Word Vectors (word2vec)

Ling 282/482: Deep Learning for Computational Linguistics

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Fall 2025

From Numbers to Language

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- In past lectures:
  - Vectors, spaces, linear transformations between spaces
  - Perceptron models, which learn to linearly separate input vectors
  - Gradient Descent, Backpropagation algorithms to learn ideal weights

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  - Perceptron models, which learn to linearly separate input vectors
  - Gradient Descent, Backpropagation algorithms to learn ideal weights
- Starting today:
  - How do we use these tools to **represent language?**
  - First step: **word vectors** (representing language in **high-dimensional space**)
  - Algorithms like **word2vec** learn these representations based on data

Word Vectors, Intro

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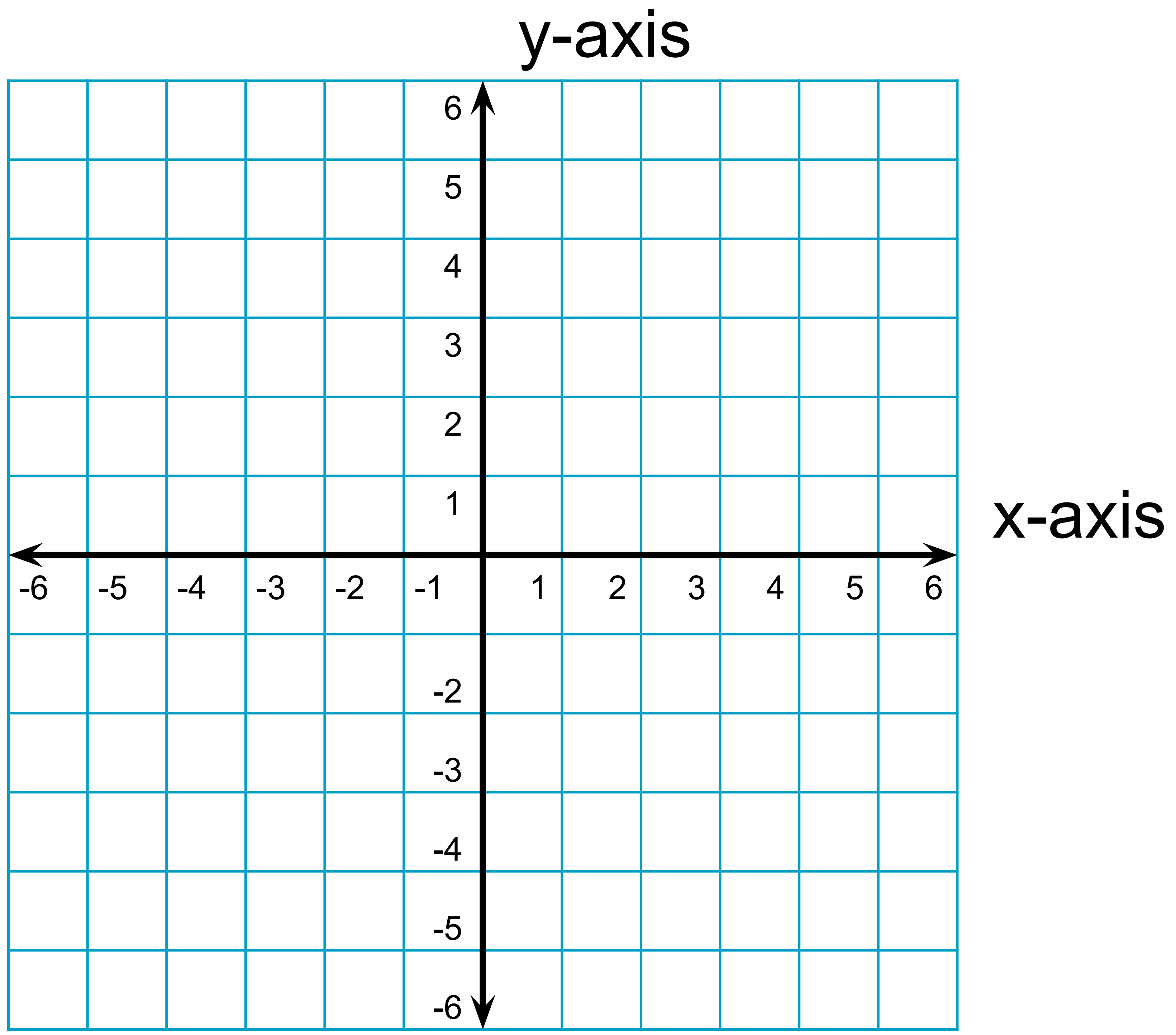

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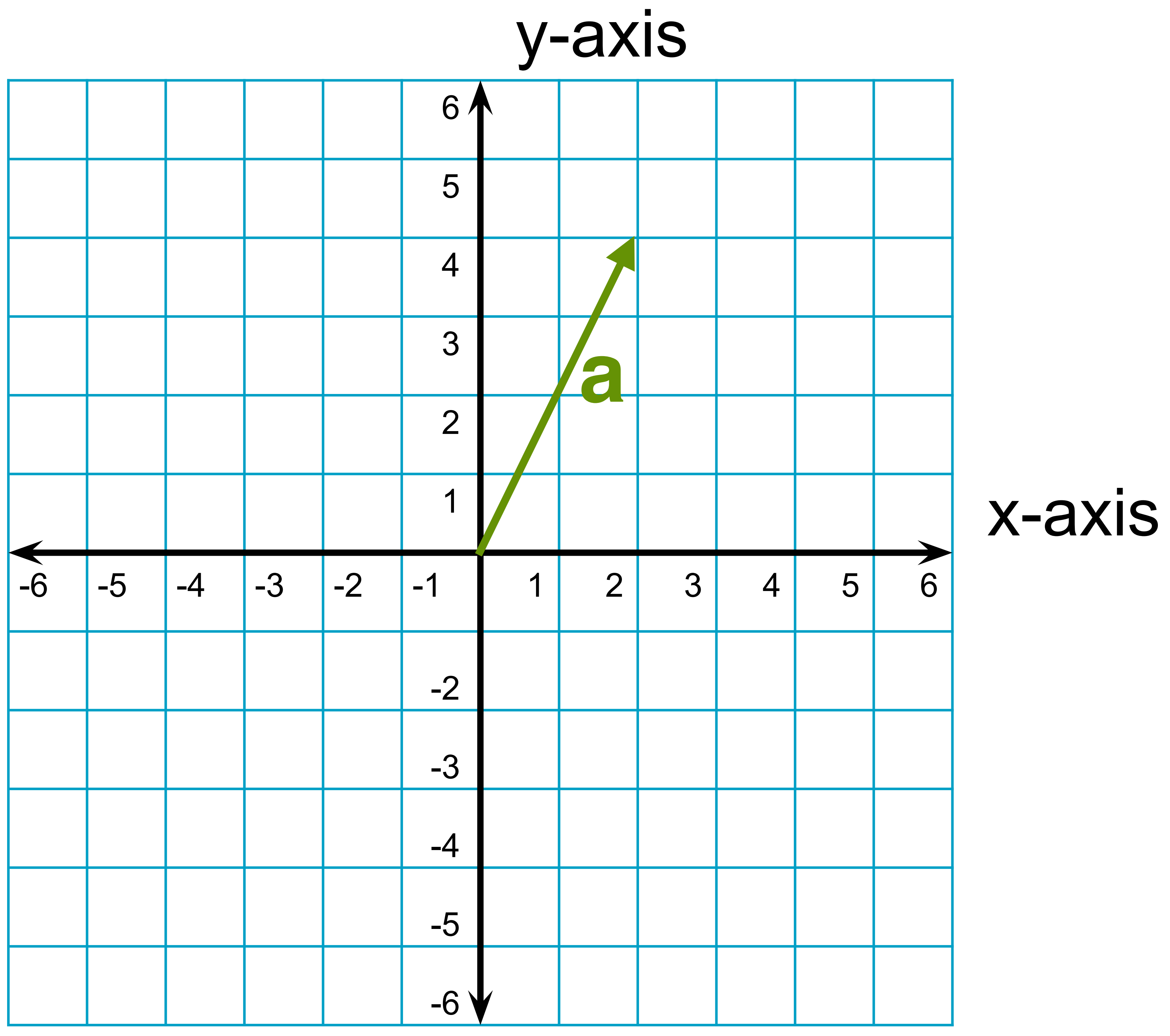

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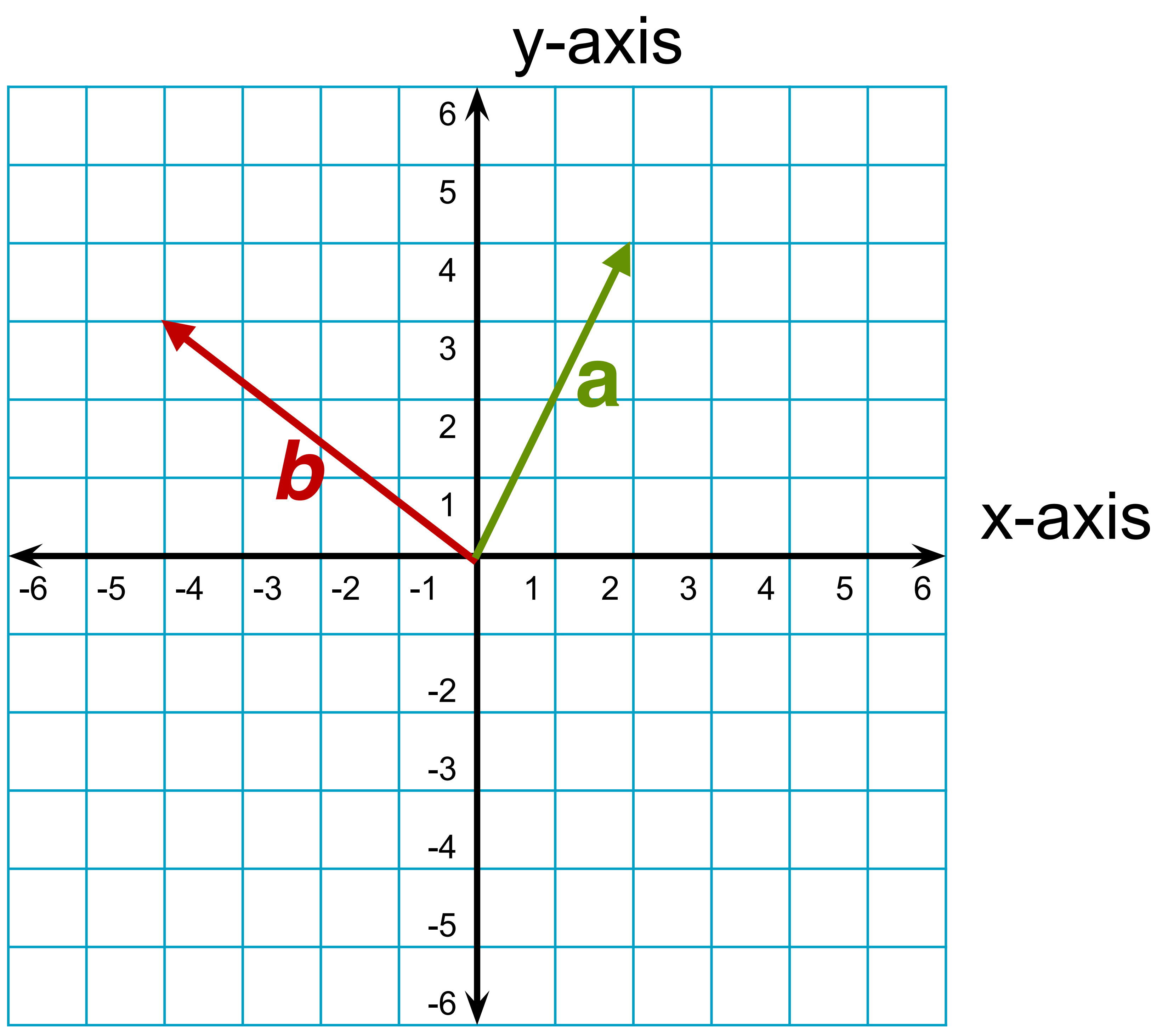

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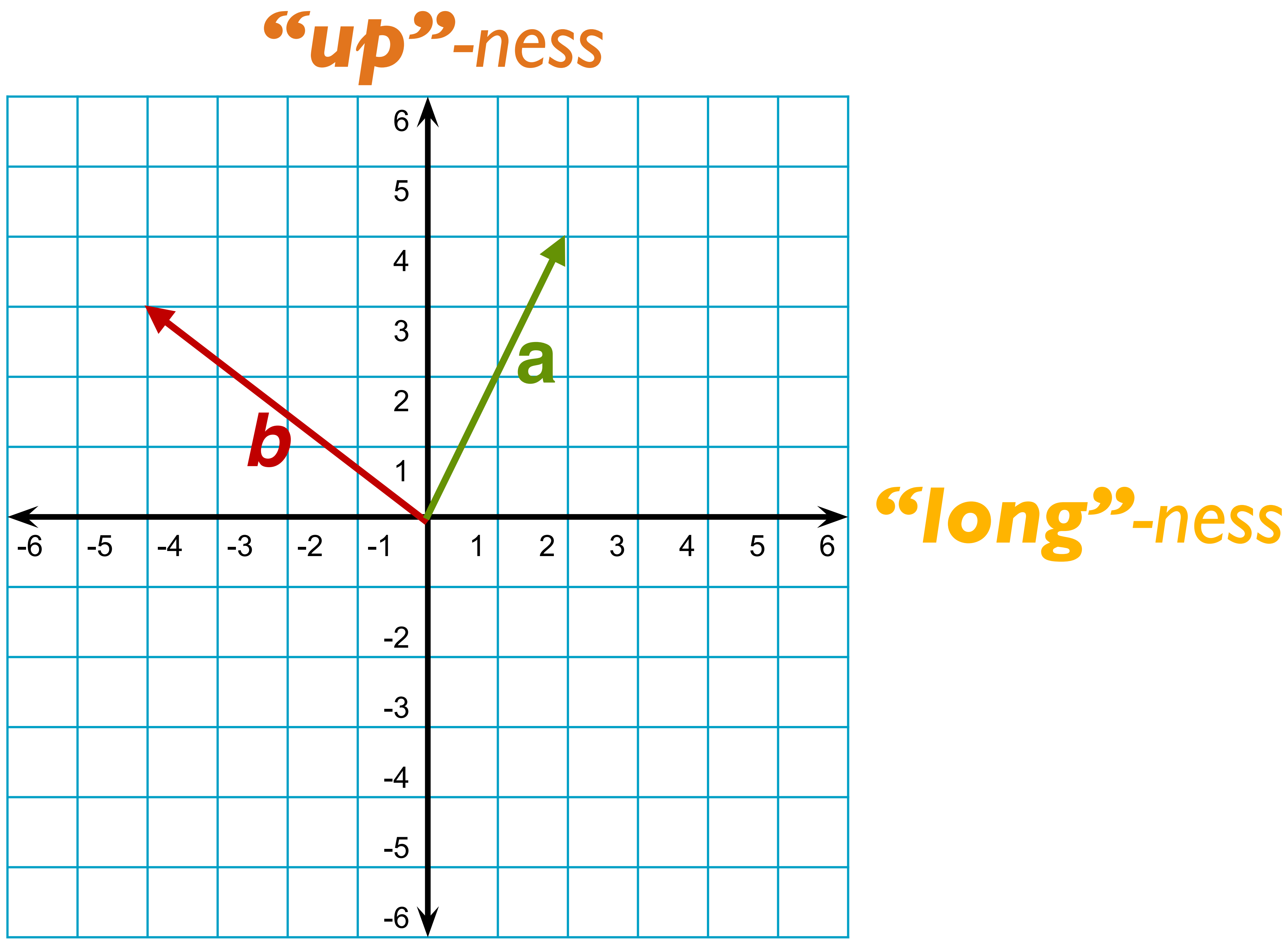

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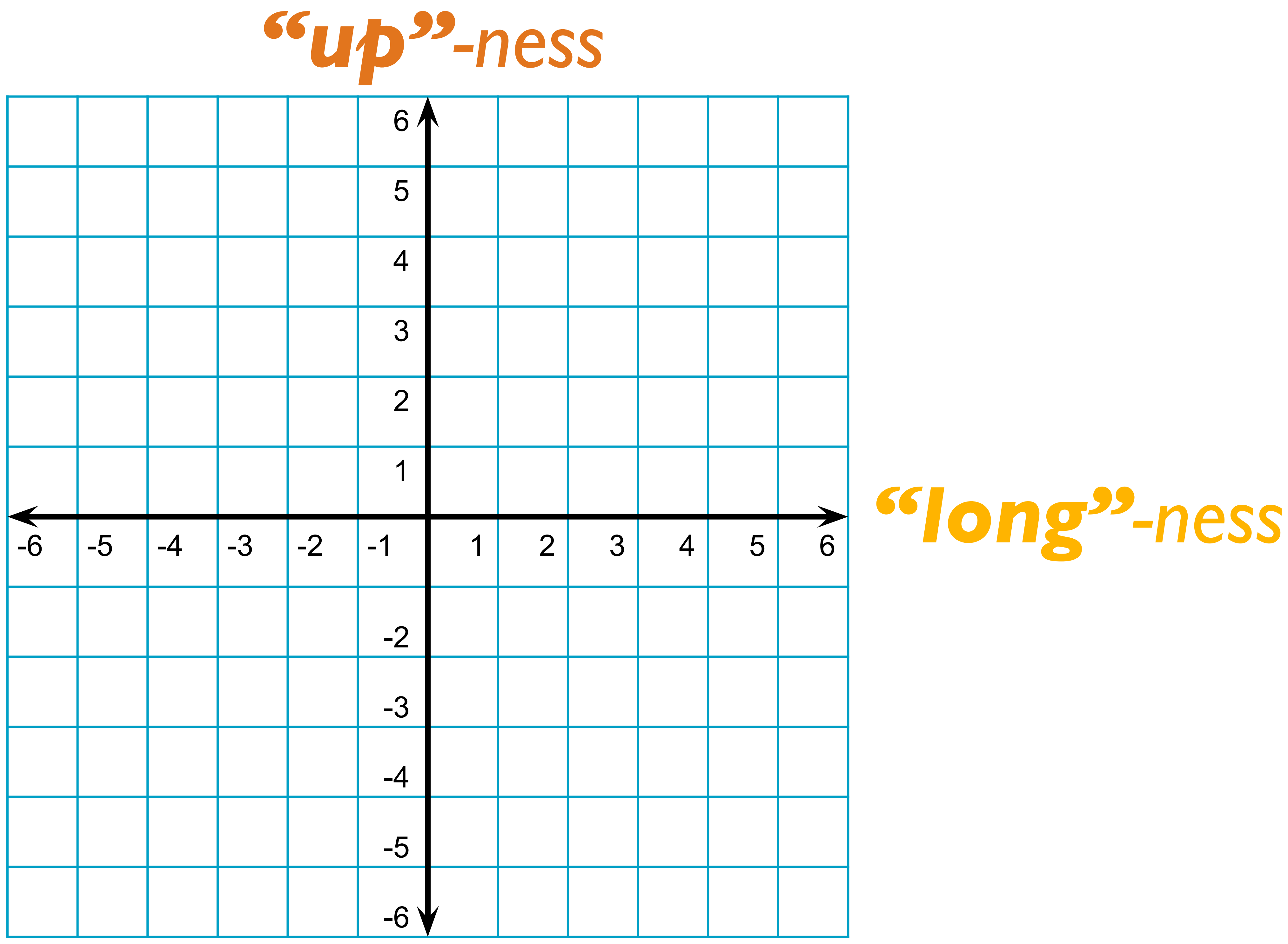

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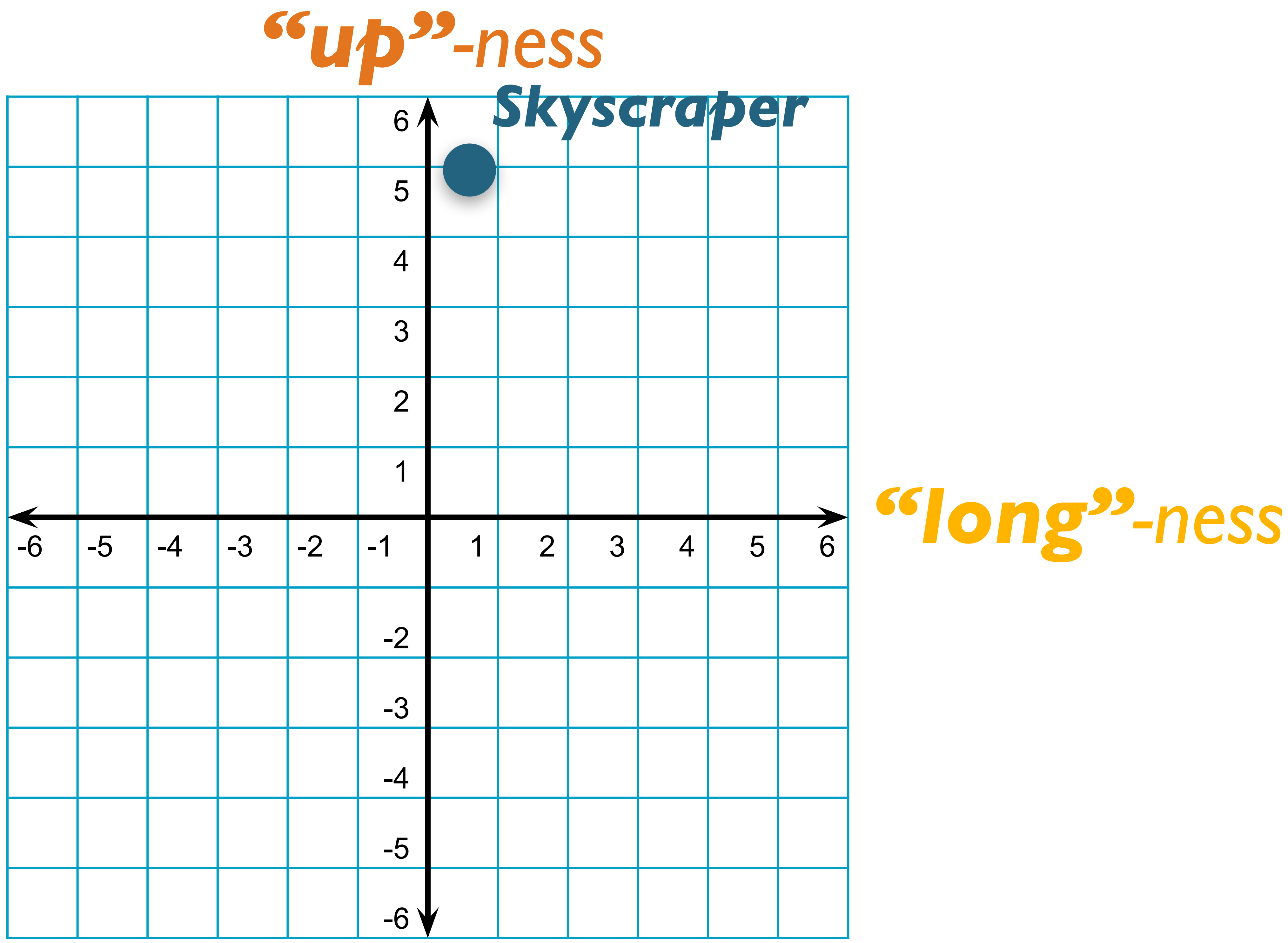

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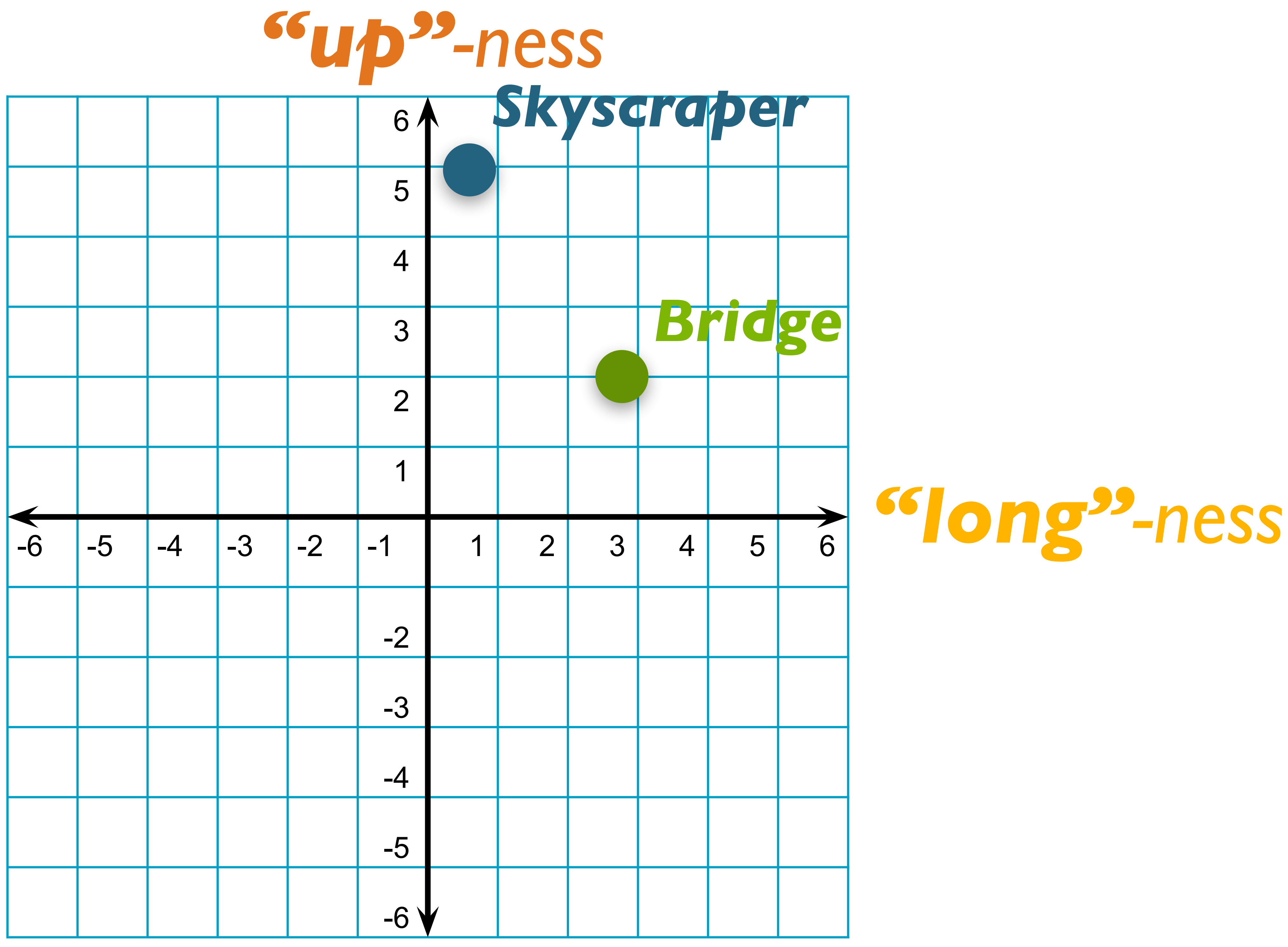

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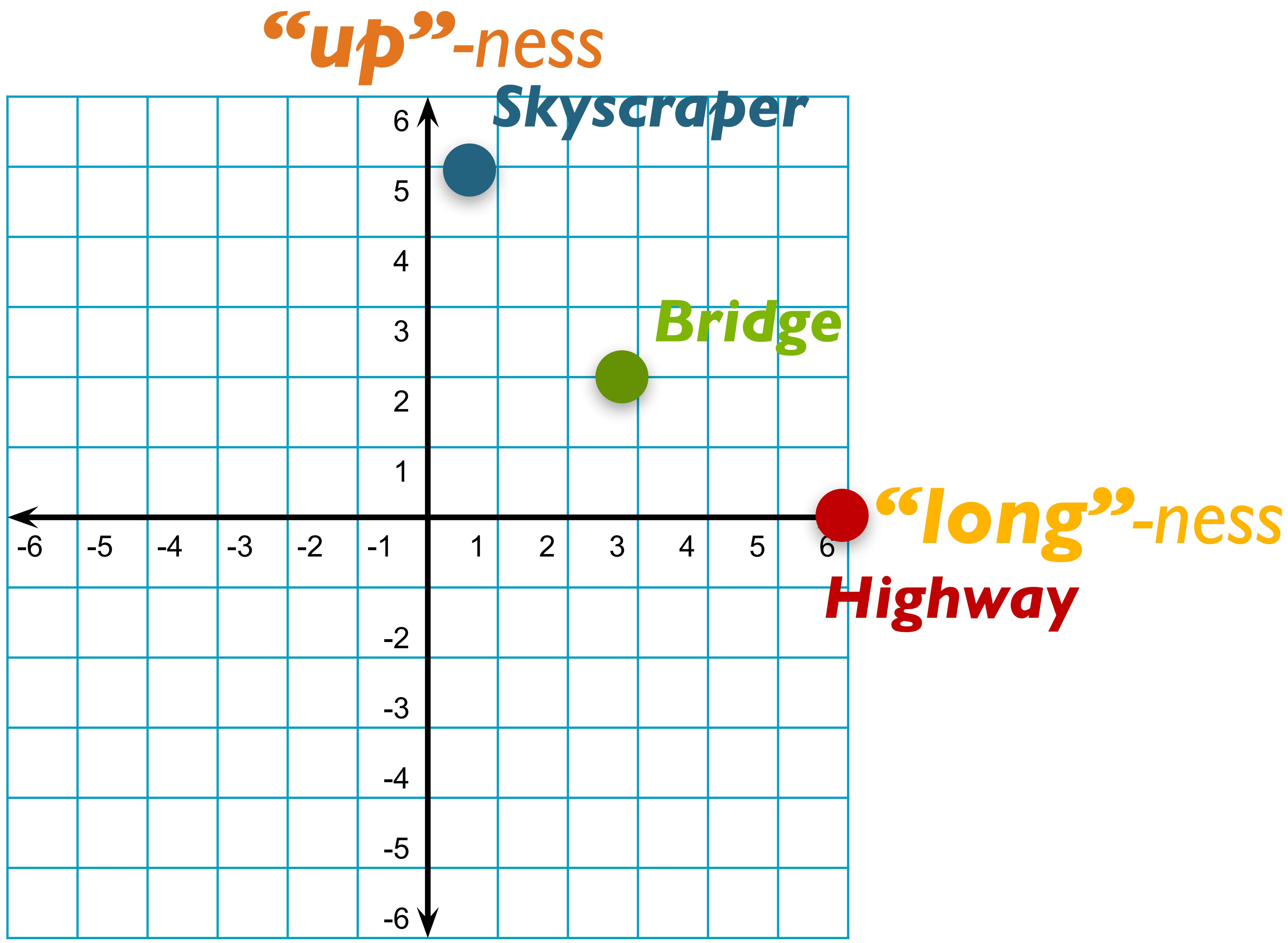

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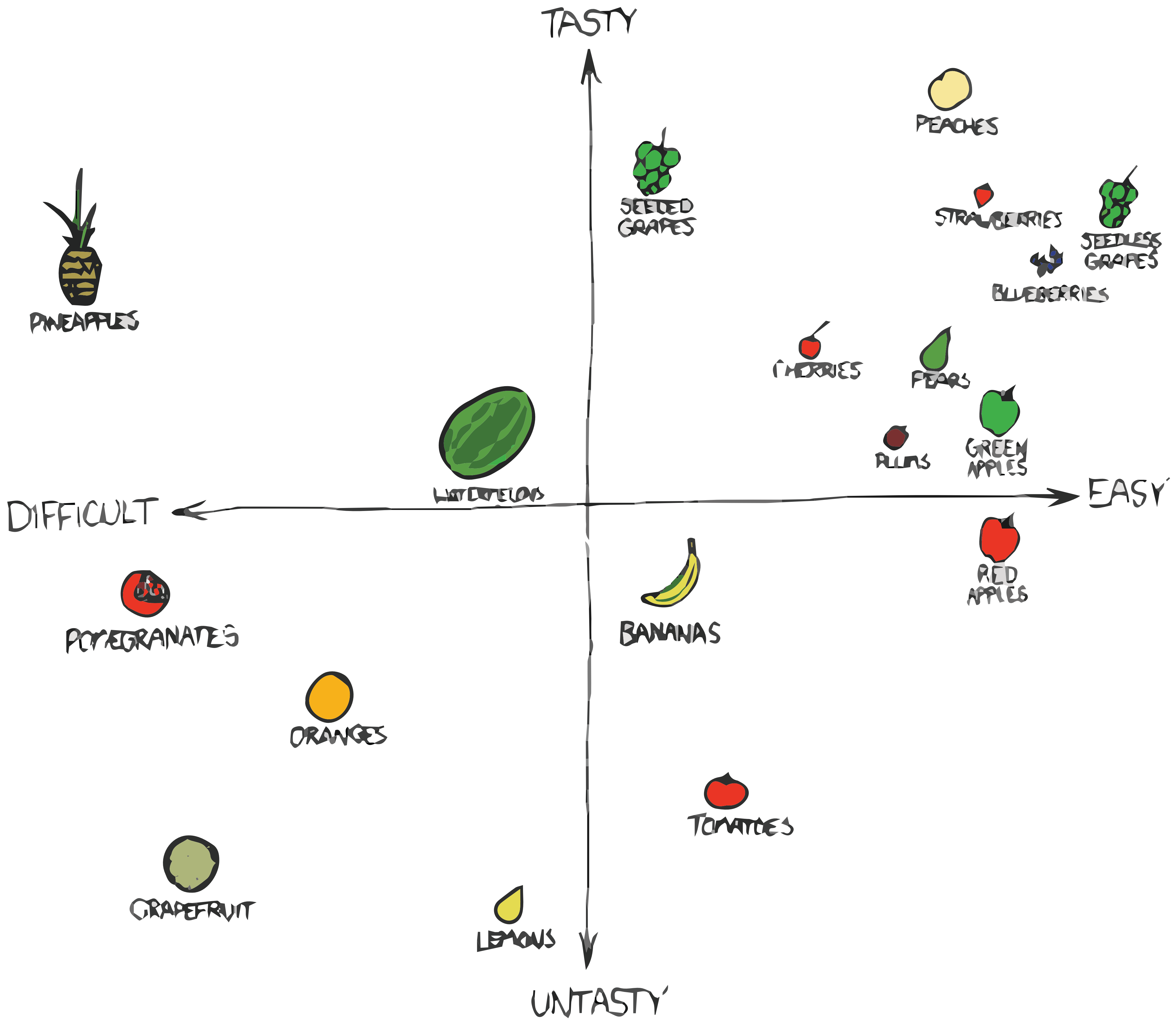

xkcd.com/388

Comparing Vectors

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  - e.g. "cat" and "kitten" are more **closely related** than "scratch" and "ministry"
  - (You can learn to categorize these relationships in a Formal Semantics course)

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  - e.g. "cat" and "kitten" are more **closely related** than "scratch" and "ministry"
  - (You can learn to categorize these relationships in a Formal Semantics course)
- How do we capture relationships between **word vectors?**
  - Luckily, we have all kinds of ways to **measure** vector relationships
  - These are mostly divided into metrics of **vector similarity** and **vector distance/ dissimilarity**
  - Which metrics are **closest to human intuitions?**

Vector Length

- A vector's **length** is equal to the *square root of the dot product with itself*
- This is an extension of the **Pythagorean Theorem** for right triangles
- Notice that **dot product** is closely related to length

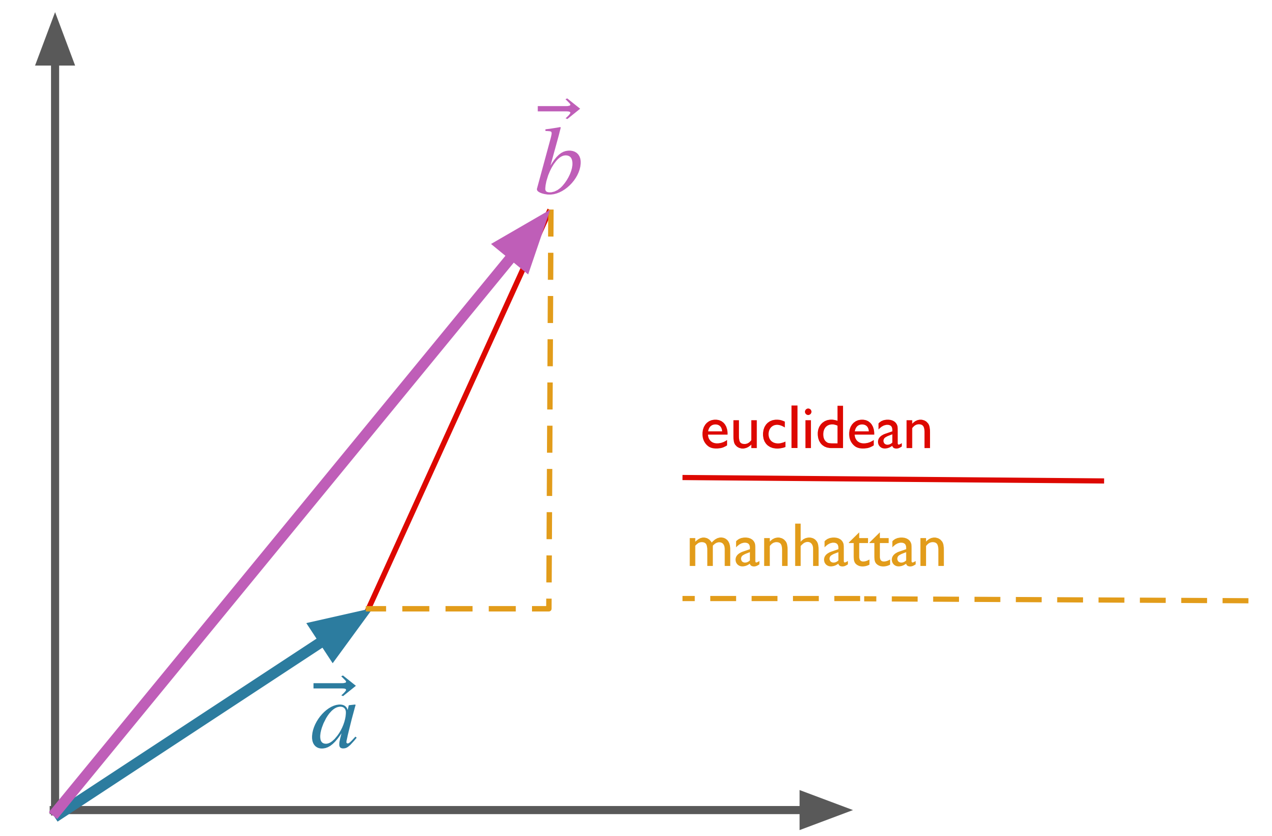
$$\text{length}(x) = \|x\| = \sqrt{x \cdot x}$$

Vector Distances: Manhattan & Euclidean

- **Manhattan Distance**
  - Distance as **cumulative horizontal + vertical moves**
  - Inspired by walking in a **city on a grid**
- **Euclidean Distance**
  - Our normal notion of distance
  - Length of a **straight line** between
- Both are sensitive to **extreme values**

$$d_{\text{manhattan}}(x, y) = \sum_i |x_i - y_i|$$

$$d_{\text{euclidian}}(x, y) = \sqrt{\sum_i (x_i - y_i)^2}$$



Vector Similarity: Dot Product

$$\text{sim}_{\text{dot}}(x, y) = x \cdot y = \sum_i x_i y_i$$

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  - This can be used as a **similarity metric**
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- Problem: gives **higher similarity** to **longer vectors**

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Vector Similarity: Cosine

$$\mathrm{sim}_{\mathrm{cosine}}(x, y) = \frac{x \cdot y}{\|x\| \|y\|} = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2} \sqrt{\sum_i y_i^2}}$$

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- This metric **correlates with human intuitions** of semantic relatedness (demonstrated through experiments)

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One-hot Vectors

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- How do we get **better representations?**

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  - We make *tezgüino* from corn.
- Tezgüino: corn-based alcoholic beverage.

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cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
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- What is the **size** of these vectors?
  - (Still $|V|$)

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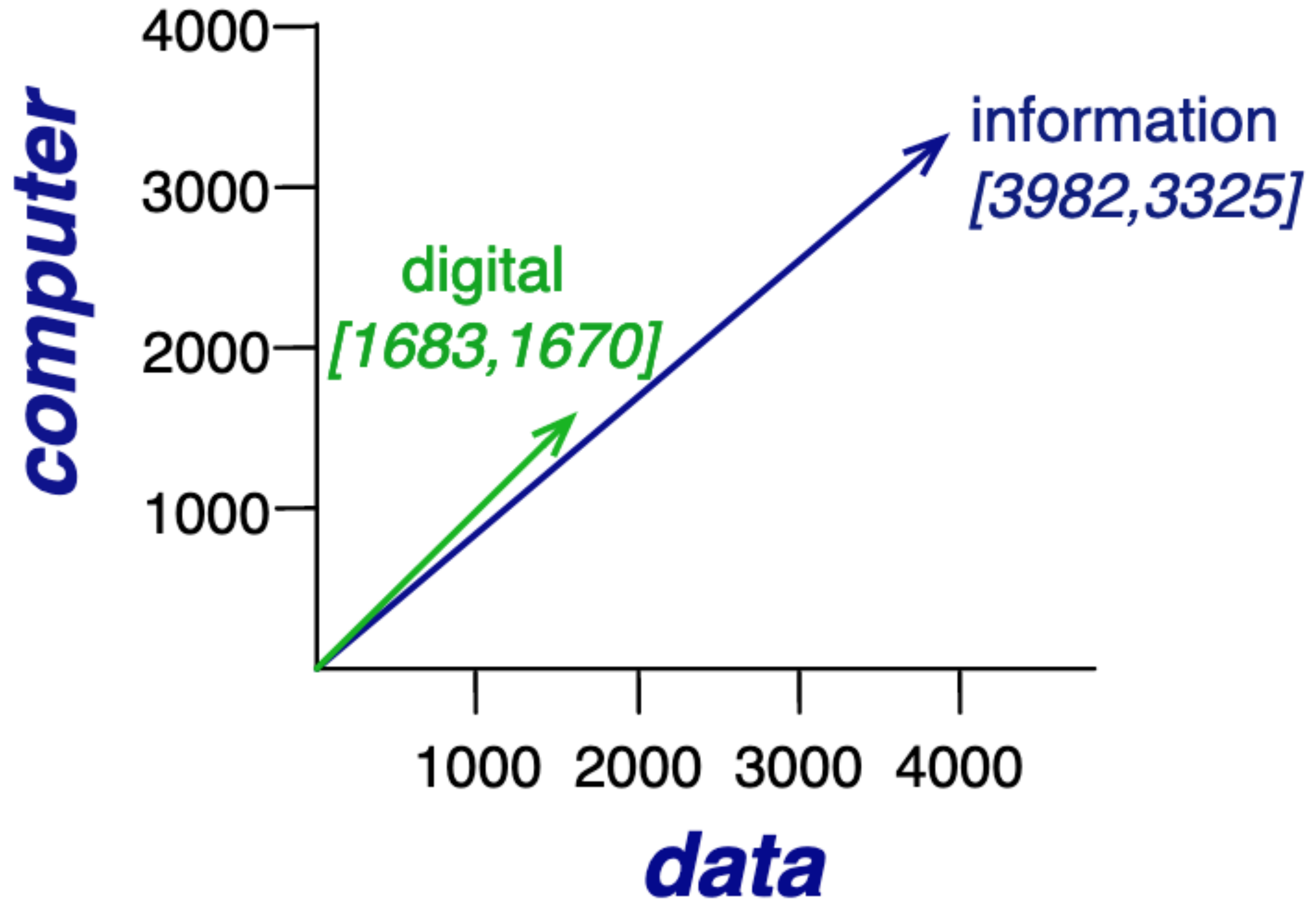

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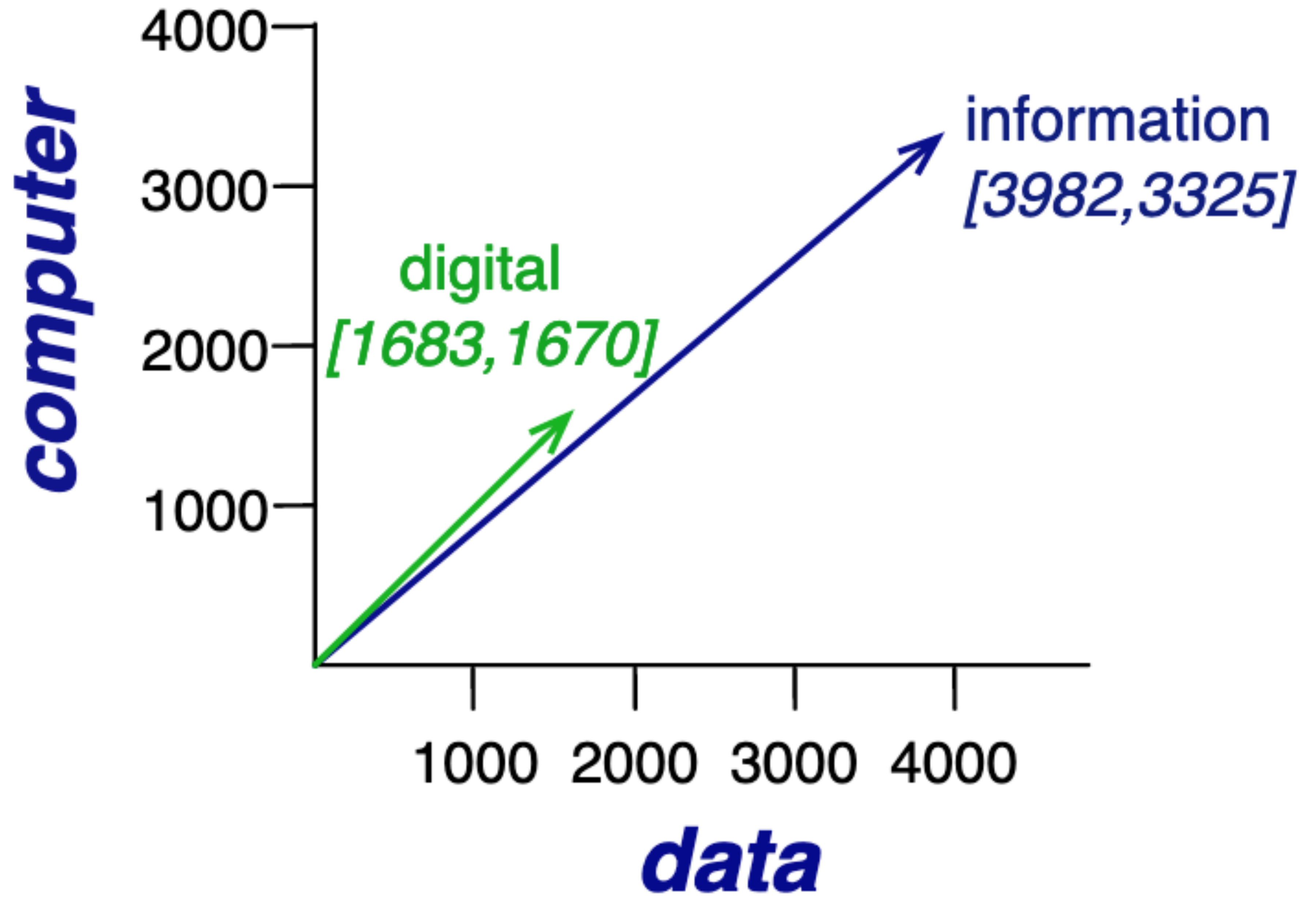

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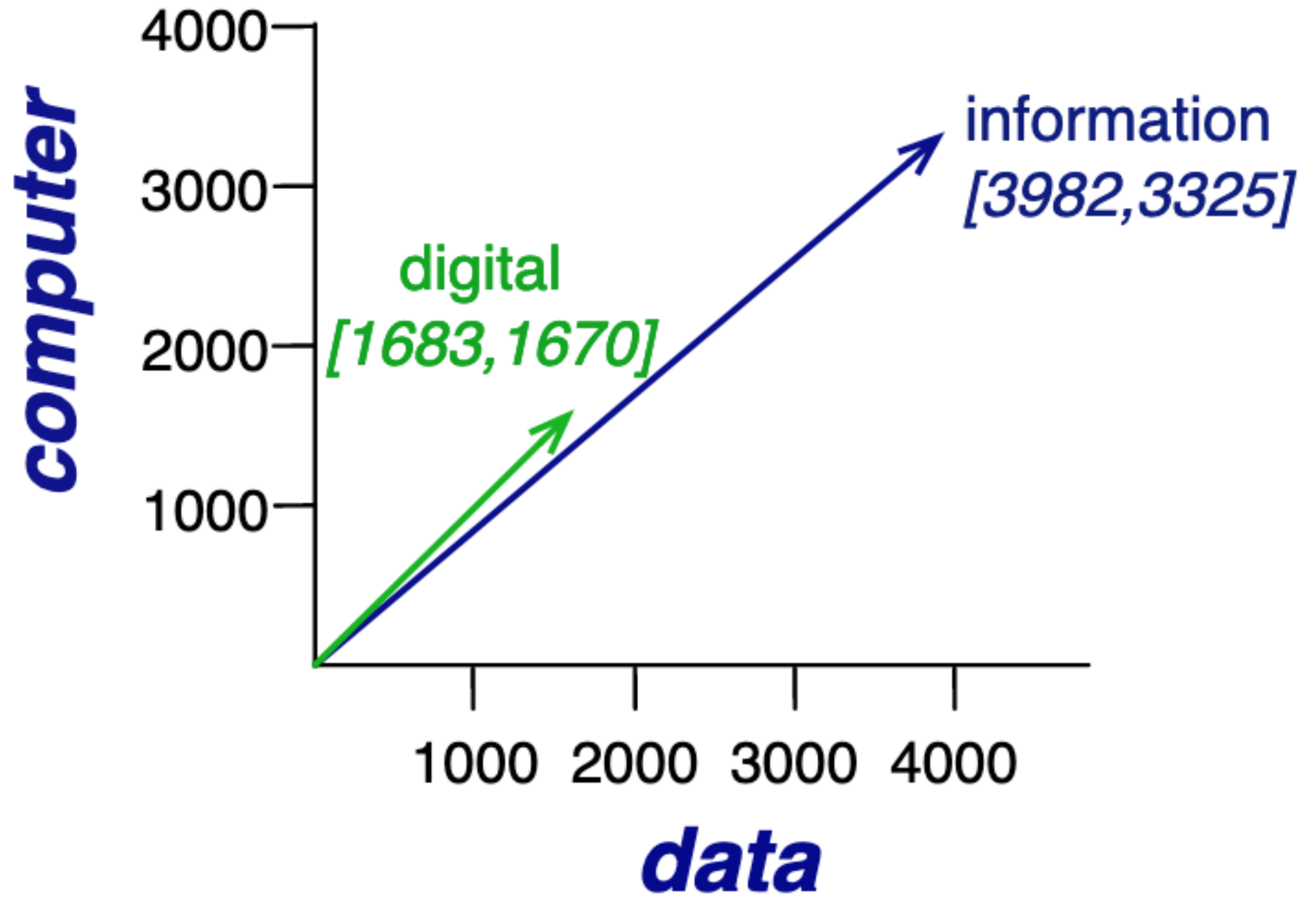

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- Problems:
  - Still very **high-dimensional** ($|V|$)
  - Still very **sparse** (most pairs of words **never co-occur**)

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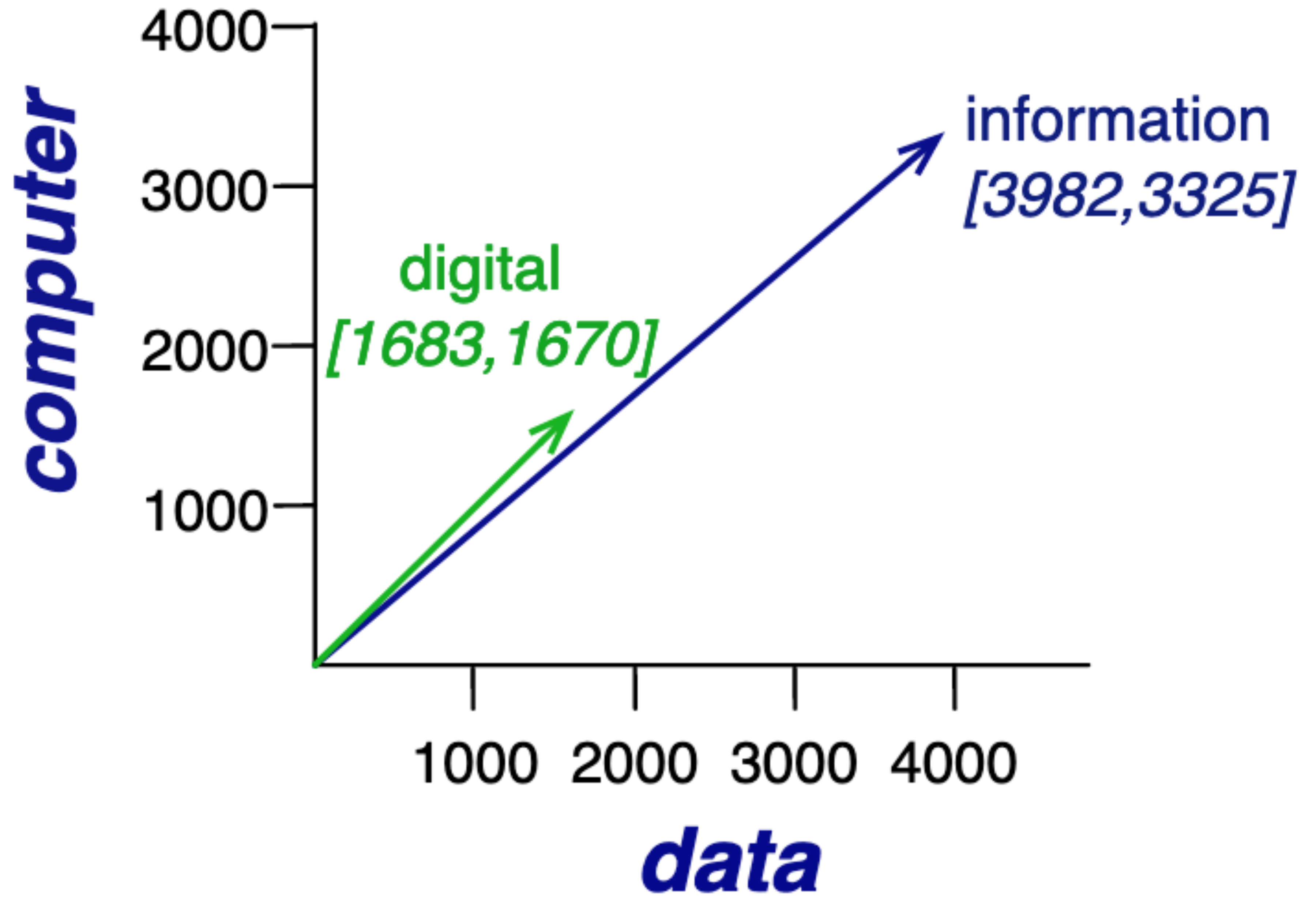

Prediction-Based Models (word2vec)

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- Words with **similar meanings** share **similar contexts**
- Instead of counting, train models to **predict context words**
- The **embeddings** (word vectors) used to accomplish the prediction become **semantically structured**

Skip-Gram vs. Continuous Bag of Words

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- Continuous Bag of Words (CBOW):
  - $P(\textit{word} \mid \textit{context})$
  - Input: $(w_{t-1}, w_{t-2}, w_{t+1}, wt_{+2} \ldots)$
  - Output: $p(w_t)$

Skip-Gram vs. Continuous Bag of Words

- Continuous Bag of Words (CBOW):
  - ***P*(*word* | *context*)**
  - Input: (w_{t-1}, w_{t-2}, w_{t+1}, w_{t+2} …)
  - Output: $p(w_t)$
- **Skip-gram:**
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Skip-Gram vs. Continuous Bag of Words

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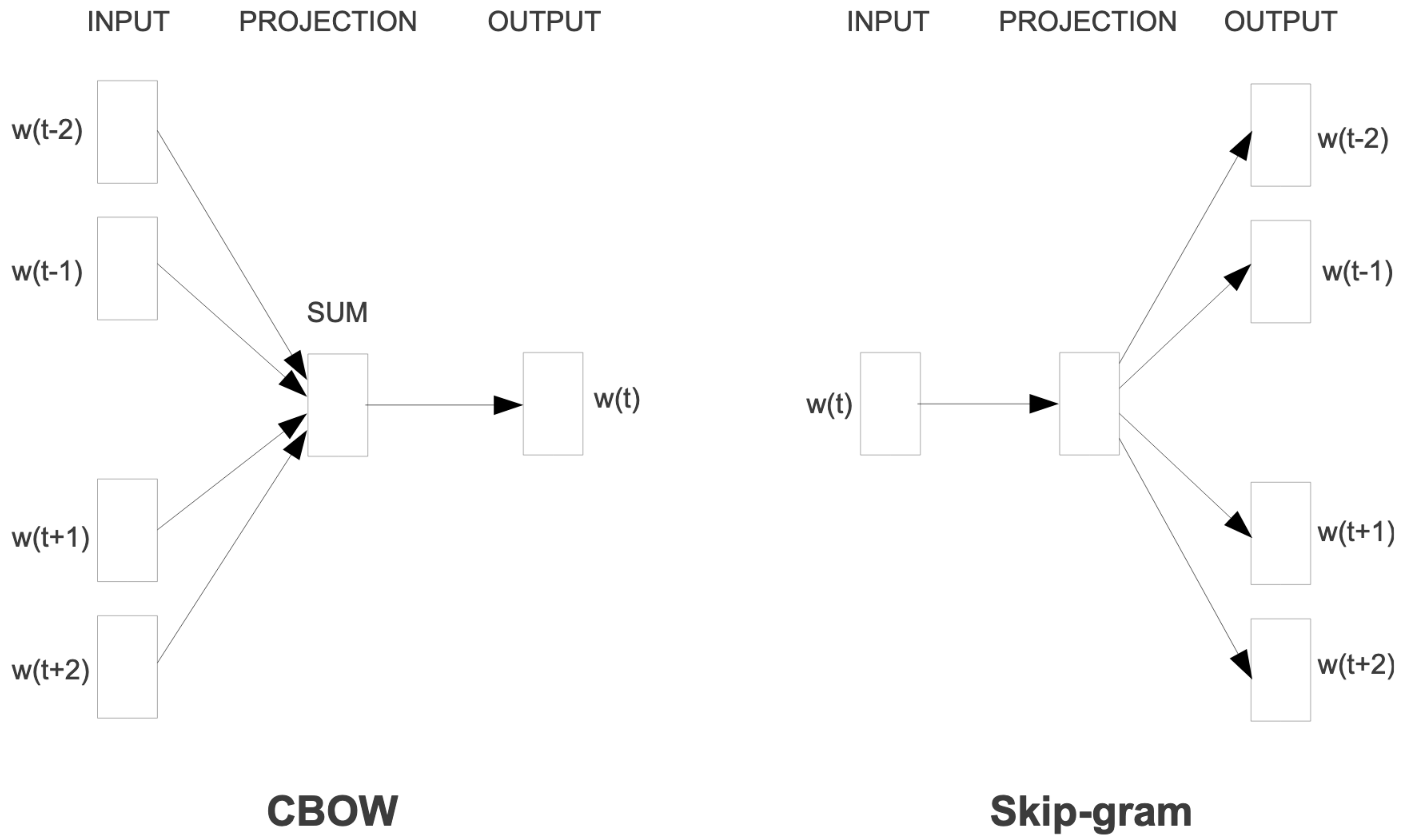

Mikolov et al 2013a (the OG word2vec paper)

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- Learns **two embedding matrices**
  - W : **word**, matrix of shape [vocab_size, embedding_dimension]
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  - W : **word**, matrix of shape [vocab_size, embedding_dimension]
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- Prediction task:
  - Given a word, **predict each neighbor** word in window
  - Compute $p(w_k \mid w_j)$ as proportional to $C_k \cdot W_j$
  - Convert to probability via Softmax
    - (Softmax is a version of Sigmoid we'll discuss more later)

$$p(w_k \mid w_j) = \frac{e^{\mathbf{C}_k \cdot \mathbf{W}_j}}{\sum_i e^{\mathbf{C}_i \cdot \mathbf{W}_j}}$$

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  - Very high-dimensional ($|V|$)
  - Sparse
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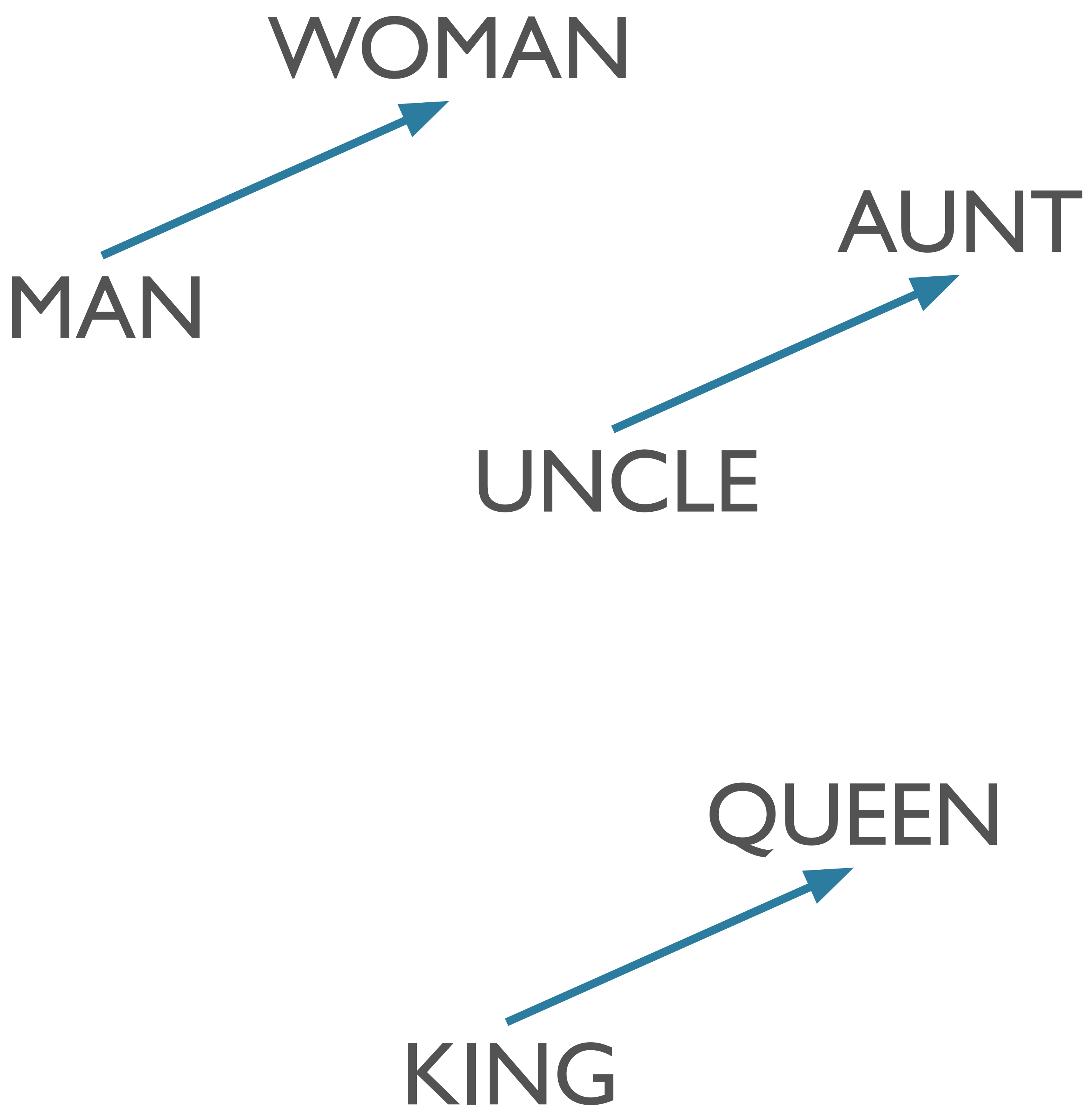
  - Pro: features are interpretable ("occurred with word W, N times in corpus")
- Prediction-based embeddings
  - **"Low"-dimensional** (typically ~256-2048)
  - **Dense**
  - Con: features are **not immediately interpretable**
    - What does "dimension 36 has value -9.63" mean?

Relationships via Offsets



Mikolov et al 2013b

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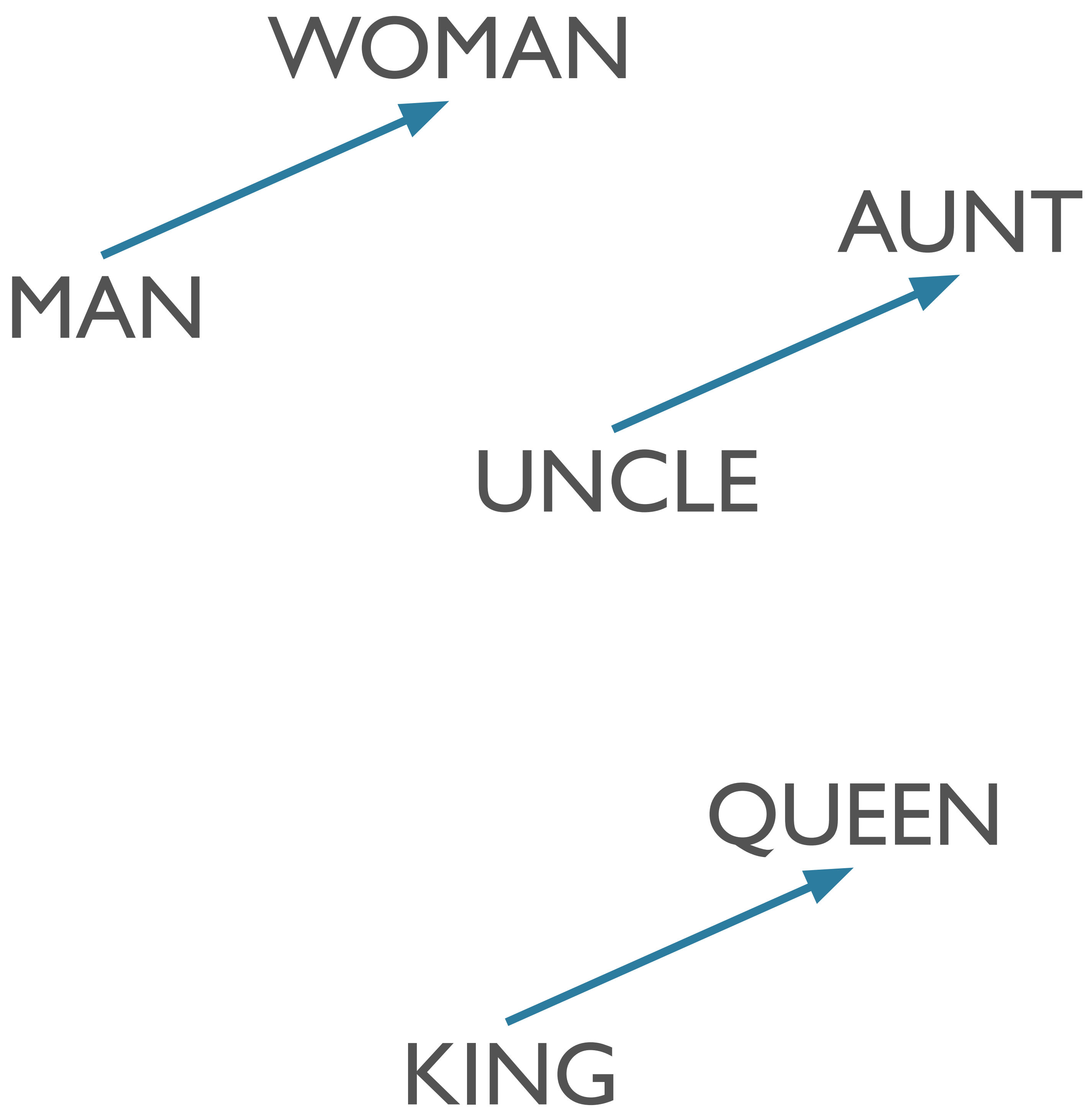

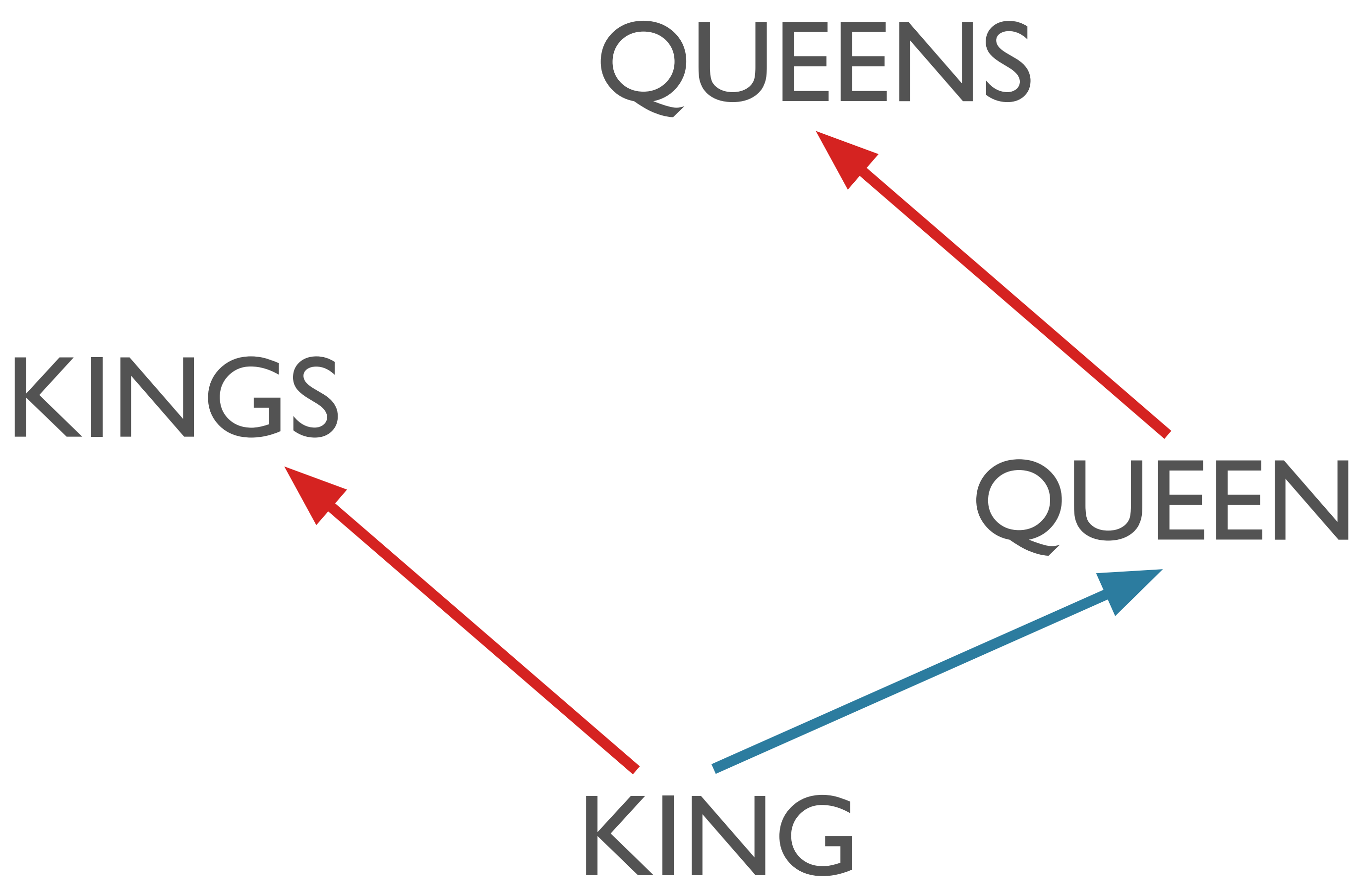

[Mikolov et al 2013b]

One More Example

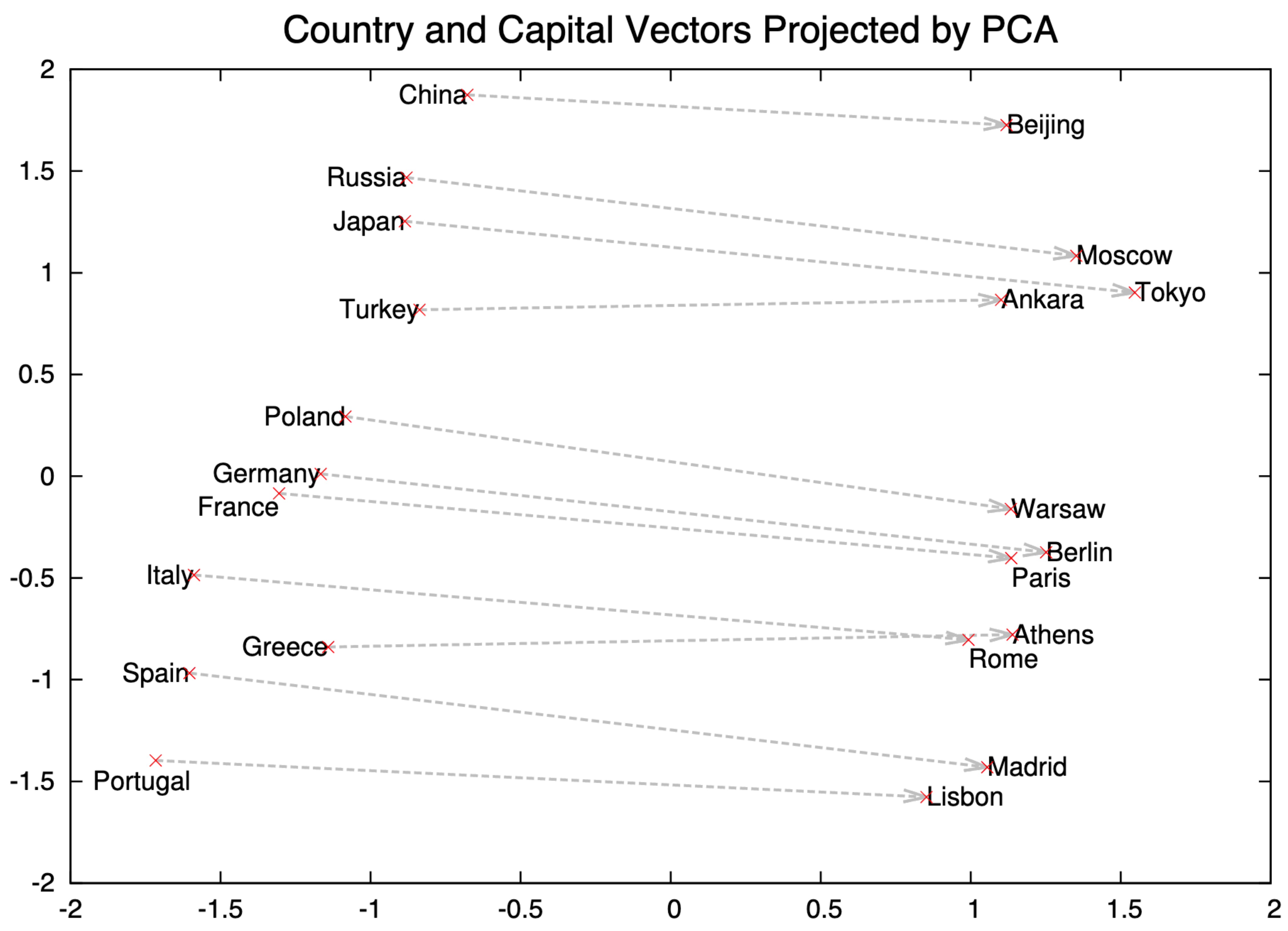

Figure 2: Two-dimensional PCA projection of the 1000-dimensional Skip-gram vectors of countries and their capital cities. The figure illustrates ability of the model to automatically organize concepts and learn implicitly the relationships between them, as during the training we did not provide any supervised information about what a capital city means.

[Mikolov et al 2013c]

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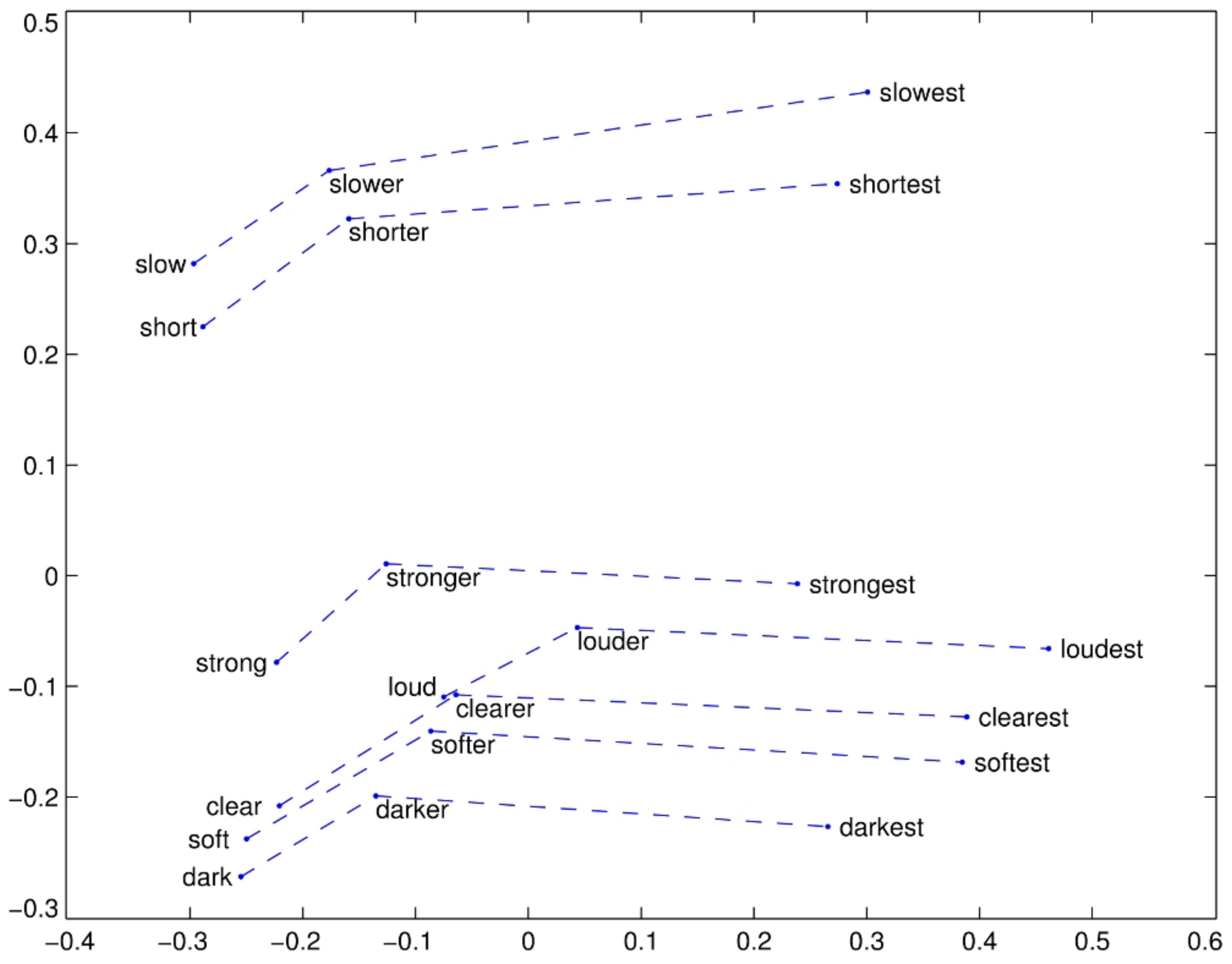

Caveat Emptor

Issues in evaluating semantic spaces using word analogies

Tal Linzen
LSCP & IJN
École Normale Supérieure
PSL Research University
tal.linzen@ens.fr

Abstract

The offset method for solving word analogies has become a standard evaluation tool for vector-space semantic models: it is considered desirable for a space to represent semantic relations as consistent vector offsets. We show that the method's reliance on cosine similarity conflates offset consistency with largely irrelevant neighborhood structure, and propose simple baselines that should be used to improve the utility of the method in vector space evaluation.

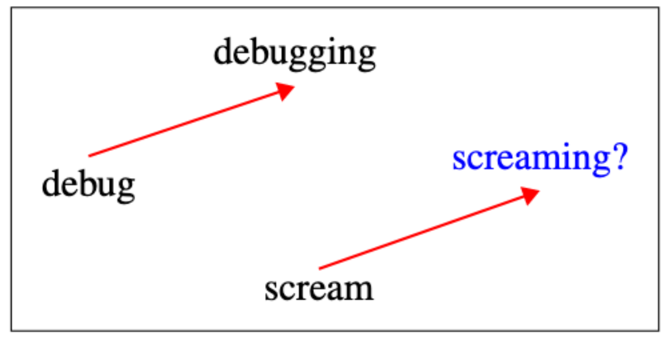

Figure 1: Using the vector offset method to solve the analogy task (Mikolov et al., 2013c).

cosine similarity to the landing point. Formally, if the analogy is given by

$$a : a^* :: b : \_\_\_ \qquad (1)$$

Linzen 2016, a.o.

Man is to Computer Programmer as Woman is to Homemaker? Debiasing Word Embeddings

Tolga Bolukbasi[1], Kai-Wei Chang[2], James Zou[2], Venkatesh Saligrama[1,2], Adam Kalai[2]
[1]Boston University, 8 Saint Mary's Street, Boston, MA
[2]Microsoft Research New England, 1 Memorial Drive, Cambridge, MA
tolgab@bu.edu, kw@kwchang.net, jamesyzou@gmail.com, srv@bu.edu, adam.kalai@microsoft.com

Abstract

The blind application of machine learning runs the risk of amplifying biases present in data. Such a danger is facing us with *word embedding*, a popular framework to represent text data as vectors which has been used in many machine learning and natural language processing tasks. We show that even word embeddings trained on Google News articles exhibit female/male gender stereotypes to a disturbing extent.

Bolukbasi et al 2016

Skip-Gram with Negative Sampling (SGNS)

Training The Skip-Gram Model

- Issue: **denominator** computation is **very expensive**
- Alternative: approximate by **negative sampling**
  - Use **"fake" examples**, and make their probability **small**
  - + example: true context word
  - – examples: *k* other words, randomly sampled

$$p(w_k \mid w_j) = \frac{e^{C_k \cdot W_j}}{\sum_i e^{C_i \cdot W_j}}$$

Negative Sampling, Idea

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- Skip-Gram
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- Skip-Gram
  - $p(w_k \mid w_j)$: what is the probability that w_k occurred in the context of w_j
  - Classifier with $|V|$ classes
- Negative sampling
  - $p(+ \mid w_k, w_j)$: what is the probability that (w_k, w_j) was a **true co-occurrence**?
  - $p(- \mid w_k, w_j) = 1 - p(+ \mid w_k, w_j)$
    - Probability that (w_k, w_j) was **not** a true co-occurrence
    - Examples of **“fake” co-occurrences** = **negative samples**
  - Binary classifier

Generating Positive Examples

```
... lemon,  a [tablespoon of apricot jam,      a] pinch ...
               c1          c2     w       c3         c4
```

positive examples +

w	c_{pos}
apricot	tablespoon
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Generating Positive Examples

- Iterate through the corpus

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Generating Positive Examples

- Iterate through the corpus
- For each word: add **all words within *window_size*** of the current word as **positive** pairs
  - *window_size* is a **hyper-parameter**

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Negative Samples

- For each positive (w, c) sample, **generate *num_negatives* samples**
  - $(w, c\_)$, where $c\_$ is different from c
  - *num_negatives* is another hyper-parameter

negative examples -

w	c_{neg}	w	c_{neg}
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

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sigmoid

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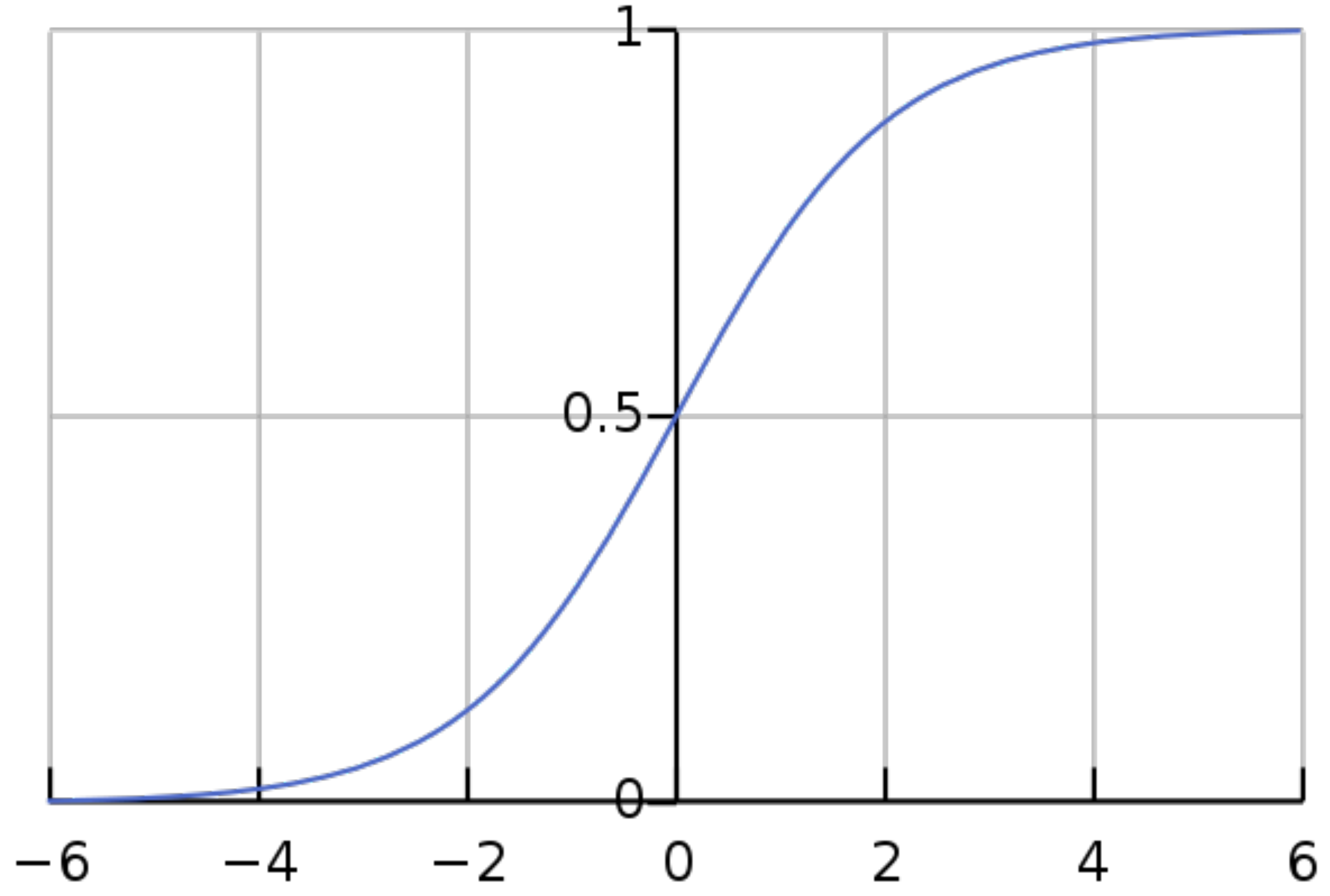
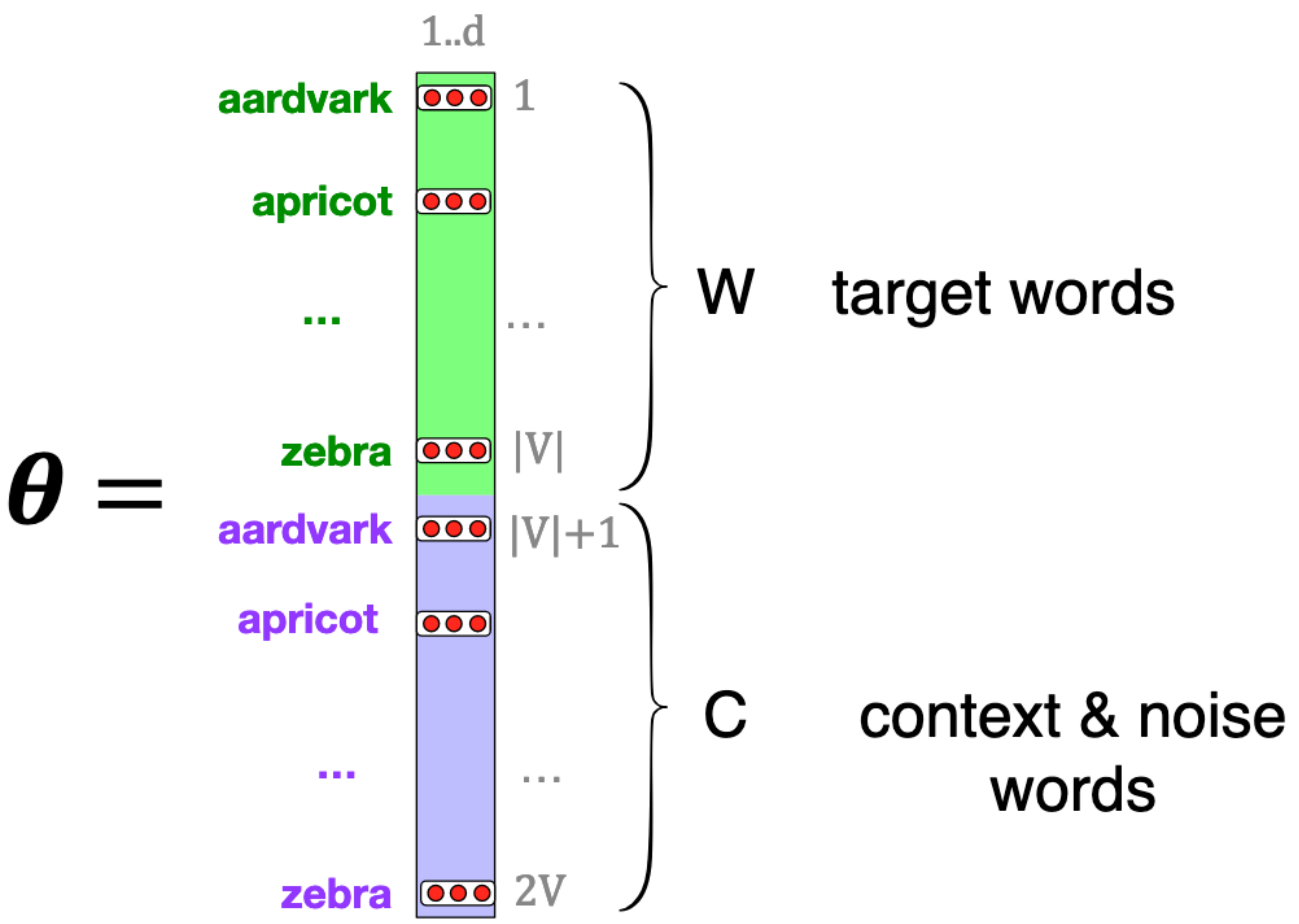
Context word embedding

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



Learning: Parameters



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- Target and context words that are **more similar** to each other (have more similar embeddings) have a **higher probability** of being a positive example

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The Model

- Target and context words that are **more similar** to each other (have more similar embeddings) have a **higher probability** of being a positive example
- We want our model to:
  - Assign high $p(1 \mid w, c_+)$ (c+ is a positive context word)
  - Assign low $p(1 \mid w, c_-)$ (c- is a negative context word)
    - Equivalently: assign high $p(0 \mid w, c_-)$

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- To make it a **loss function**, take the **negative log probability** $-log(p)$

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- I.e. the **negative log probability** that the model assigns to the true label
- BCE loss **incorporates both** into the closed form (y is either 1 or 0)

$$\ell_{BCE}(\hat{y}, y) := -y \log \hat{y} - (1 - y)\log(1 - \hat{y})$$

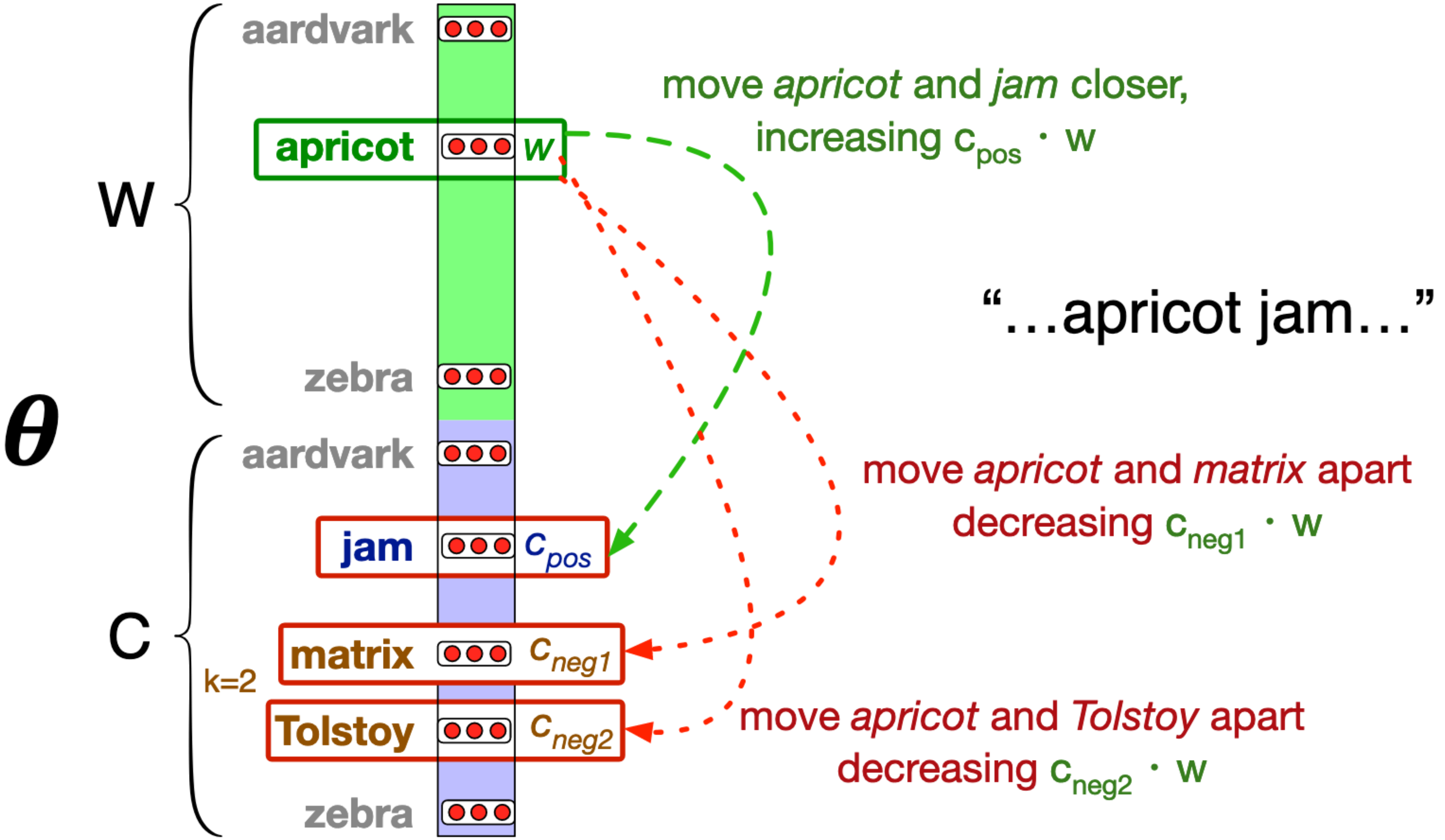
Training Loop w/ Negative Samples

```
initialize parameters / build model

for each epoch:

 positives = shuffle(positives)

 for each example in positives:

  positive_output = model(example)
  generate k negative samples
  negative_outputs = [model(negatives)]
  compute gradients
  update parameters
```

Combo Loss

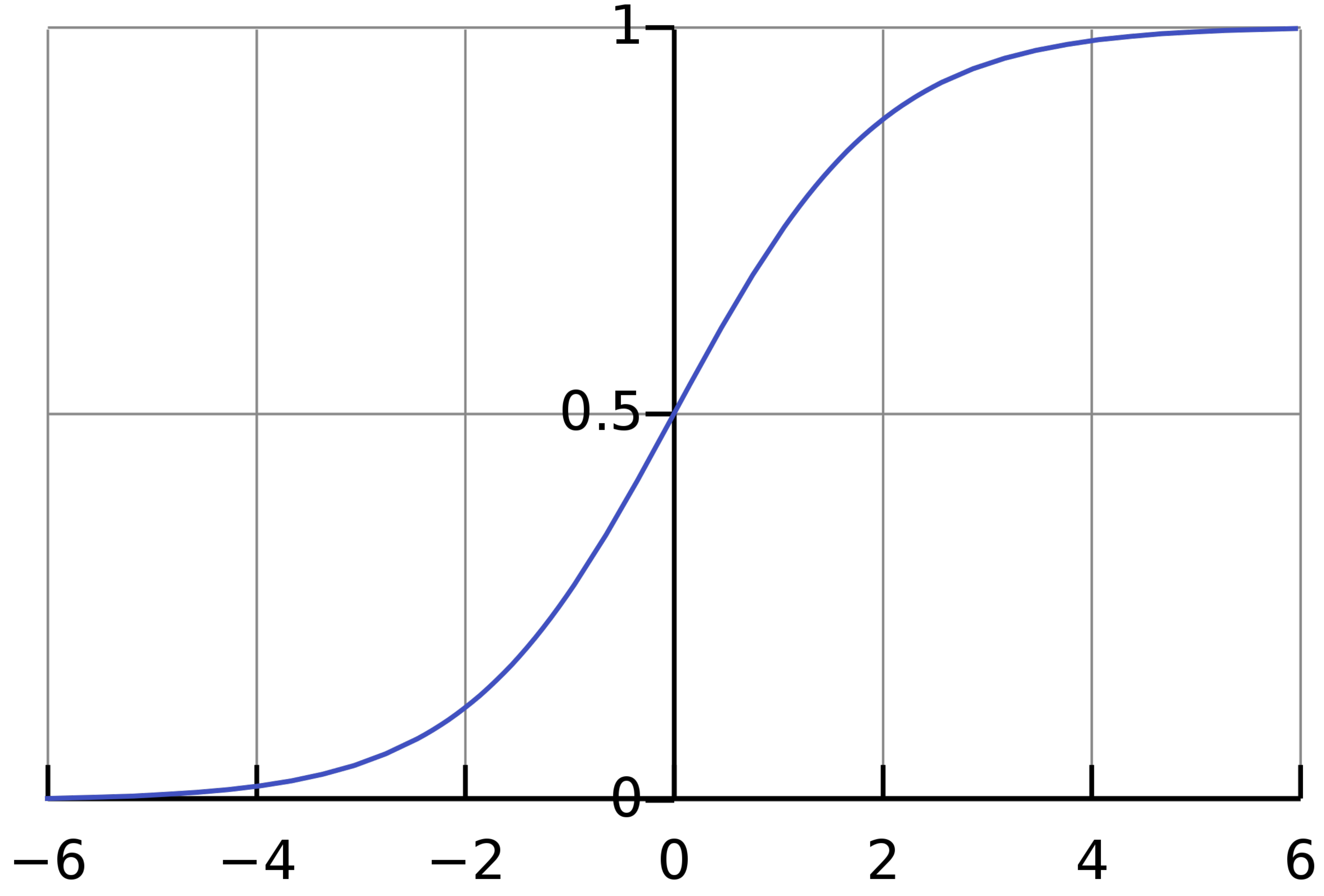
$$
\begin{aligned}
L_{CE} &= -\log P(1,0,0,\ldots,0 \mid w, c_+, c_{-1}, c_{-2}, \ldots, c_{-k}) \\
&= -\log\left(P(1 \mid w, c_+) \prod_{i=1}^{k} P(0 \mid w, c_{-i})\right) \\
&= -\log P(1 \mid w, c_+) - \sum_{i=1}^{k} \log P(0 \mid w, c_{-i})
\end{aligned}
$$

Learning: Intuitively



Appendix: Sigmoid and Softmax

Sigmoid



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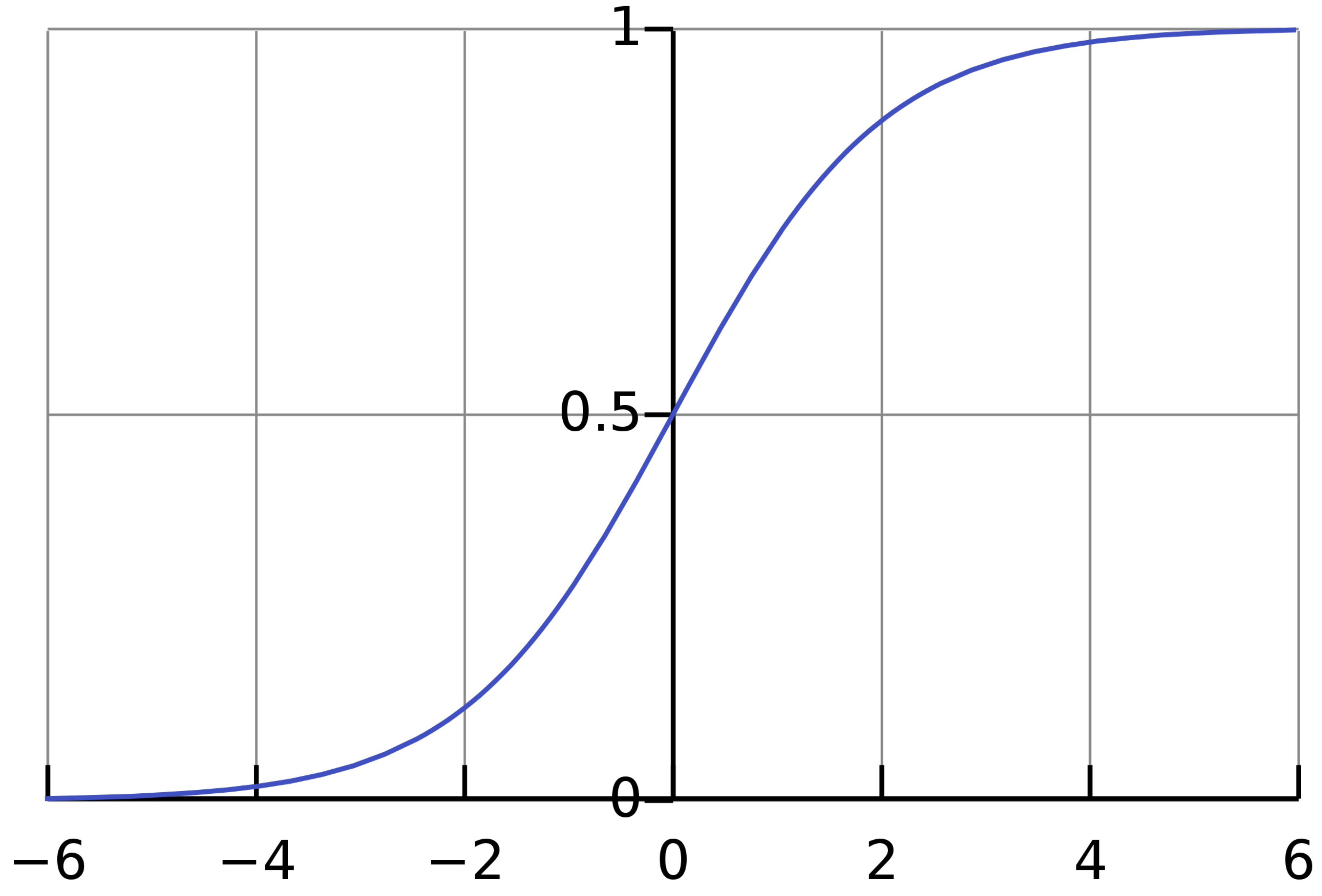

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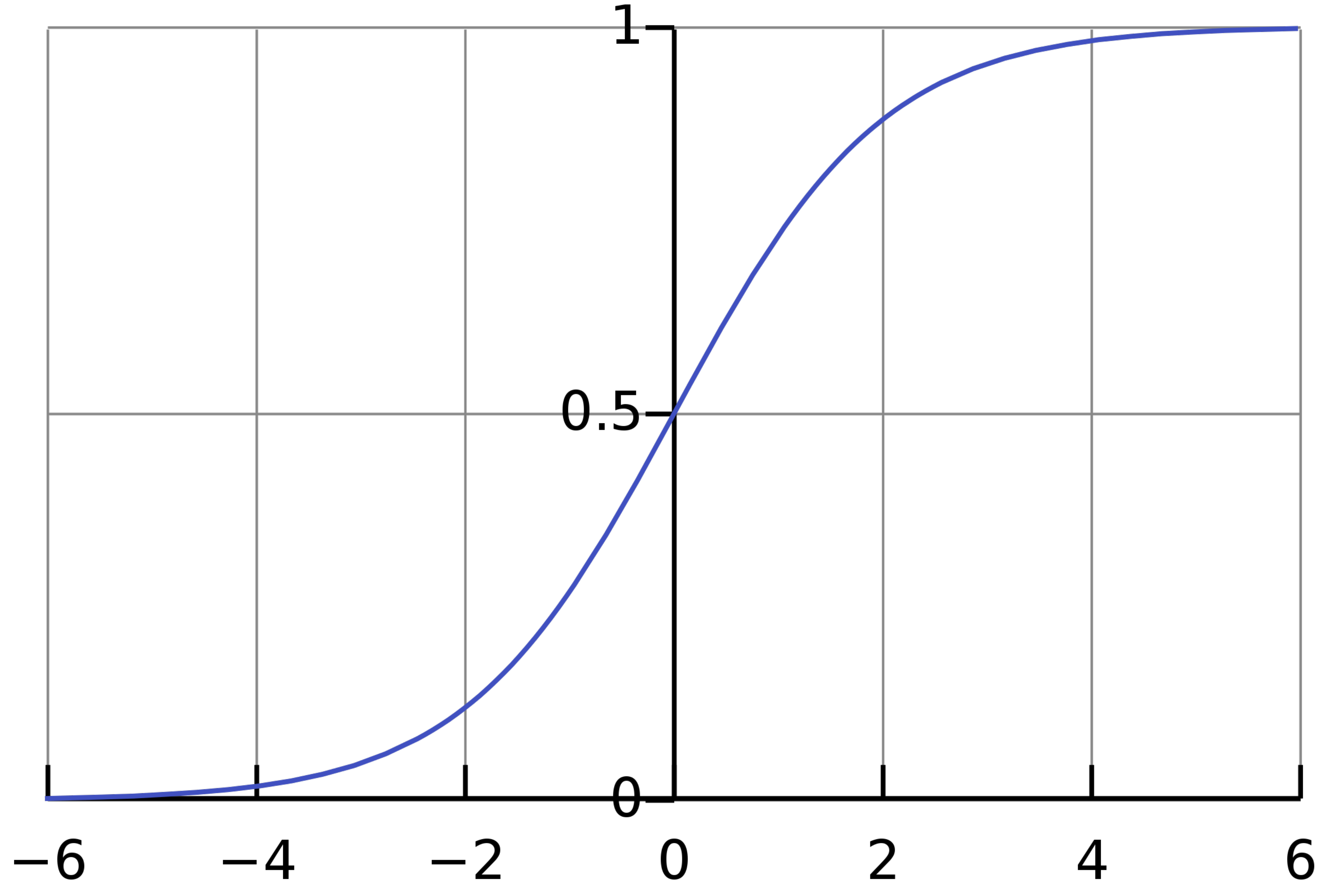

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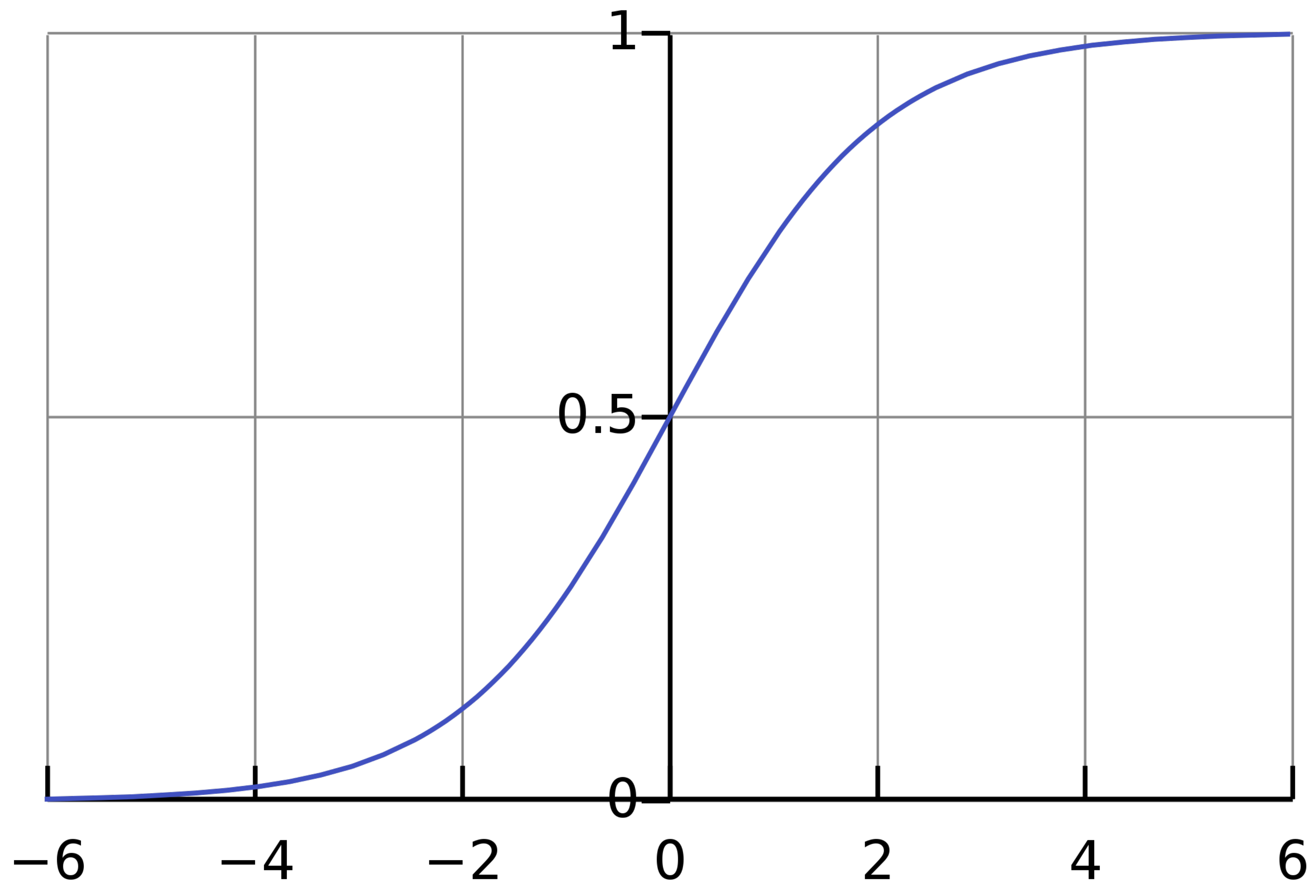

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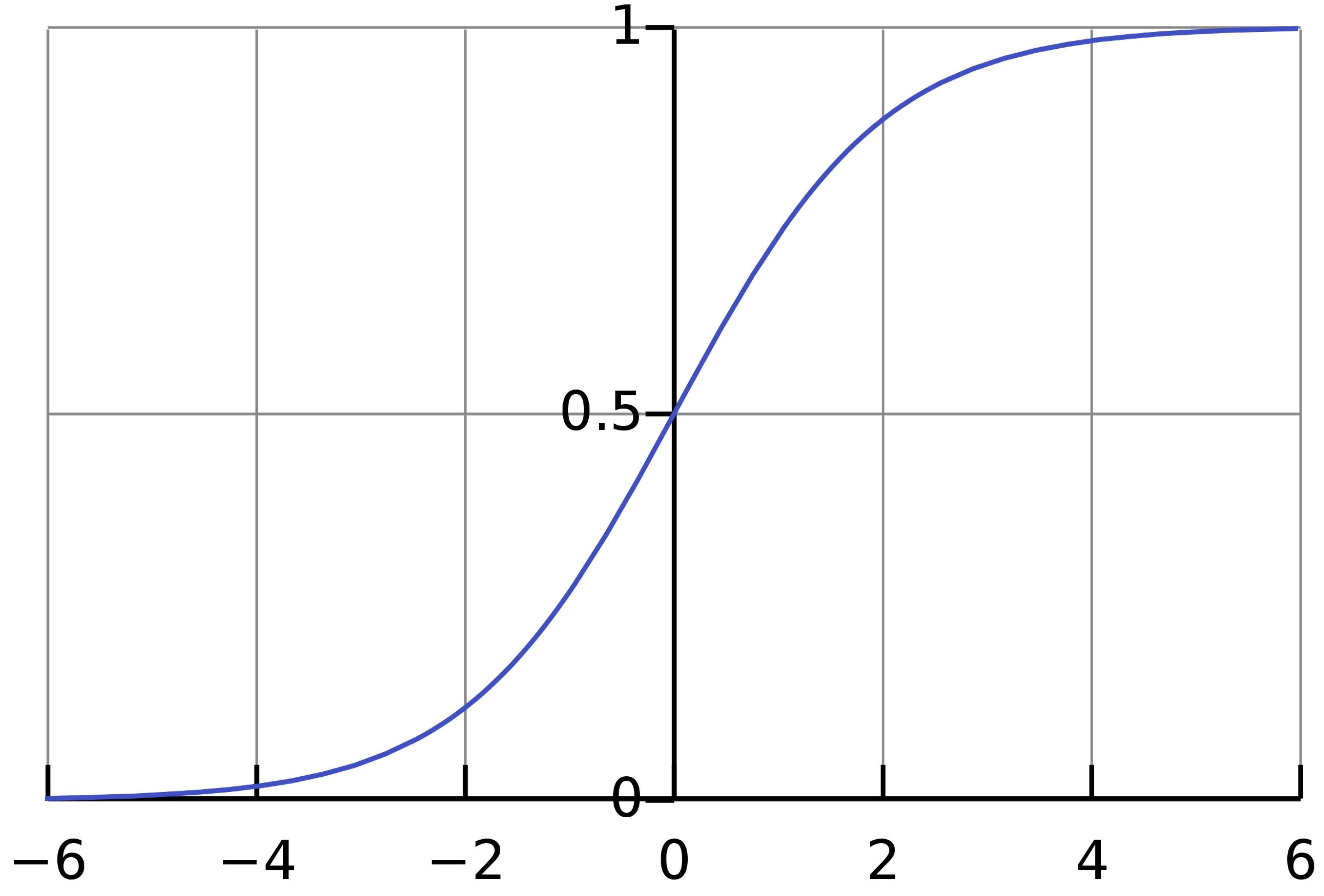

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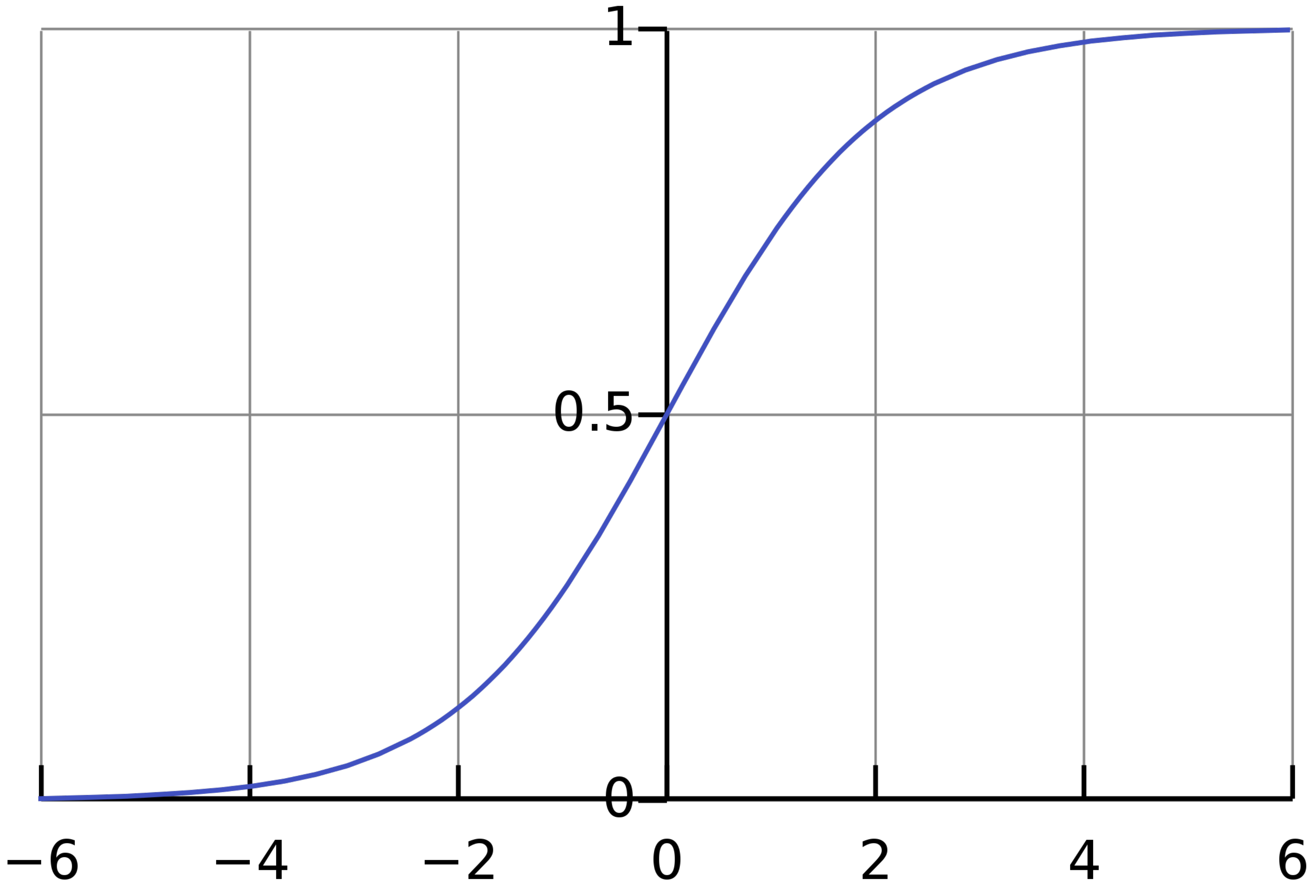

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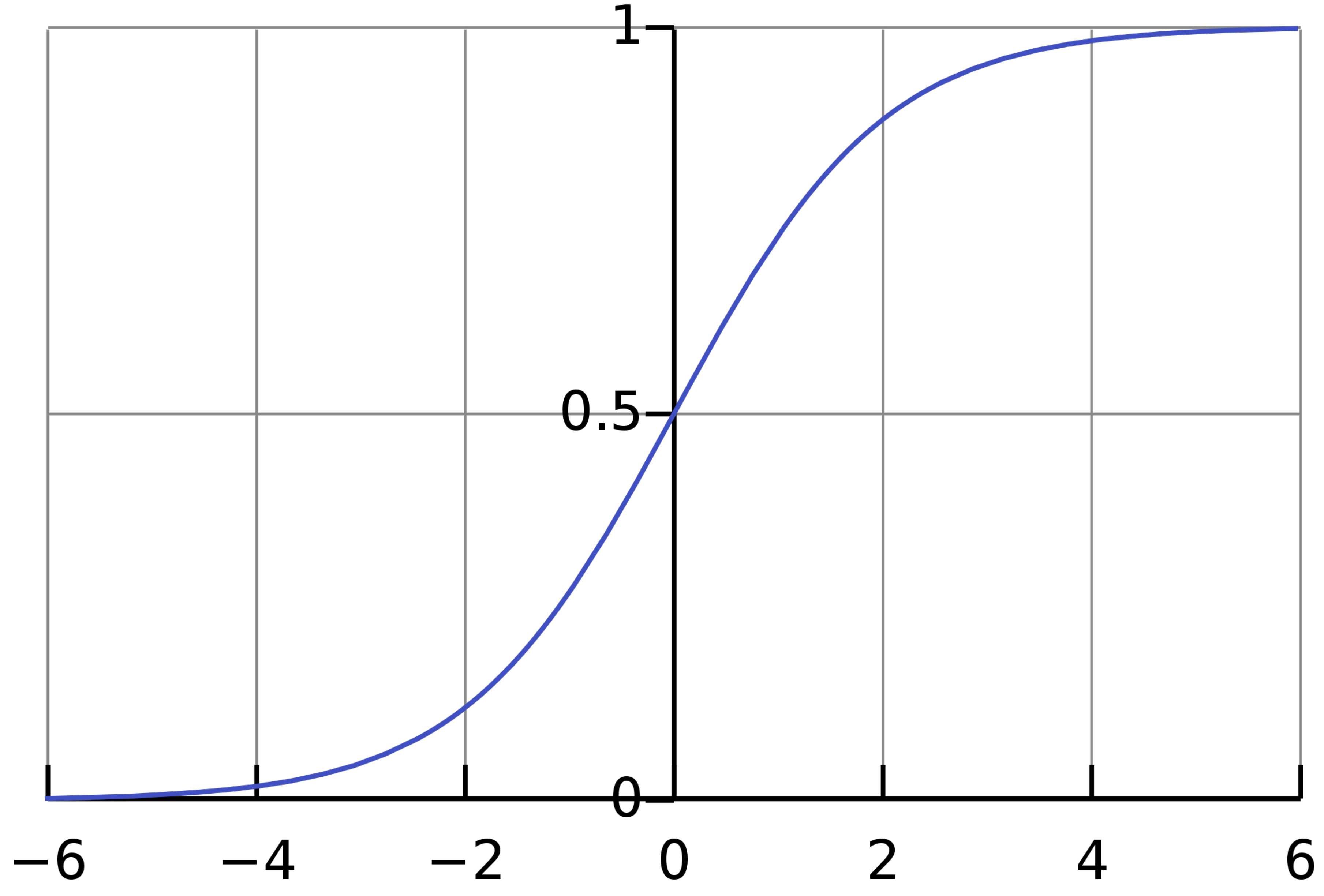

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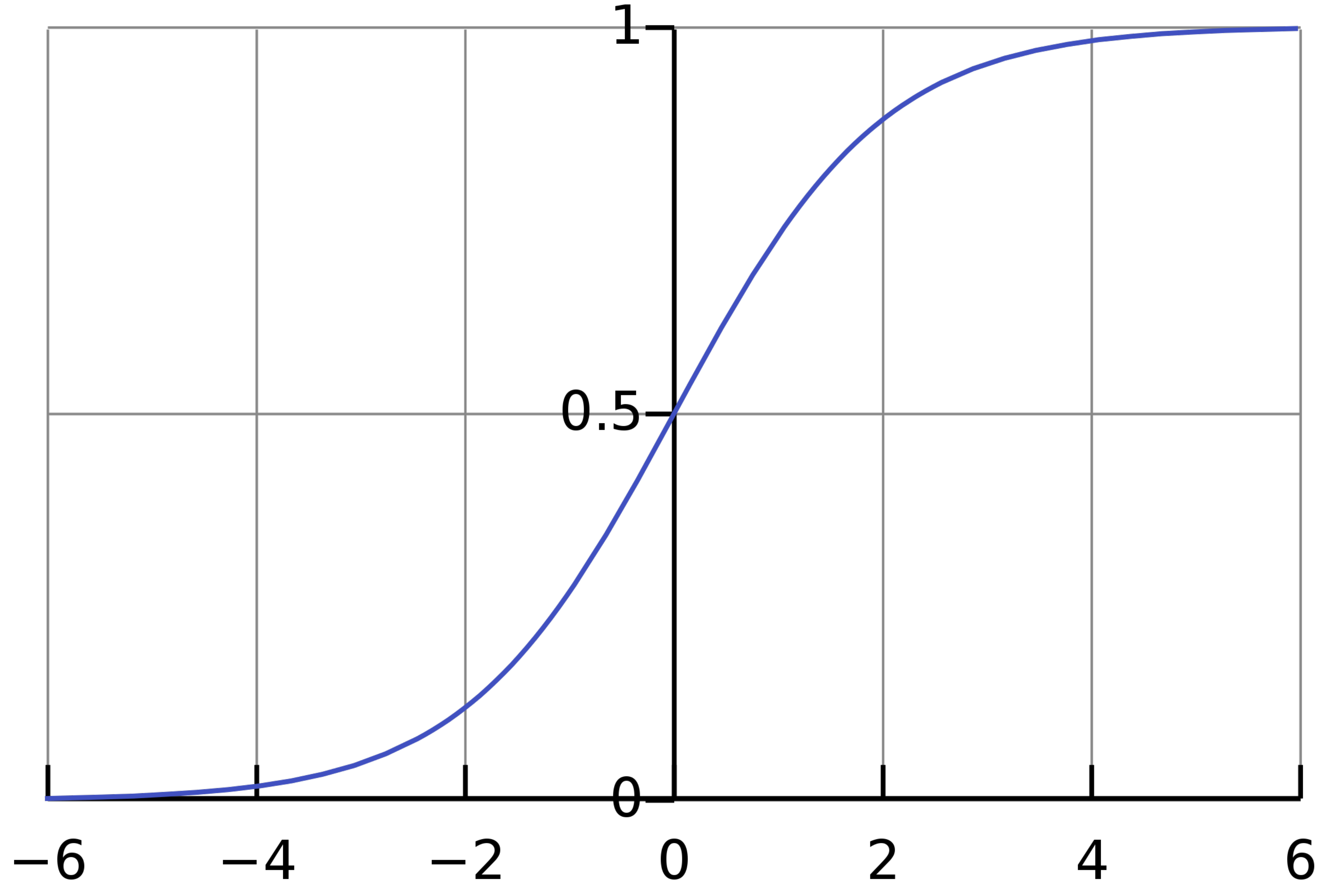
- $x < 0 \rightarrow 0 \leq e^x < 1$ (True odds **less than** False)
- $x > 0 \rightarrow 1 < e^x \leq \infty$ (True odds **greater than** False)



$$\sigma(x) = \frac{e^x}{e^x + 1} = \frac{e^x}{e^x + e^0}$$

"True" odds

"True" odds

"False" odds

Softmax

$$\text{Softmax}(x)_i = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

odds of class i

summed odds of
all classes

Softmax

- Softmax is the **extension of Sigmoid** for classification into **more than 2 classes**
  - Input: a **vector** x
    - x_i is the **"score"** for class i
  - Output: a **vector of probabilities** p
    - p is the **same size** as x
    - p_i is the **probability** of class i
    - The probabilities **sum to 1.0** (this is a valid **probability distribution**)

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- The input "scores" x are often called the **logits**

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