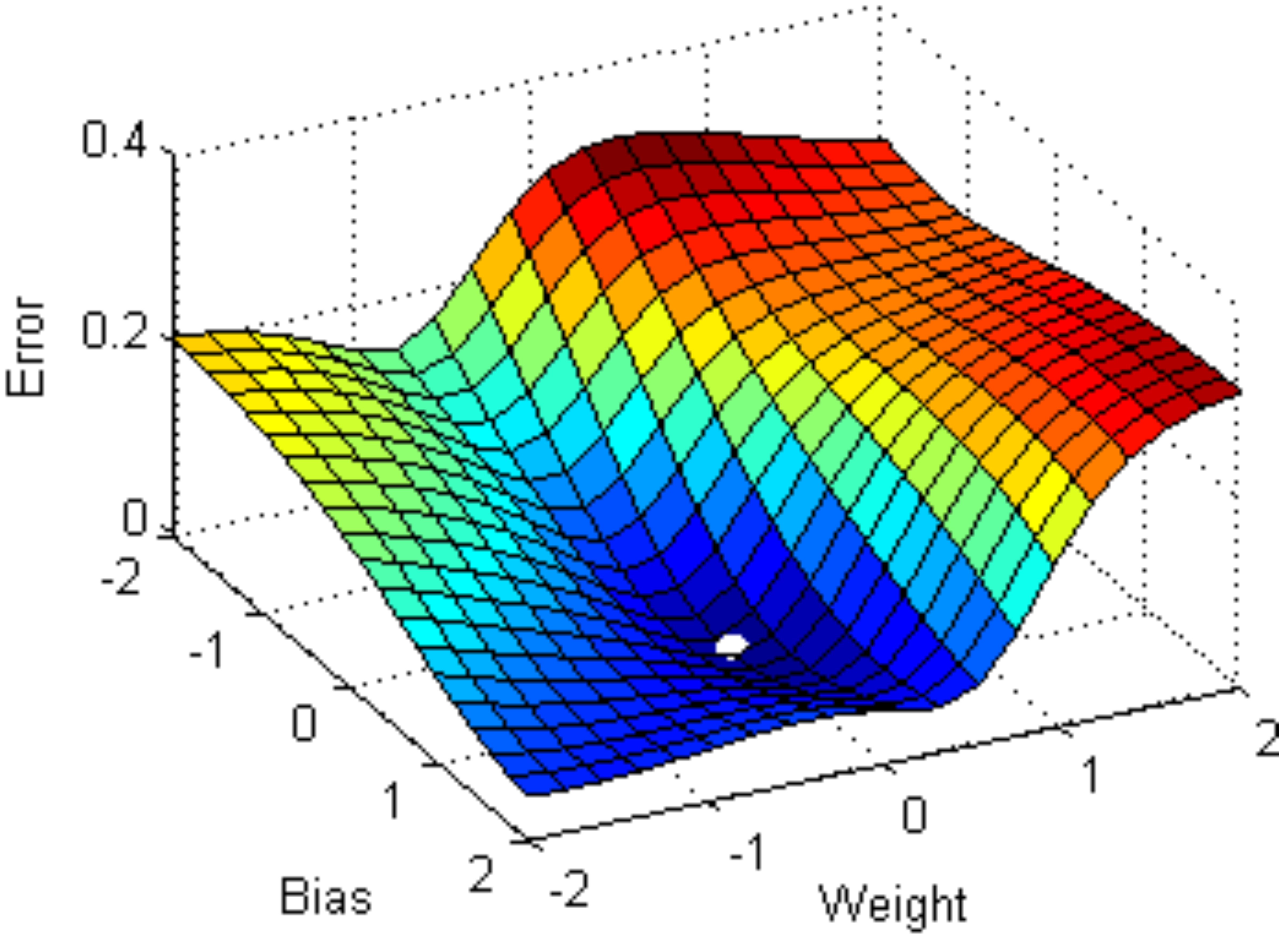

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"derivative of f with respect to y"

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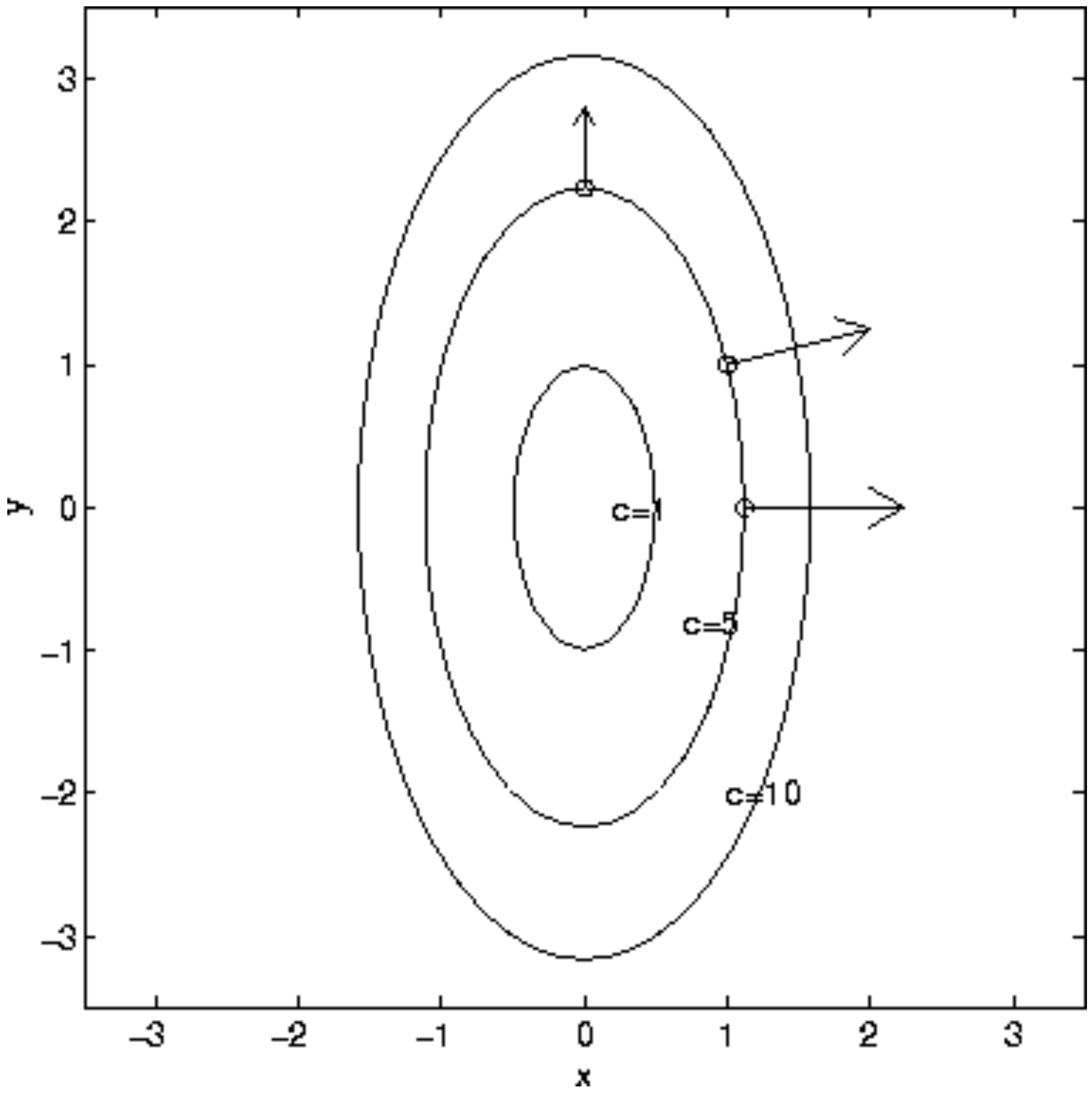
- The gradient points in the direction of **greatest increase** of f

Gradient and Level Curves



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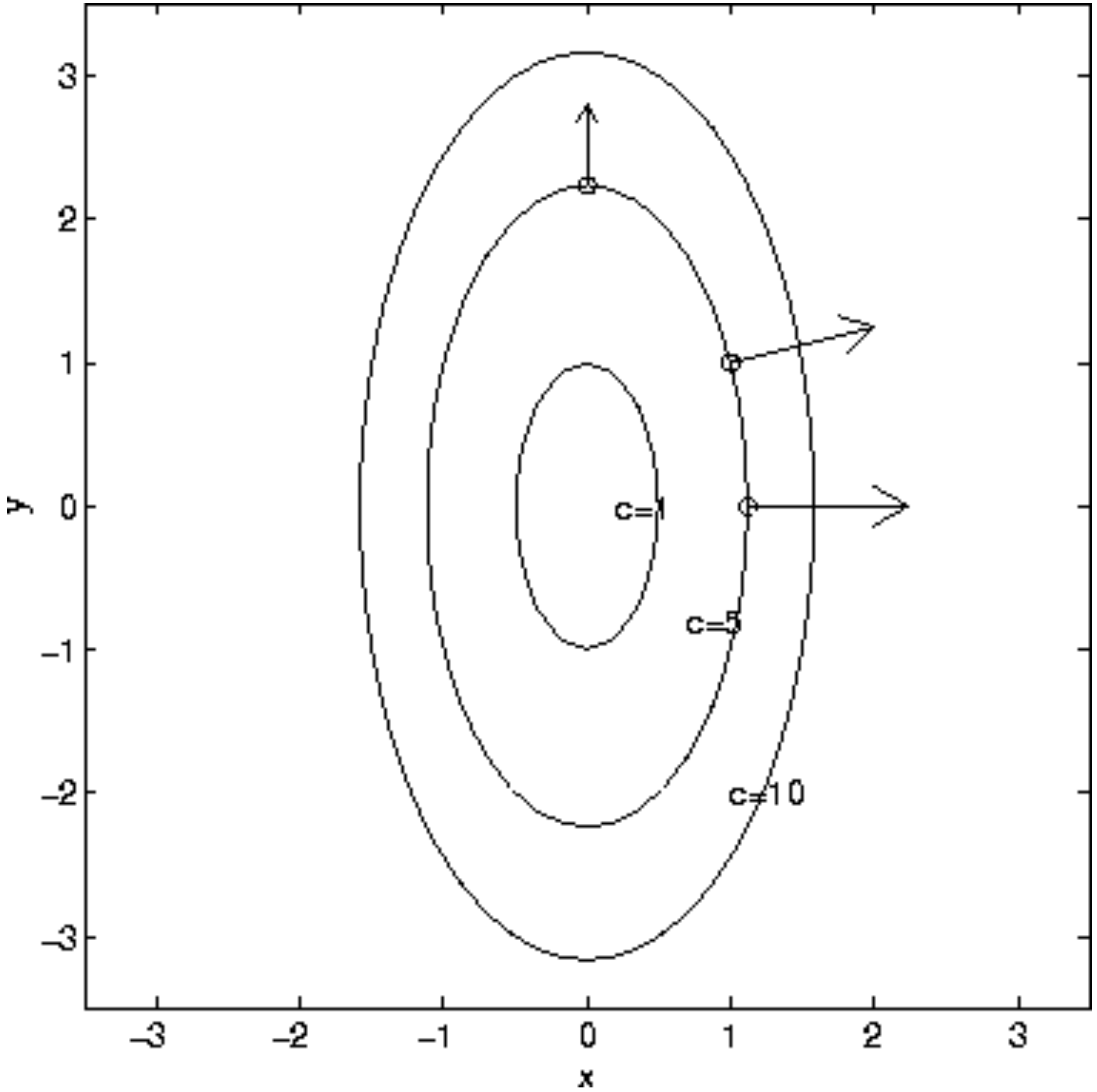

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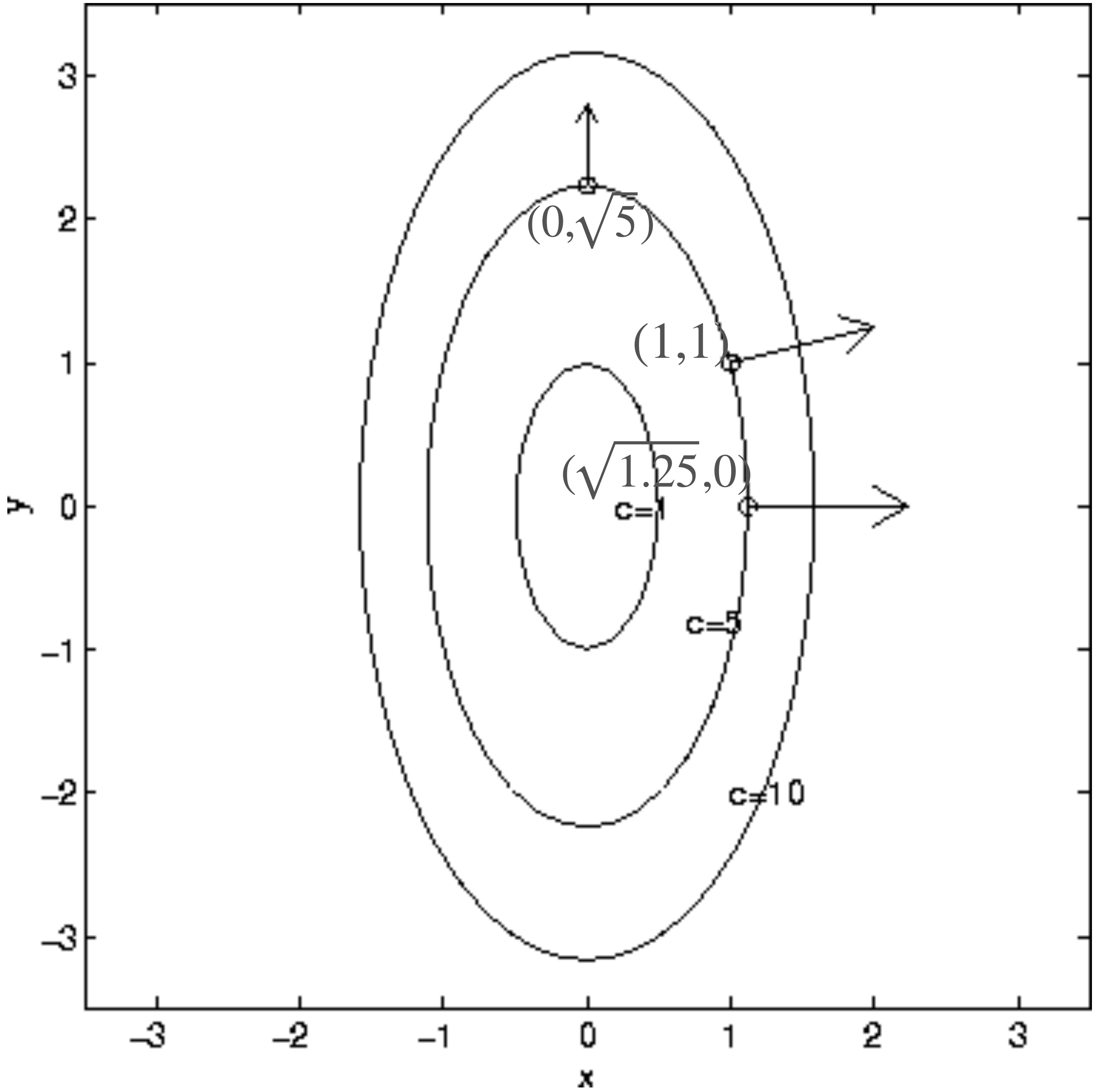

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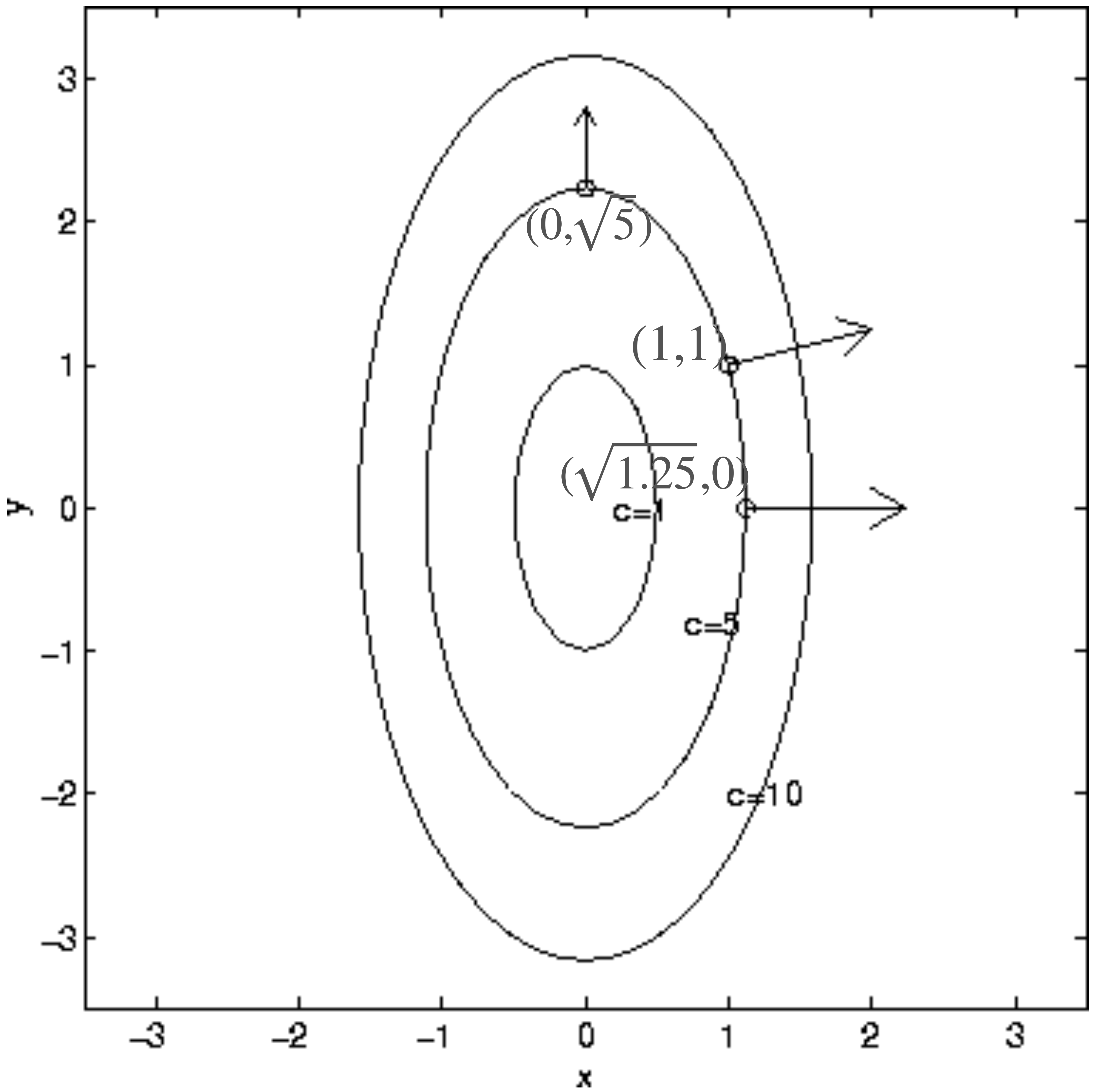

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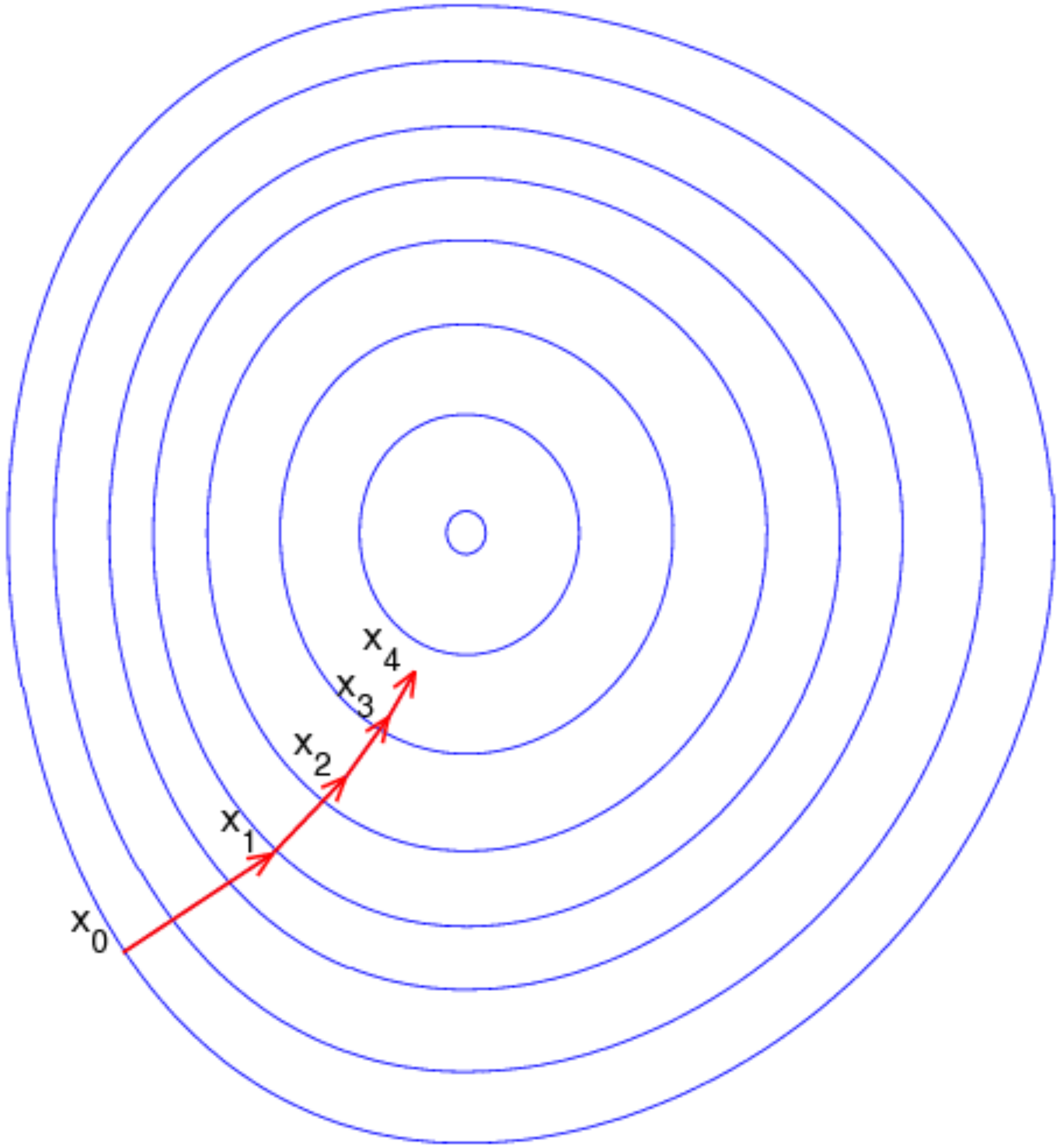
Q: what are the actual gradients at those points?

Gradient Descent and Level Curves



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- Initialize θ_0
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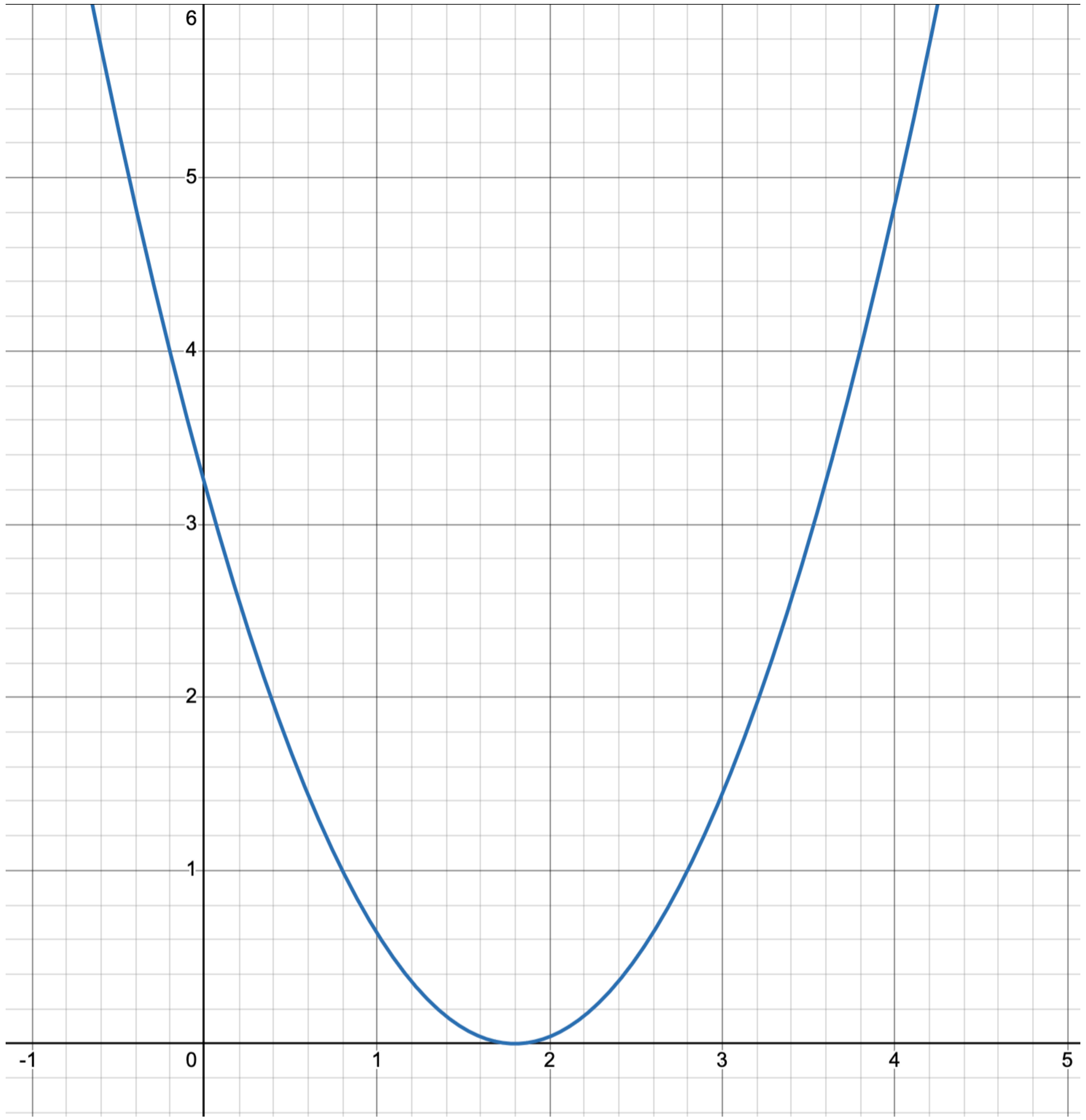

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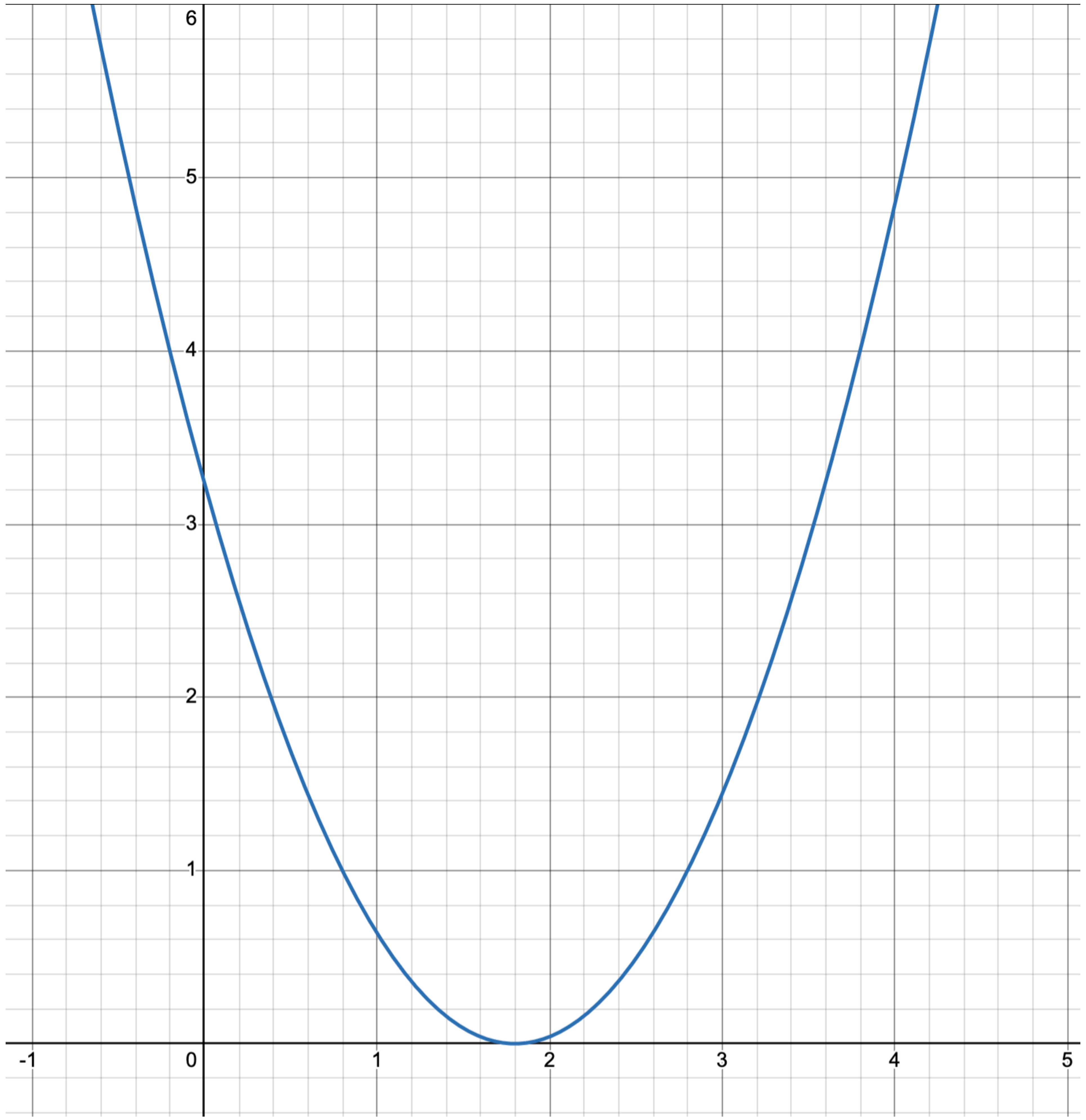

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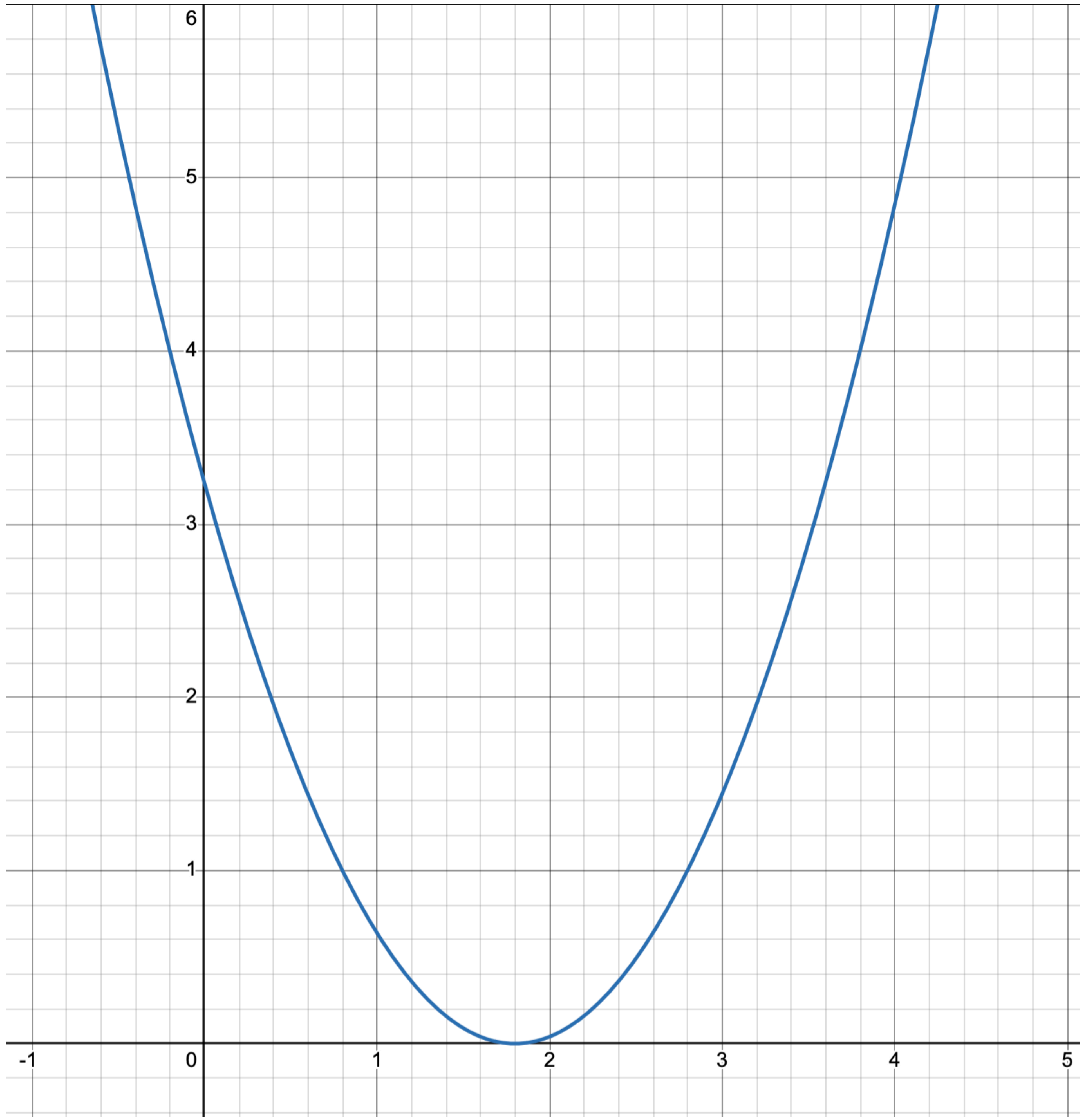

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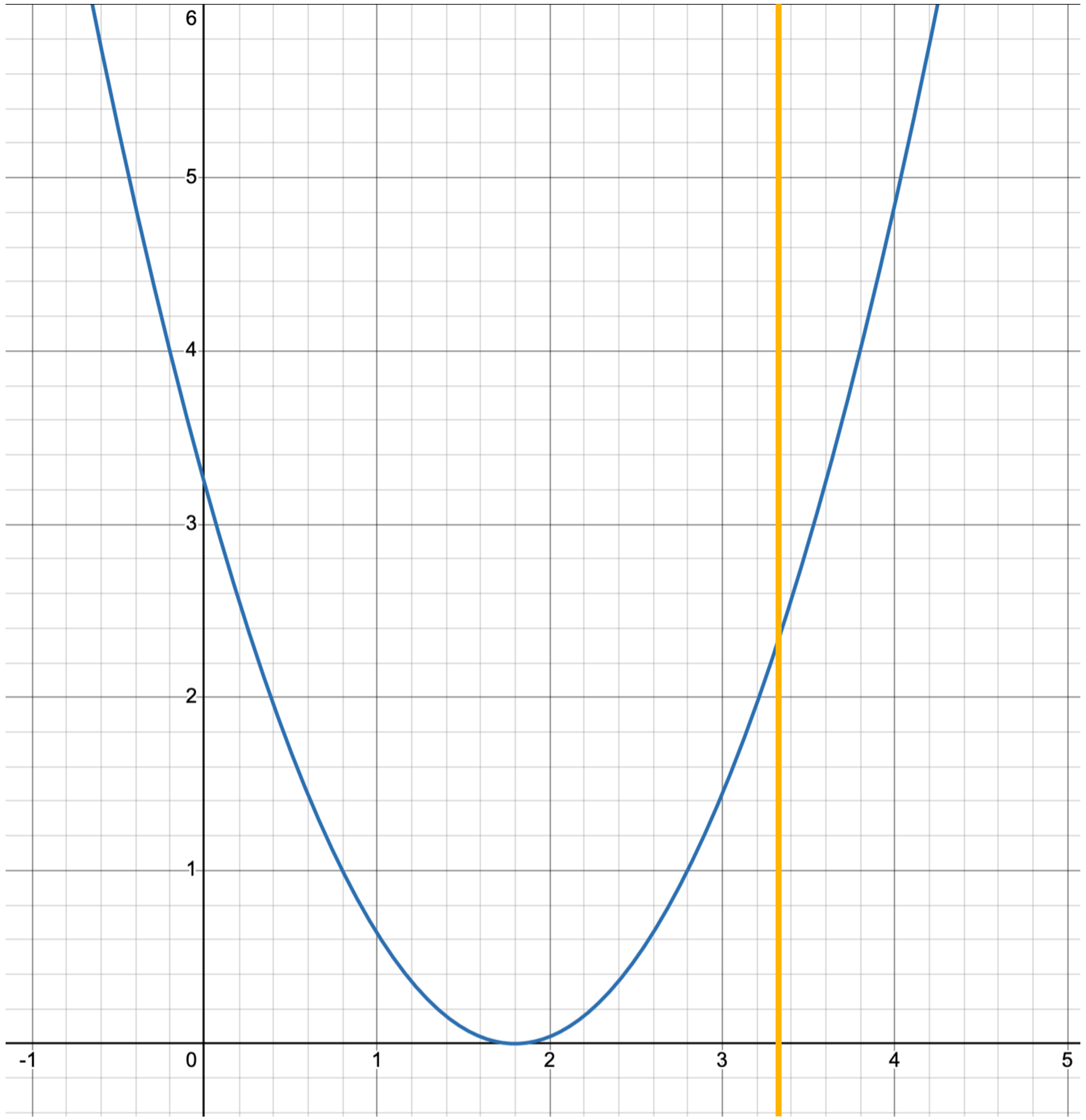

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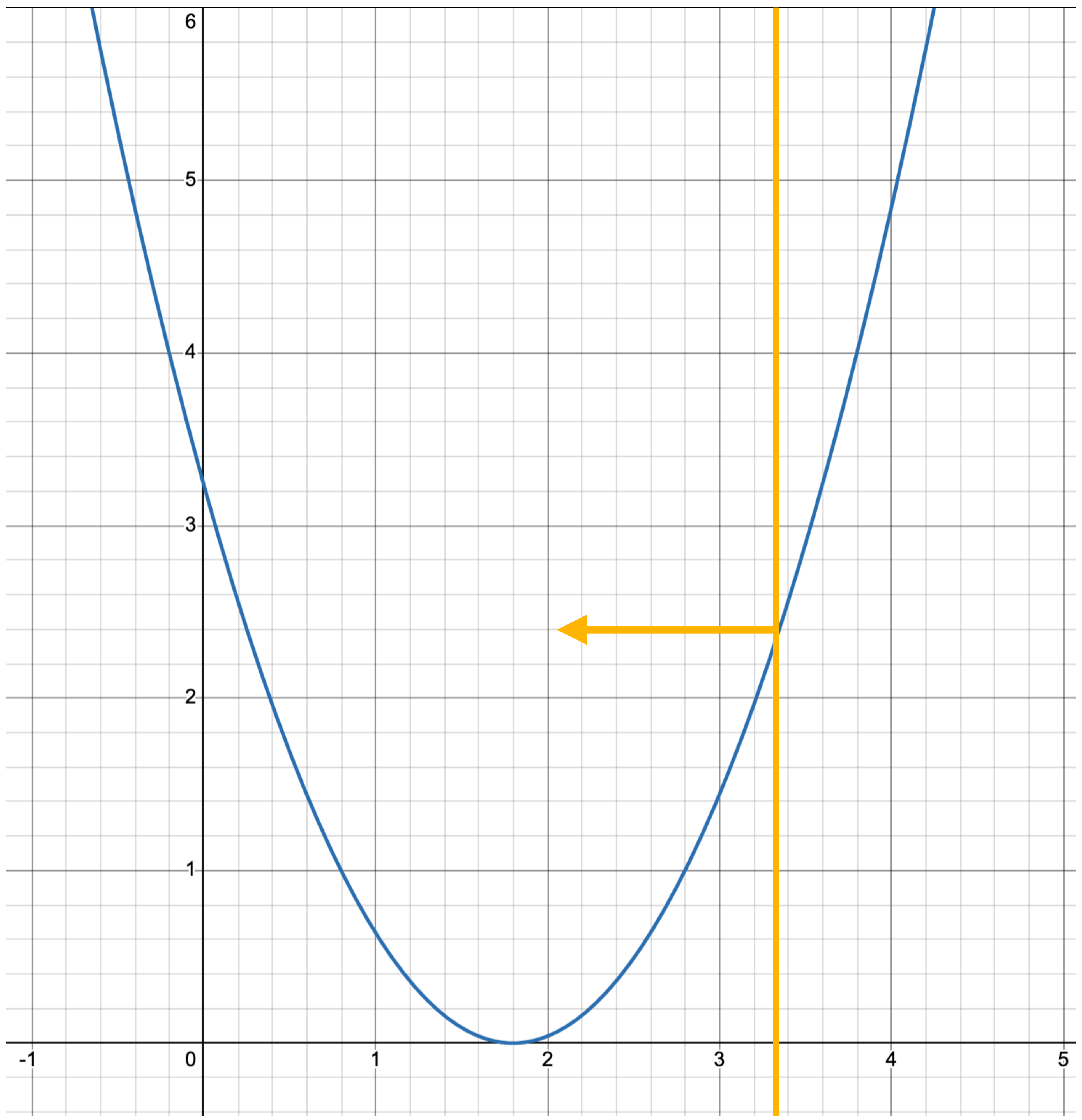

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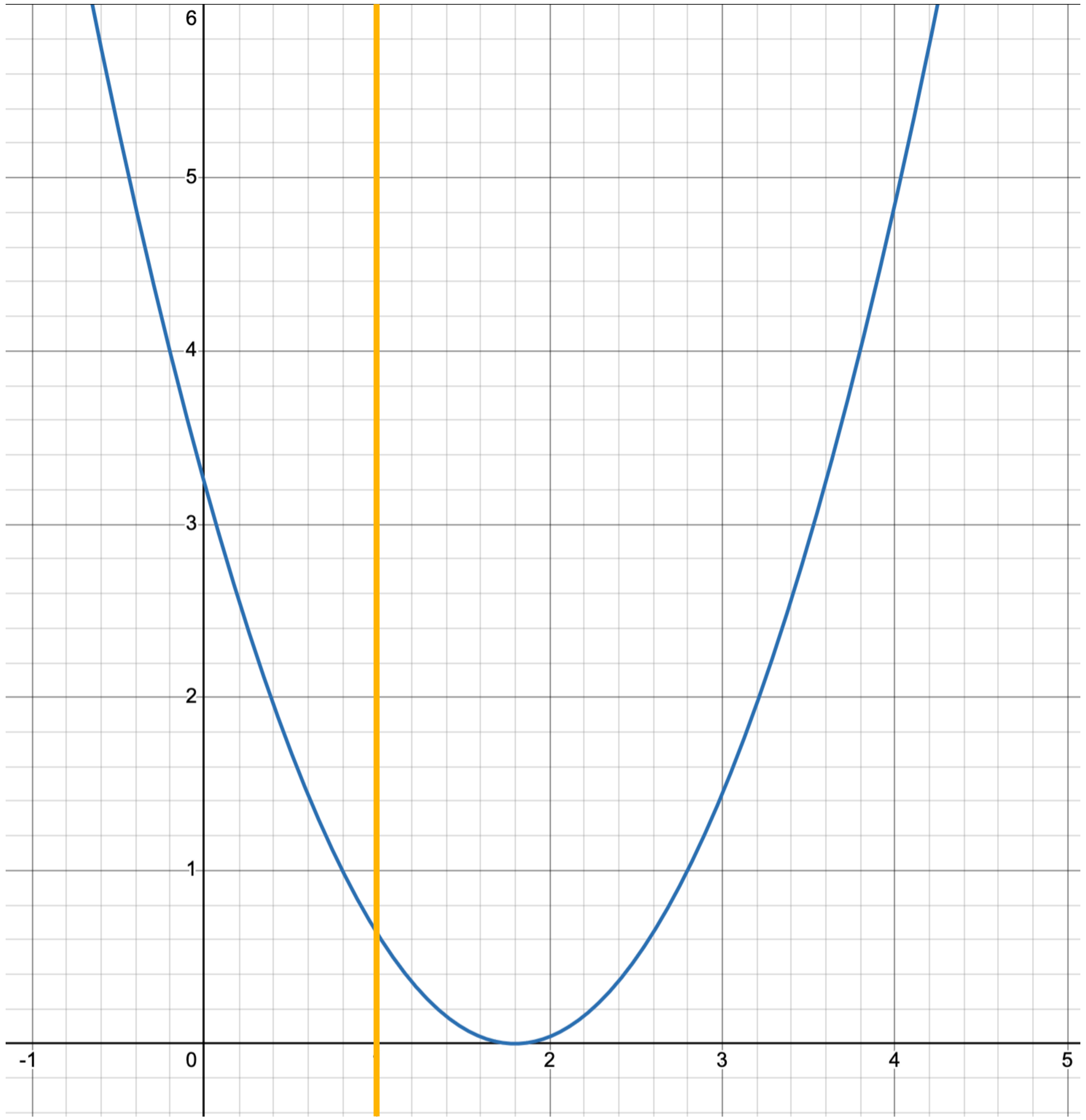

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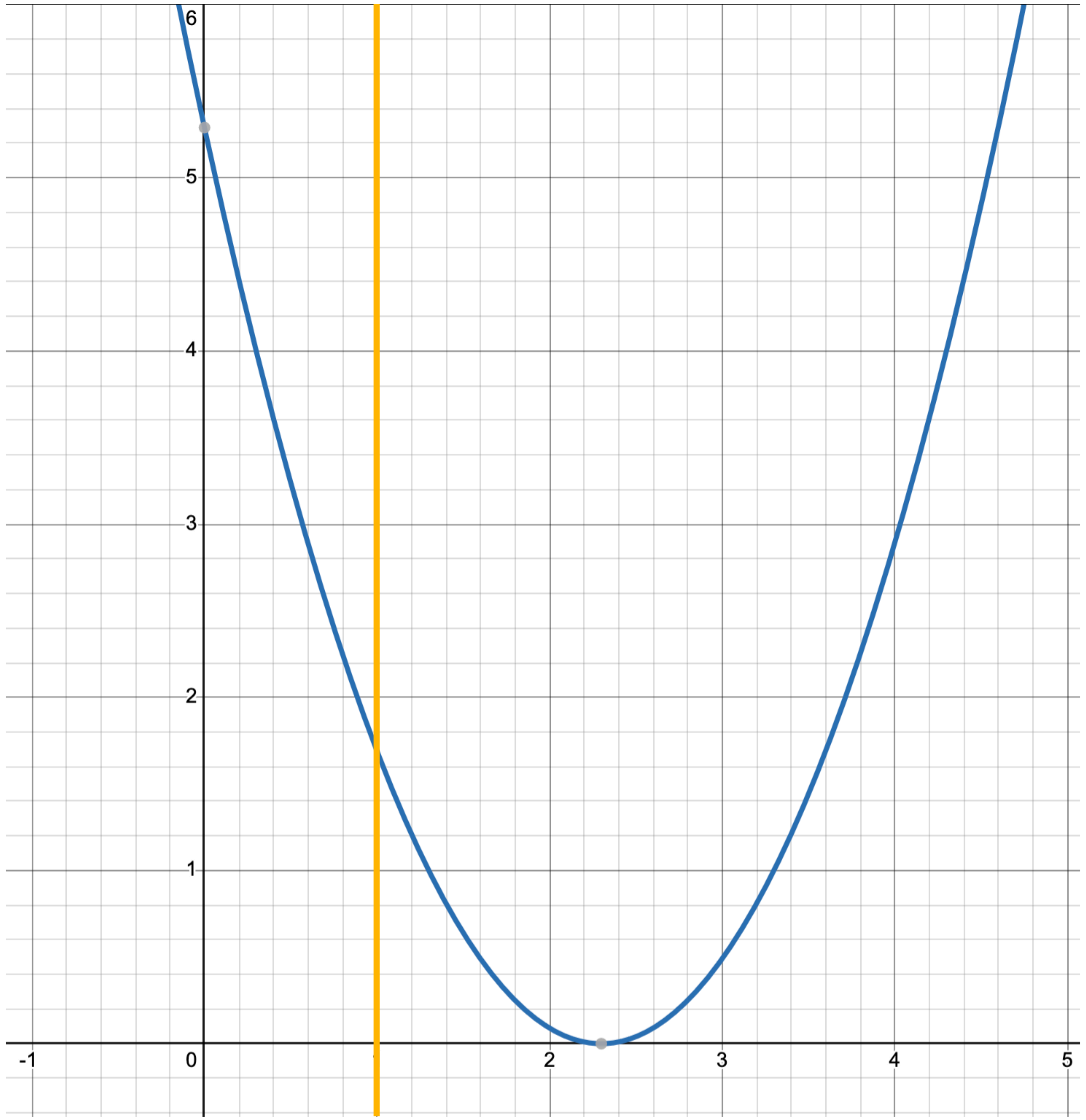

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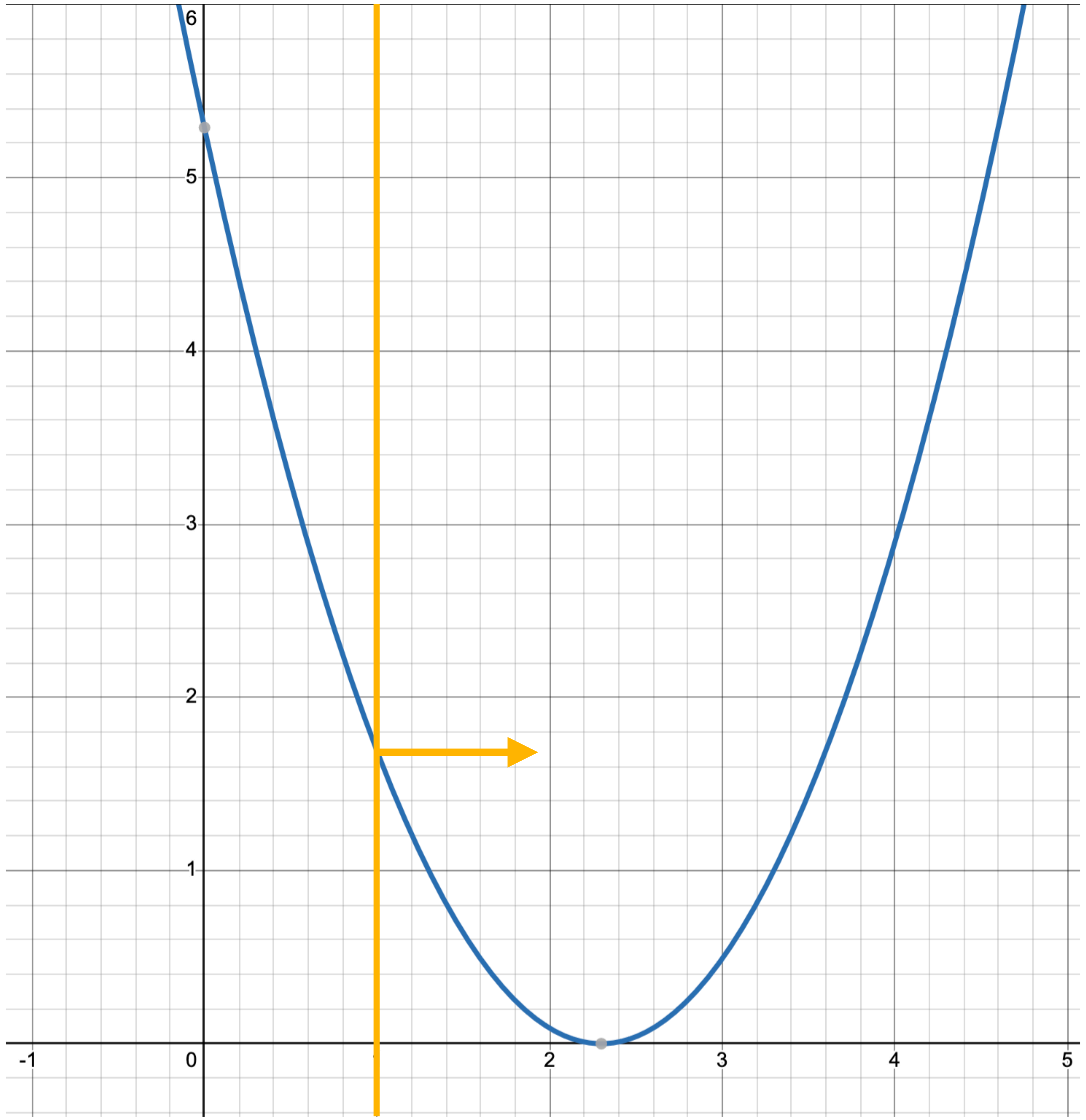

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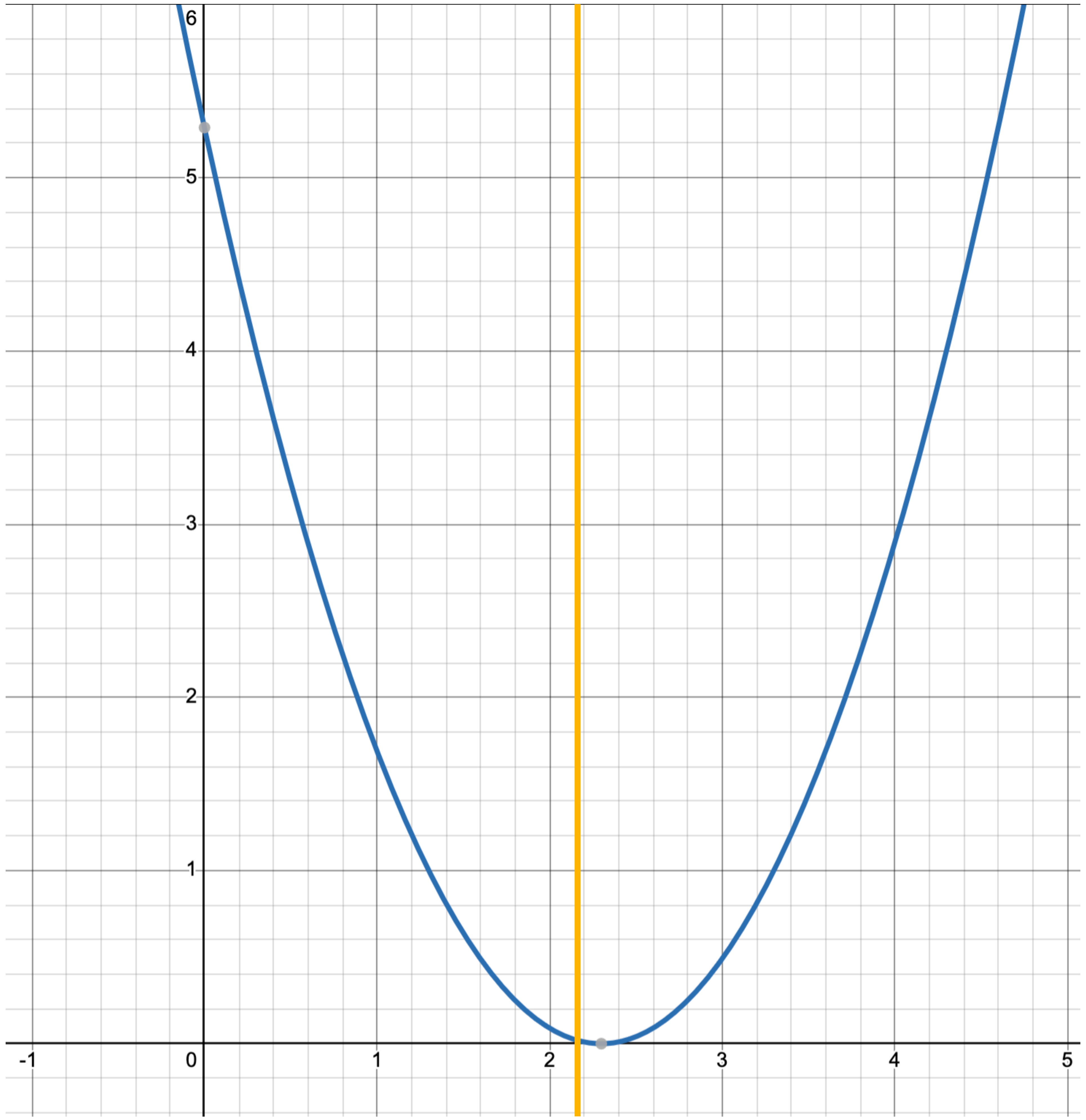

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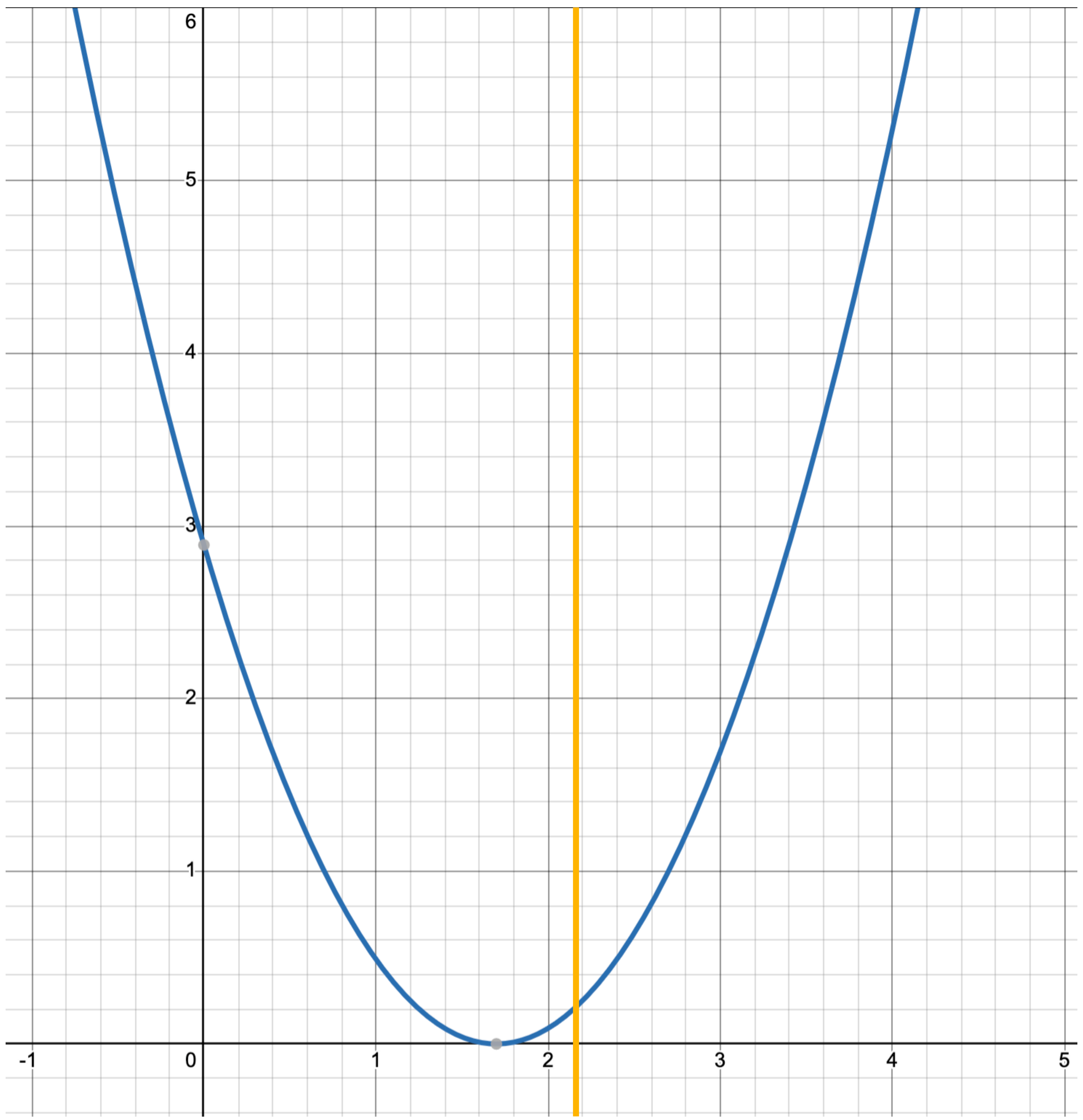

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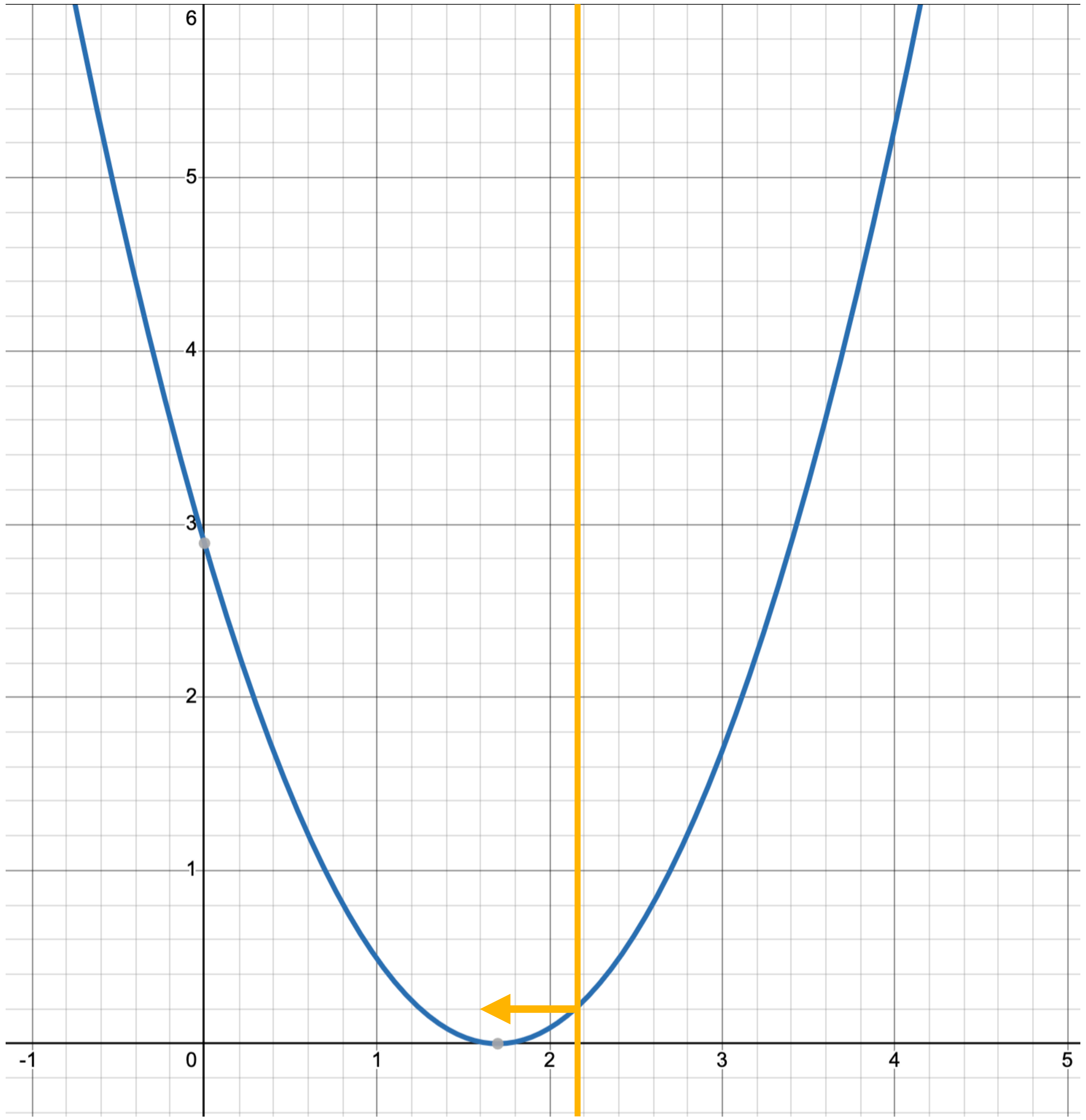

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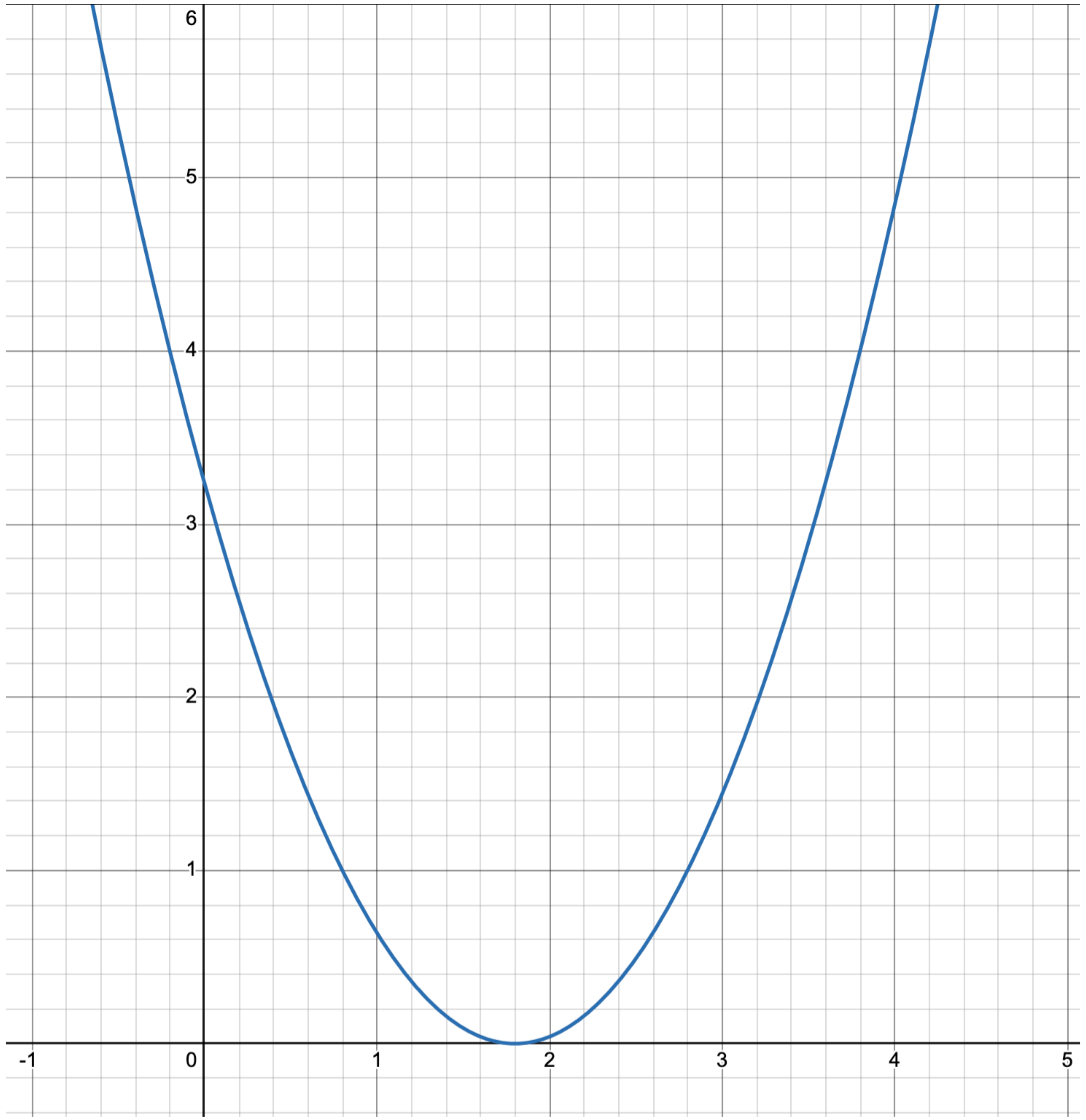

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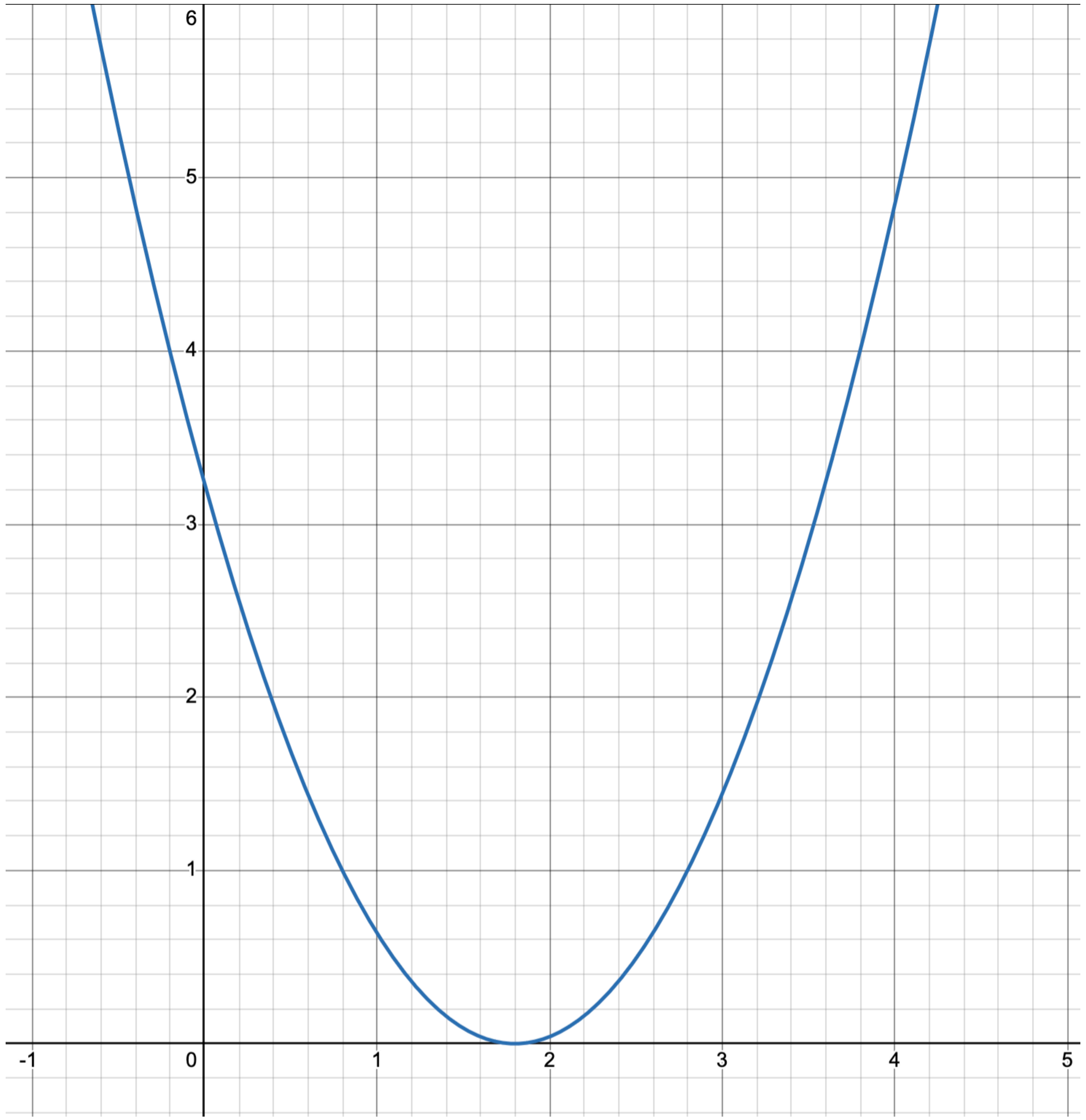

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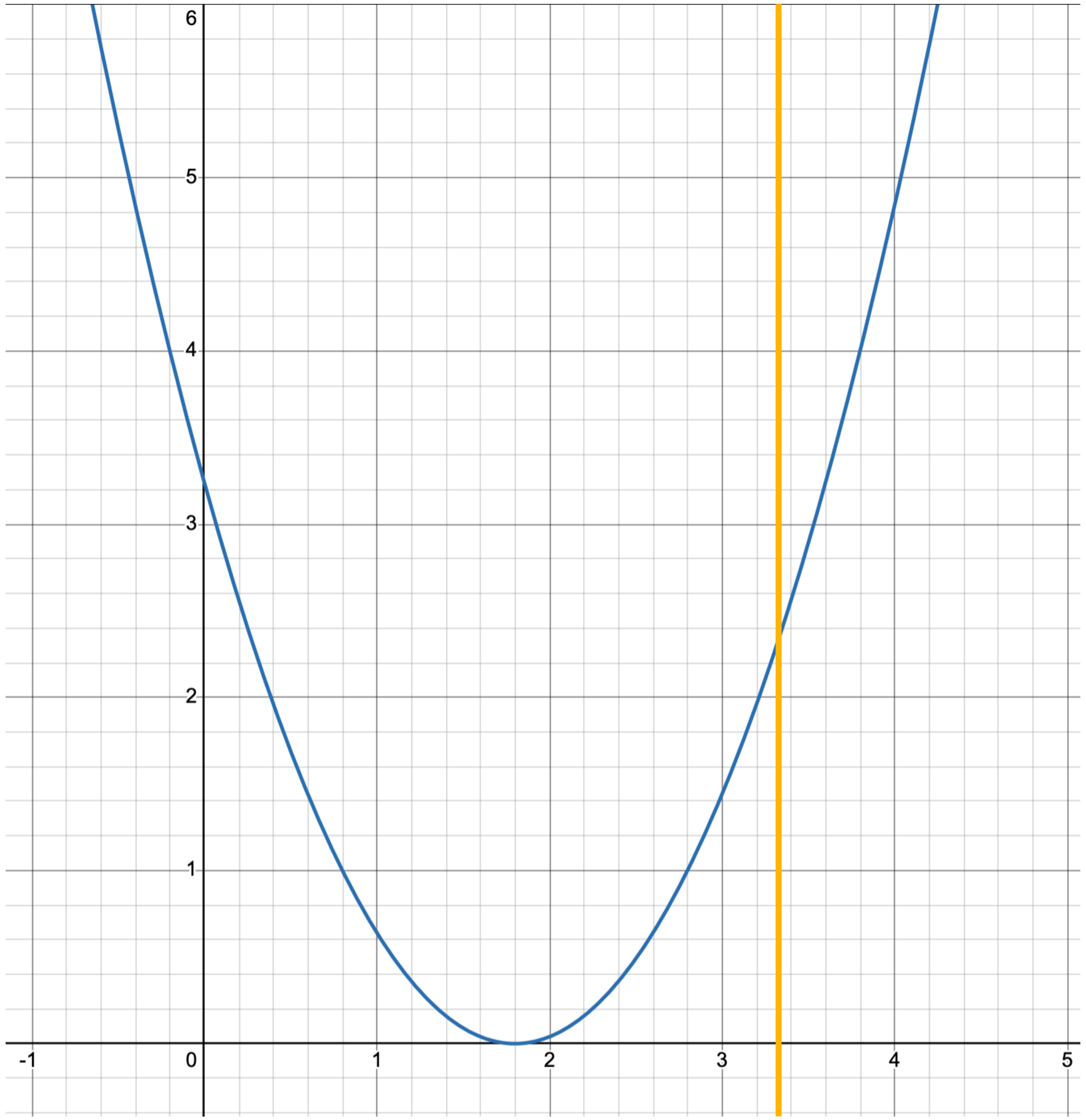

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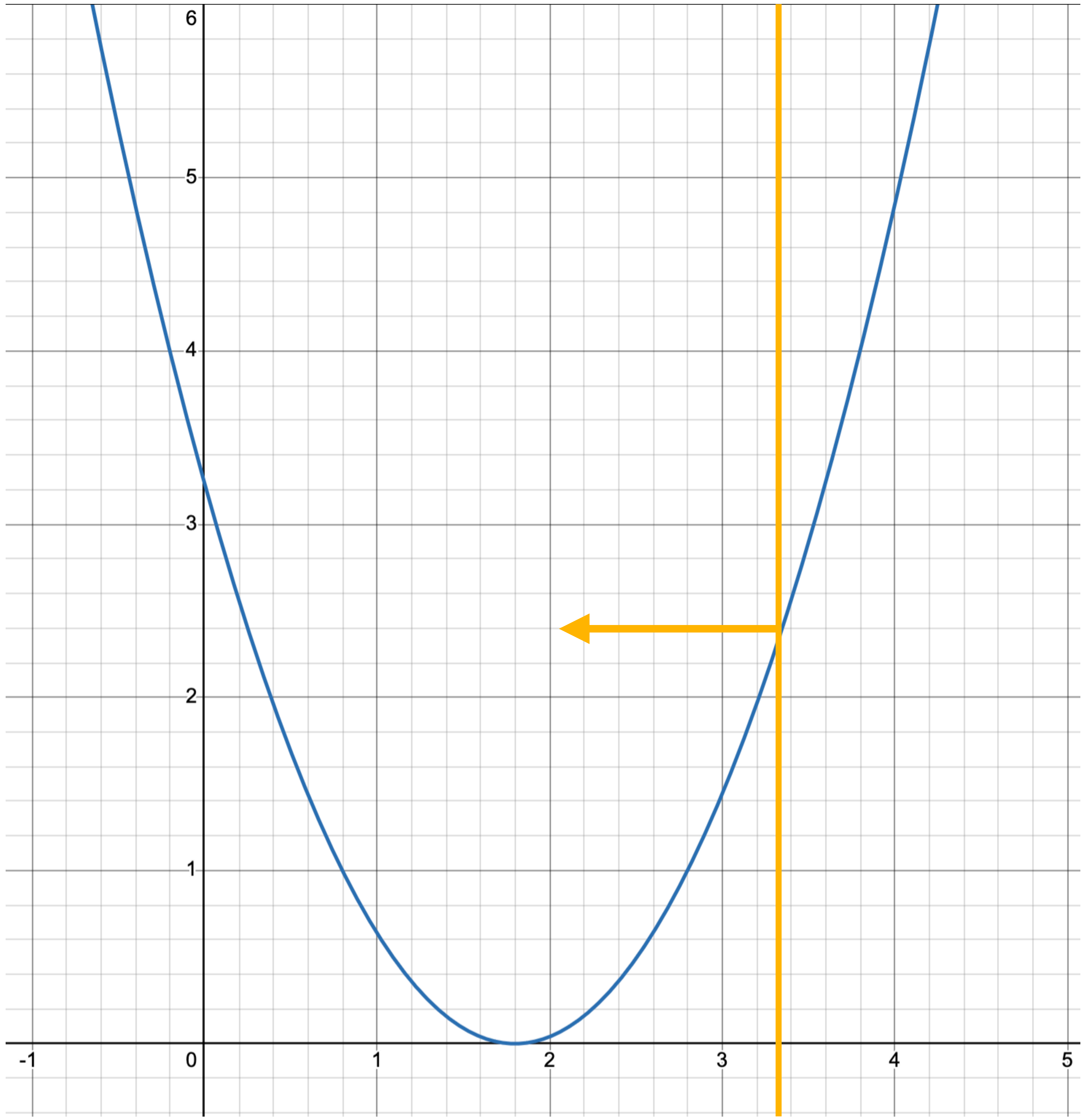

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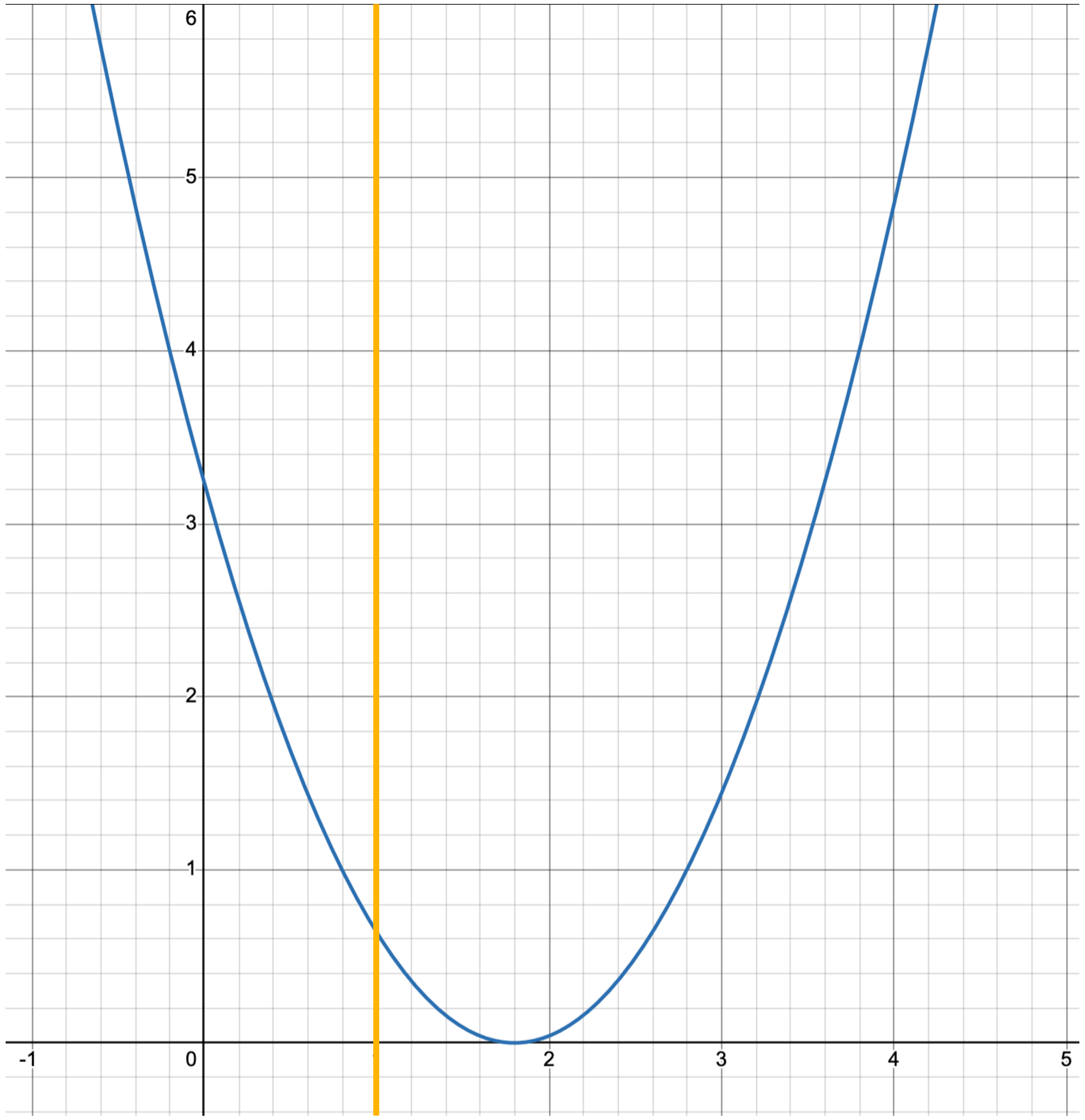

Mini-batch Gradient Descent

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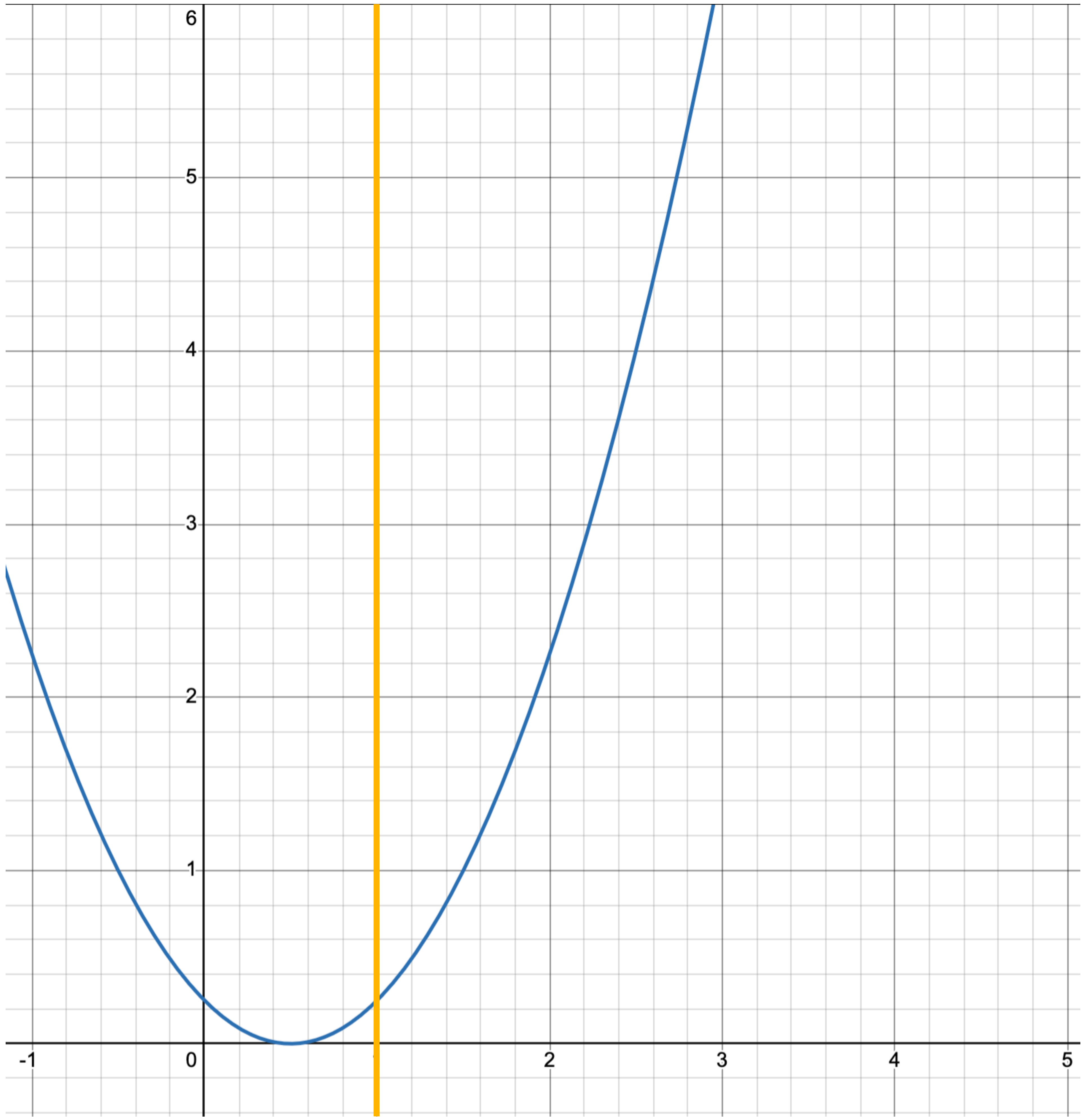

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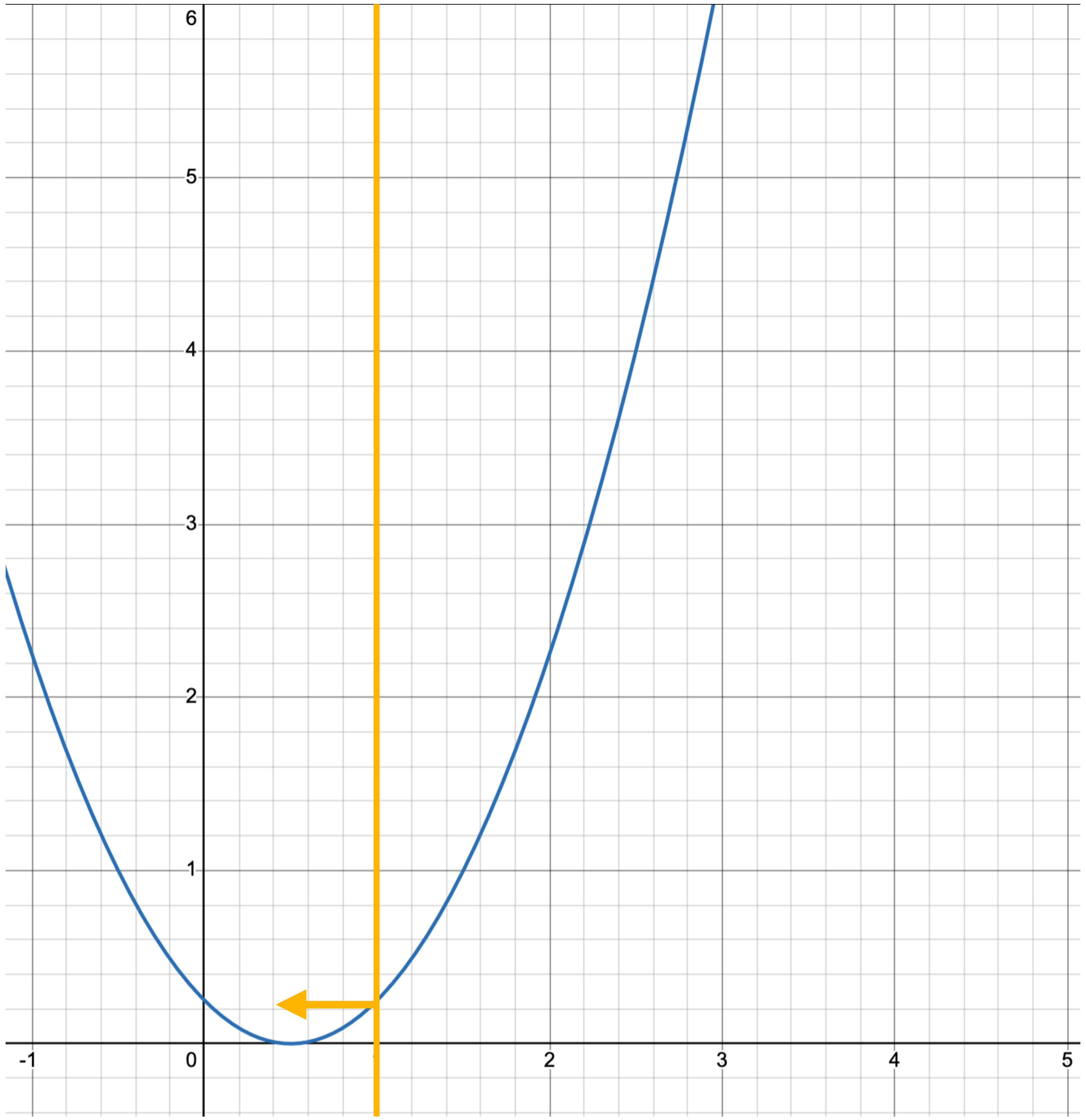

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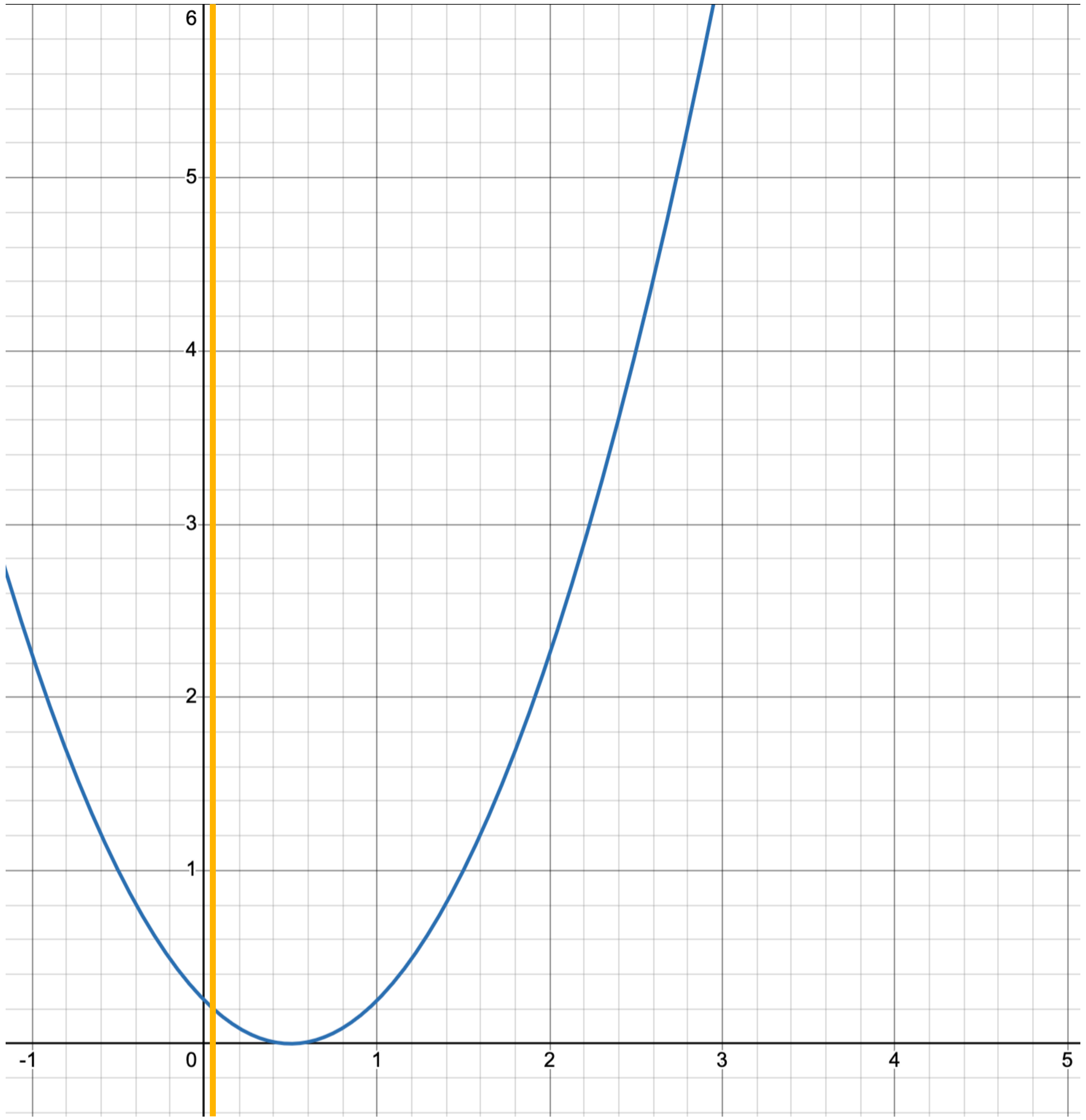

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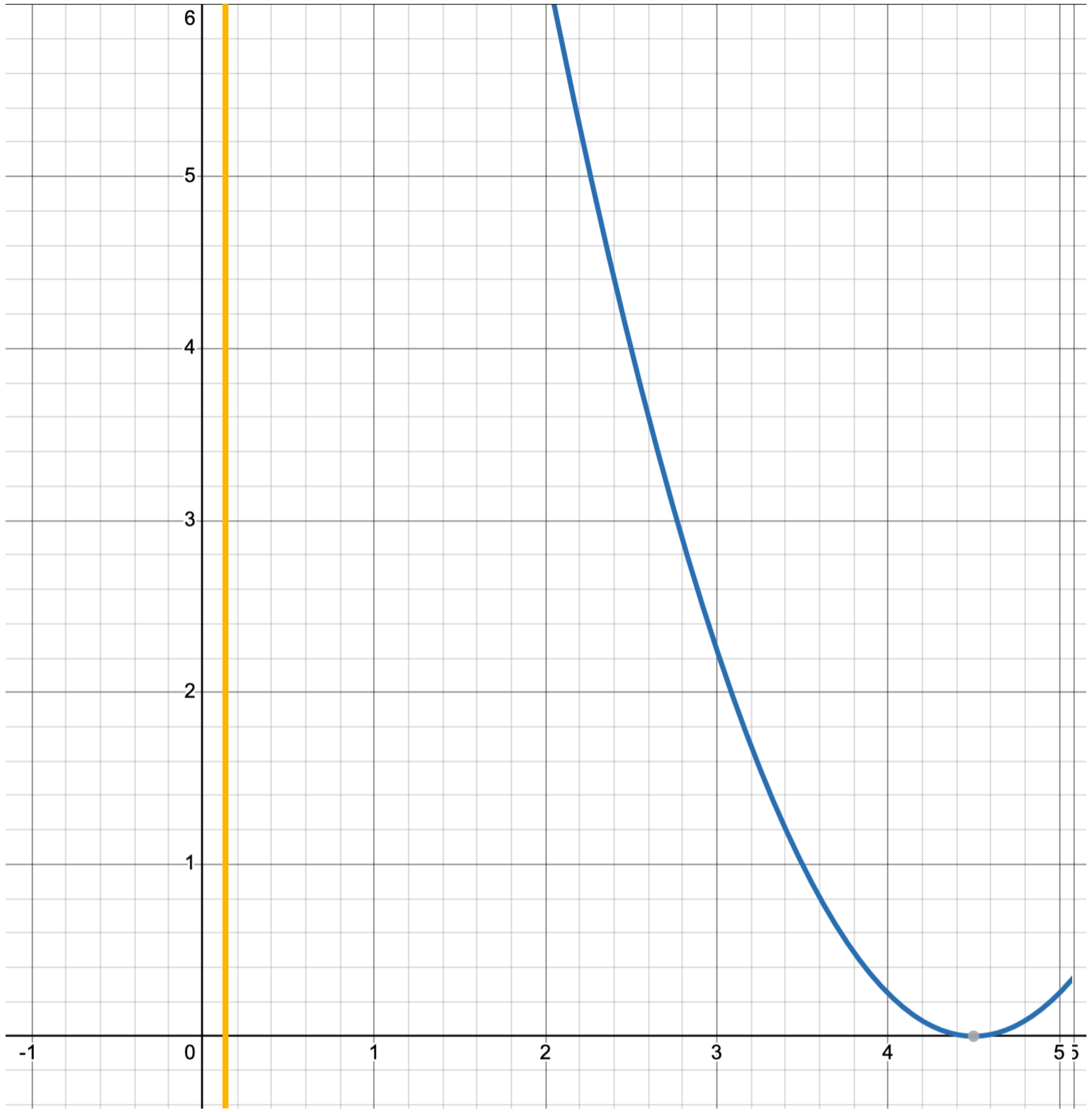

Mini-batch Gradient Descent

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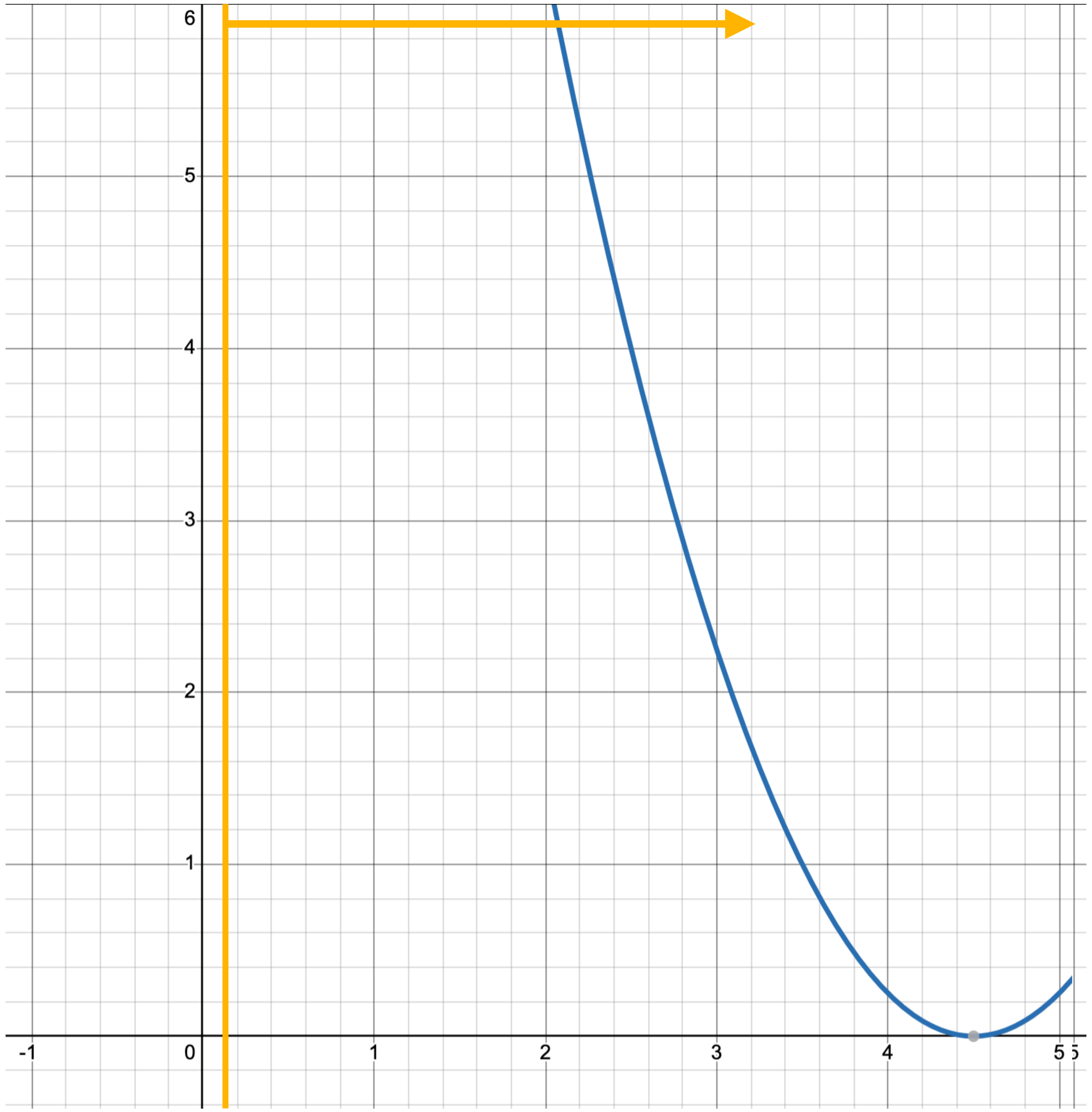
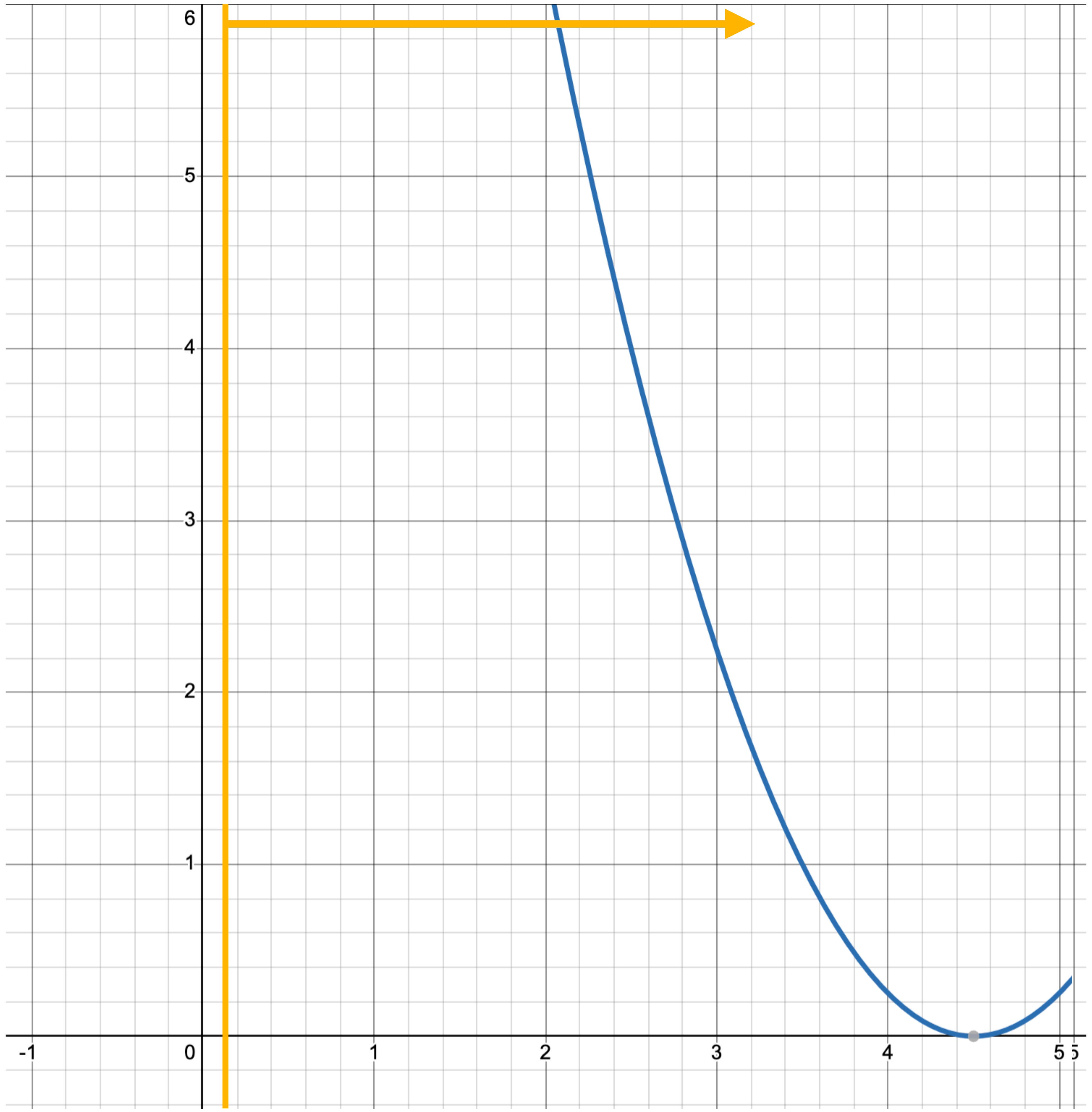


Mini-batch Gradient Descent

- Risk: SGD introduces **too much noise**
  - Parameters **"bounce around"**
- Solution: **Mini-batch GD**
  - Compute gradient for a **certain number of examples**, rather than the whole dataset
  - Gives a **better approximation** of the global gradient
  - **More efficient** than computing for the whole dataset
  - **Batch size** is a design-choice. Anywhere from a few dozen to tens of thousands



Mini-batch Gradient Descent

```
initialize parameters / build model

for each epoch:

  data = shuffle(data)
  batches = make_batches(data)

  for each batch in batches:

    outputs = model(batch)
    loss = loss_fn(outputs, true_outputs)
    compute gradients
    update parameters
```