Word Vectors; Gradient Descent

LING 575j: Deep Learning for NLP

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Spring 2023

Announcements

- max_size: not including special tokens like <unk> (cf lines 43-44 in vocabulary.py)
- Python 3.9:

```python
def save_to_file(self, filename: str) -> None:
    """Write the vocab to a file, including frequencies.

    Args:
        filename: name of file to save to.
    """
```

  - The code makes heavy use of type hinting
    - Improves readability / future-proofing
    - Works well with code completers, static type checkers like mypy
  - Including *native* type hinting for many data structures, which is new to 3.9
  - So be sure to run in the environment we provide, which includes 3.9

Announcements

- Developing locally:
  - Download `requirements.txt` from our dropbox
  - `conda create —name 575j python=3.9`
  - `conda activate 575j`
  - `pip install -r requirements.txt`
  - This should give you a local environment that matches the one we provide on patas

Today's Plan

- Word Vectors
- Machine Learning Terminology / Notation
- Gradient Descent

Word Vectors, Intro

Distributional Similarity

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  - We make ***tezgüino*** from corn.
- Tezguino; corn-based alcoholic beverage. (From *Lin, 1998a*)

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- How can we make similar words have similar representations?

Why use word vectors?

- With words, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires exact same word to be in training and test
  - **One-hot vectors:**
    - “terrible”: [0 0 0 0 0 0 1 0 0 0 … 0]
    - Length = size of vocabulary
  - All words are as different from each other
    - e.g. “terrible” is as different from “bad” as from “awesome”

Why use word vectors?

- With embeddings (= vectors):
  - Feature is a word vector
  - “The previous word was vector [35,22,17, …]”
  - Now in the test set we might see a similar vector [34,21,14, …]
  - We can generalize to similar but unseen words!

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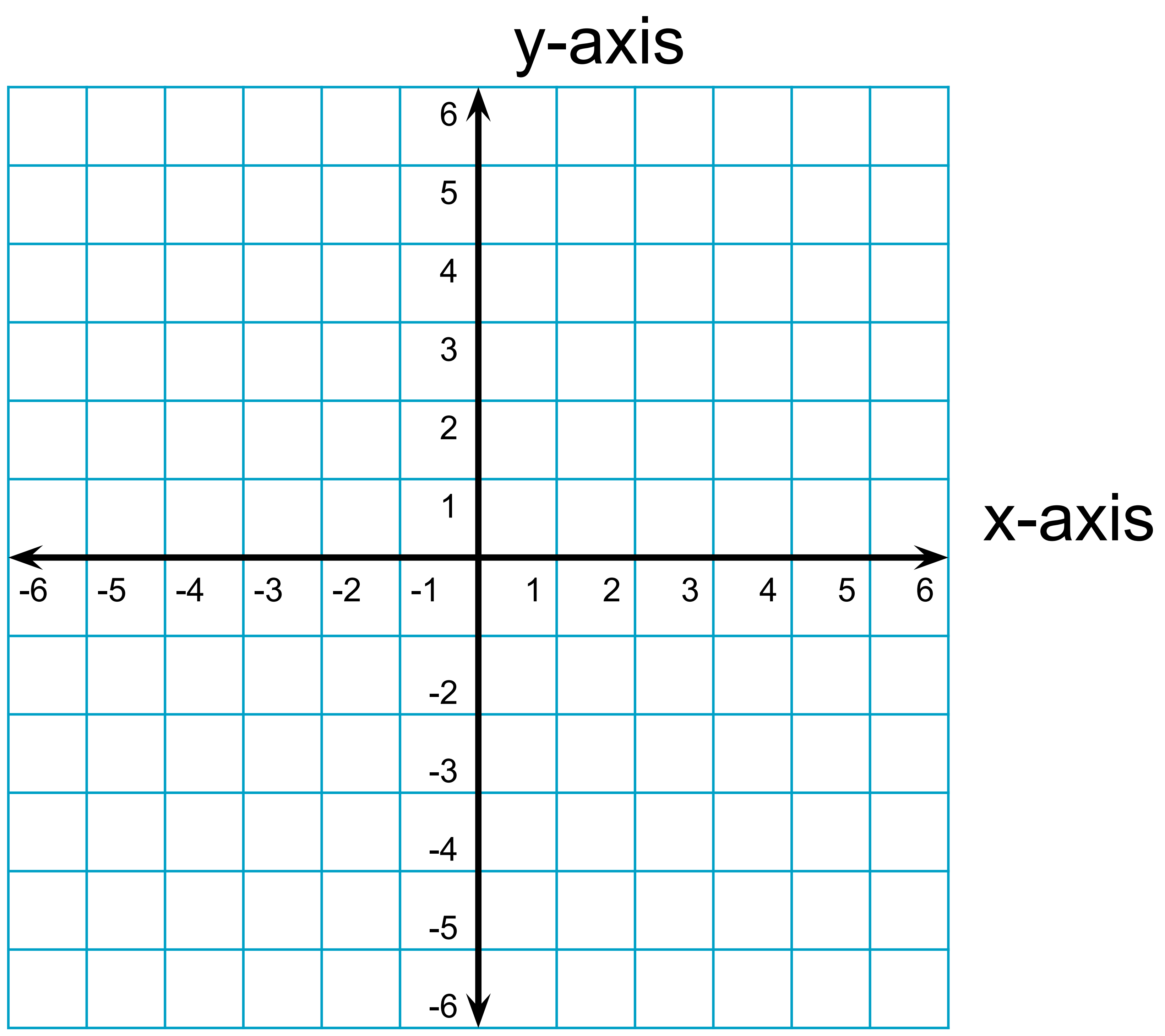

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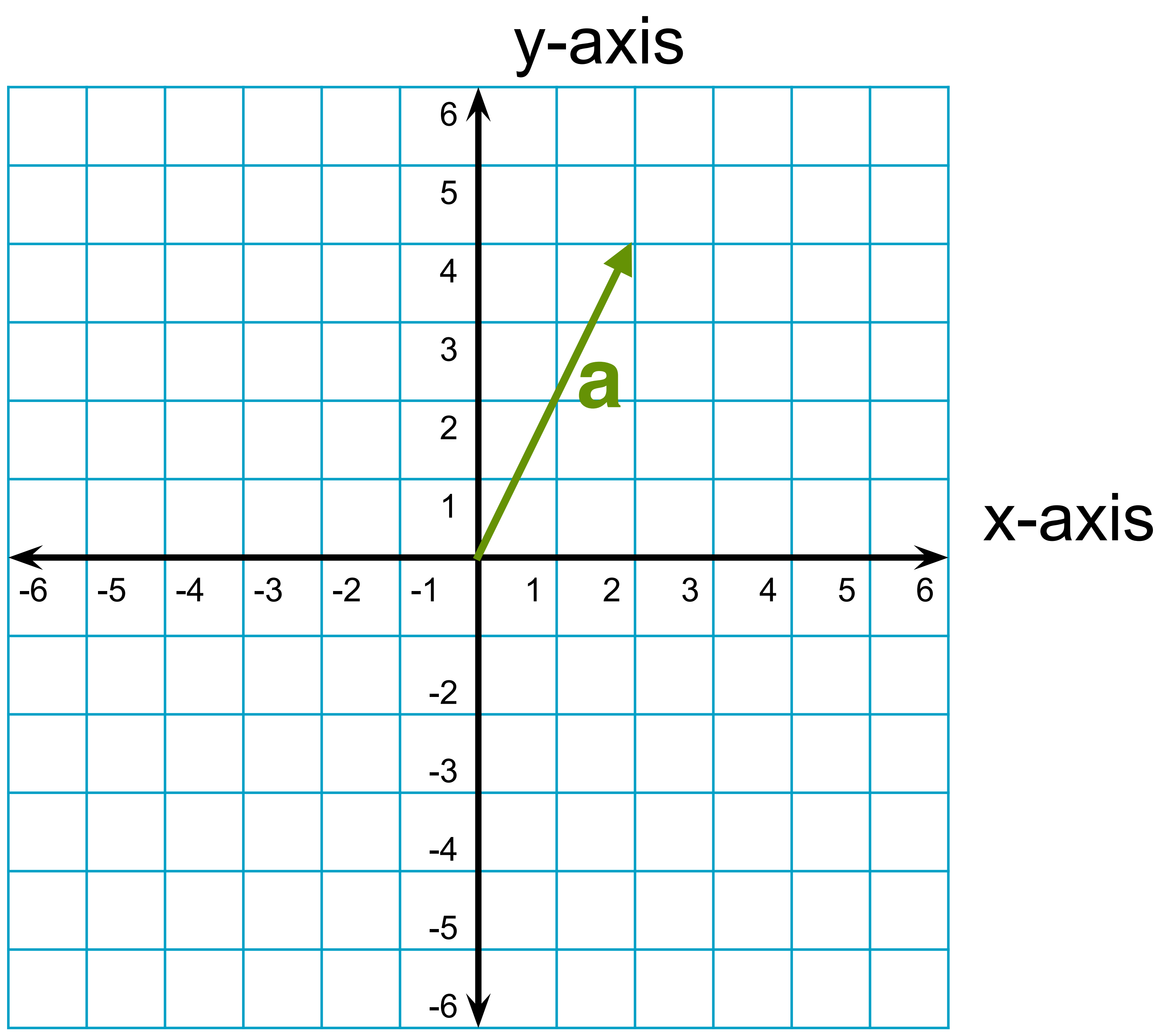

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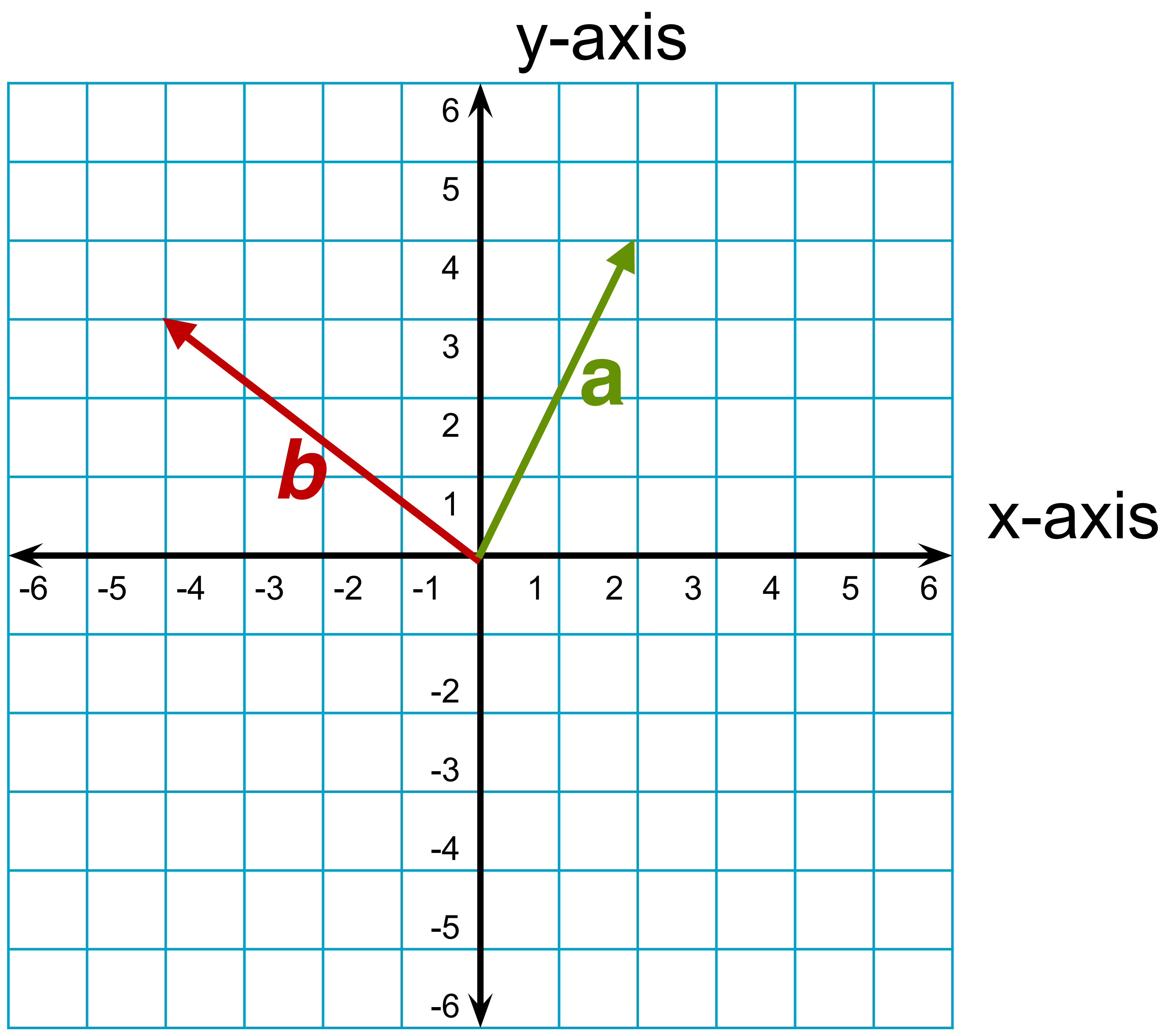

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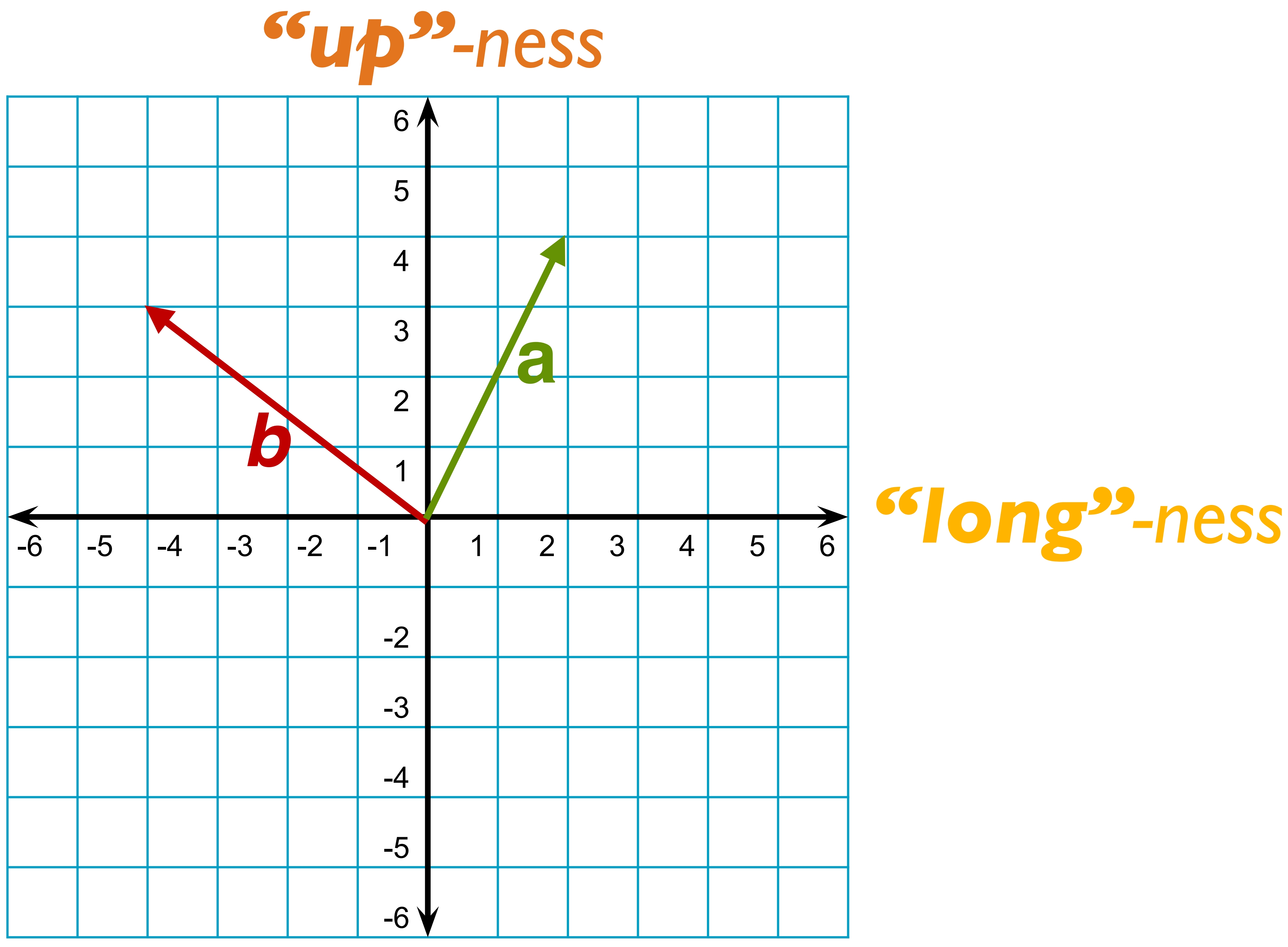

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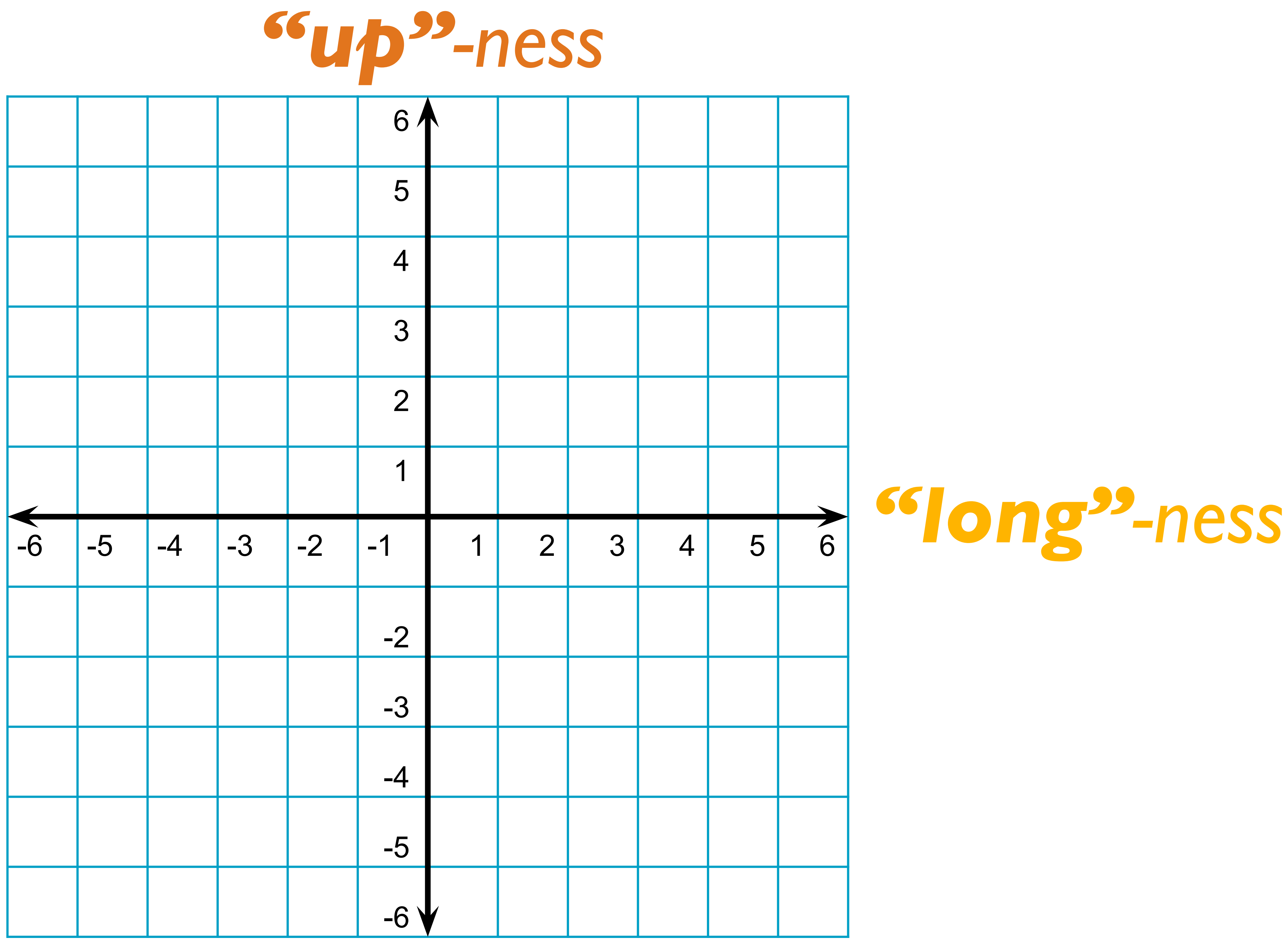

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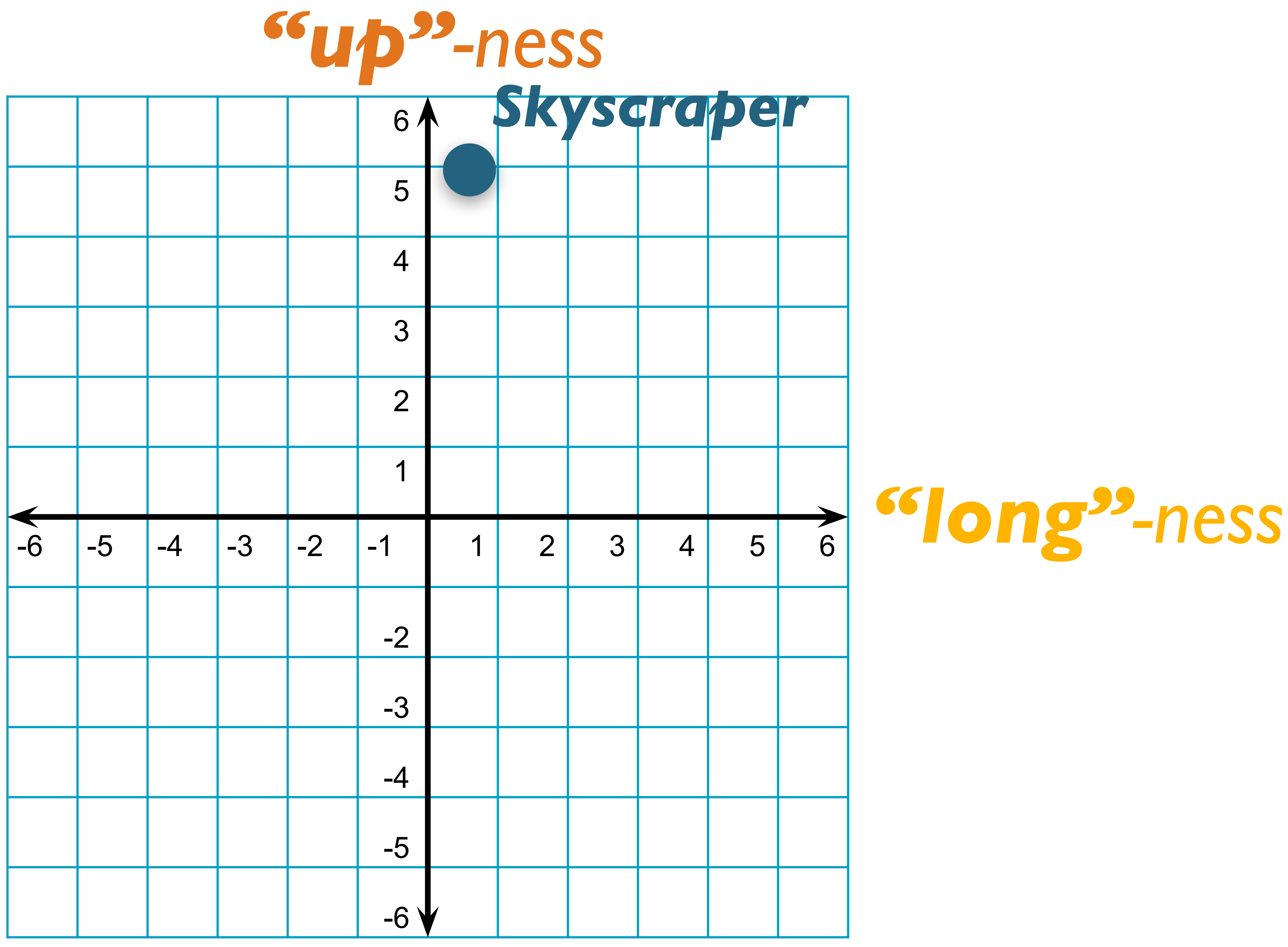

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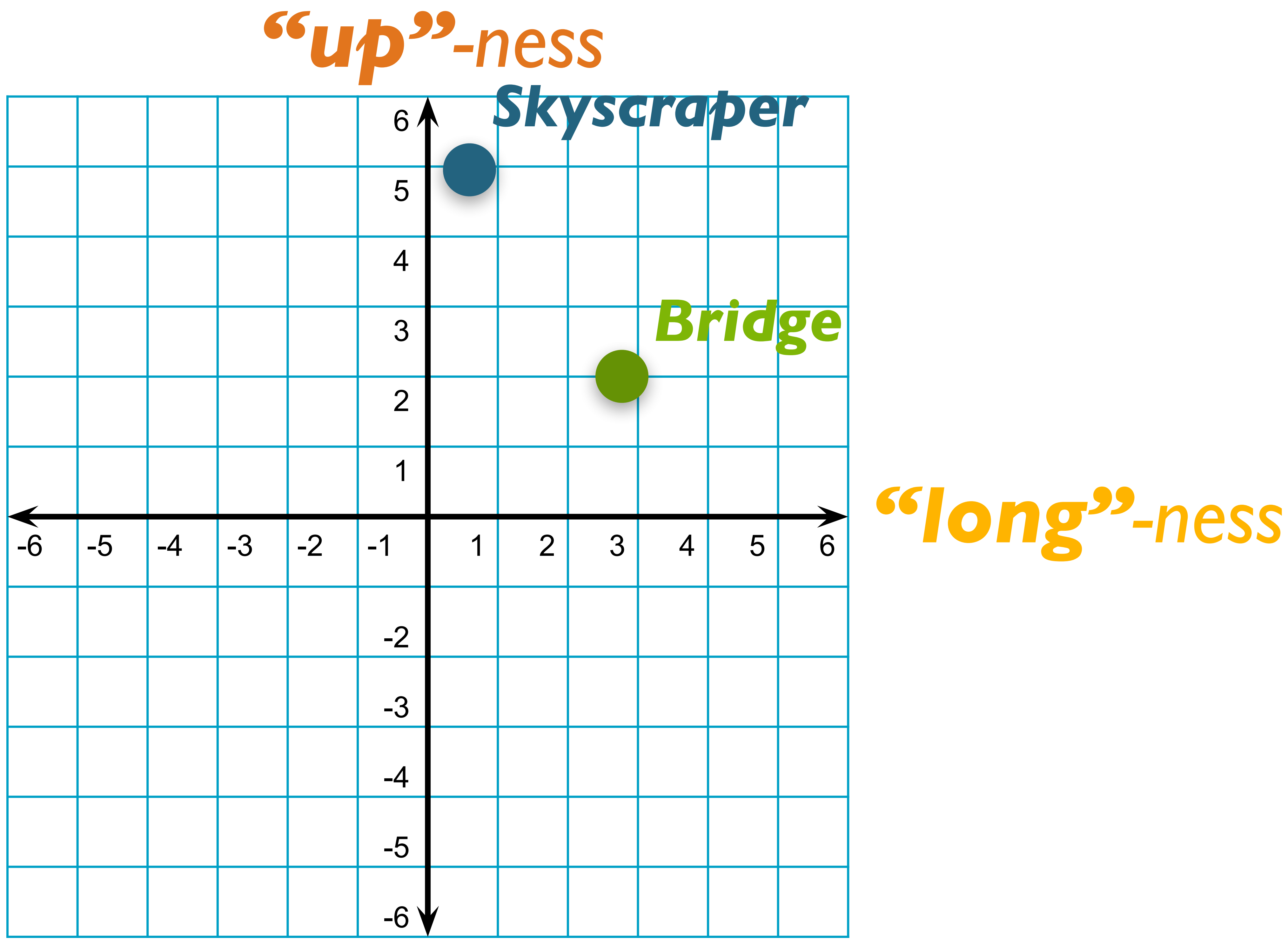

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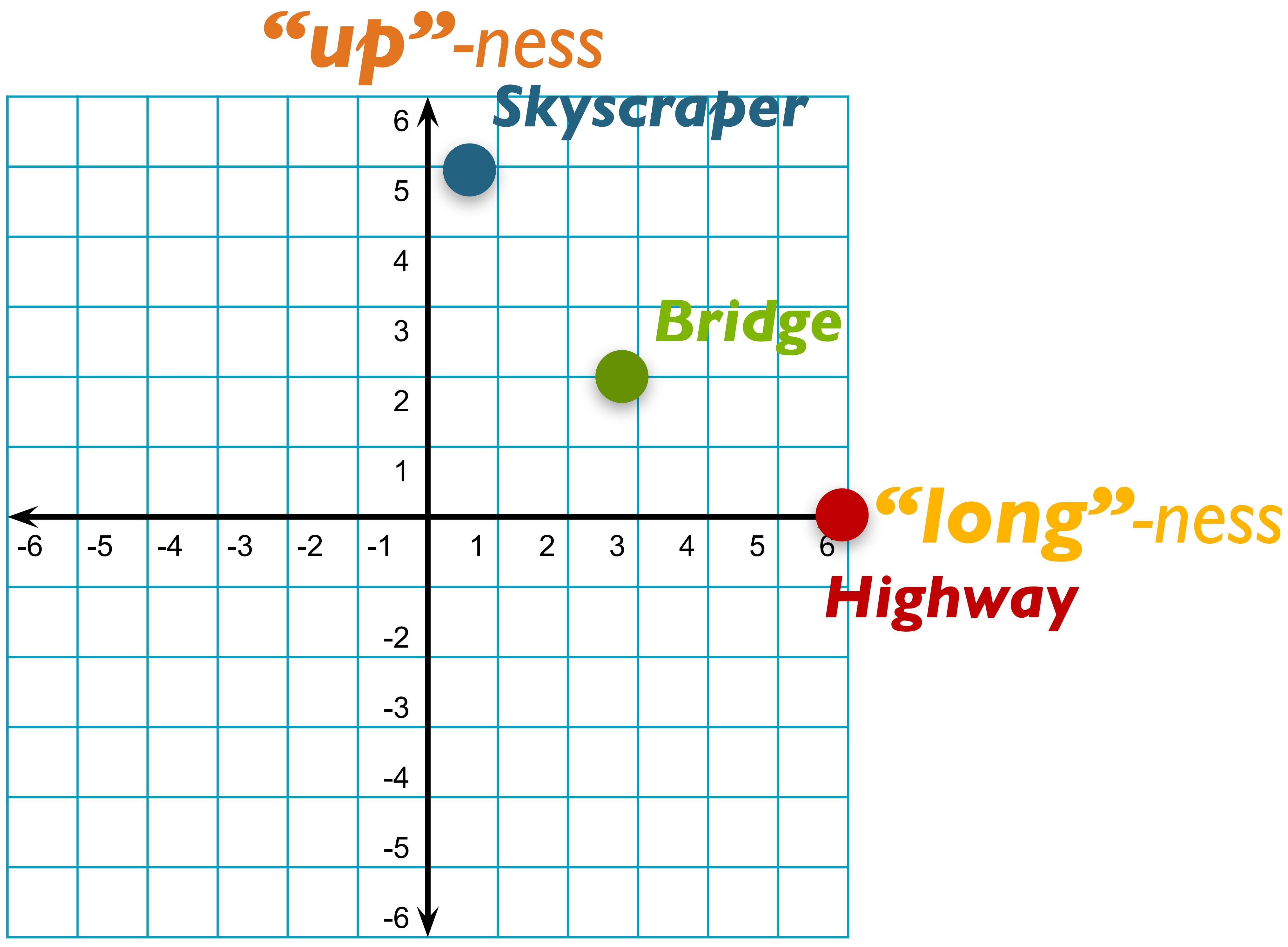

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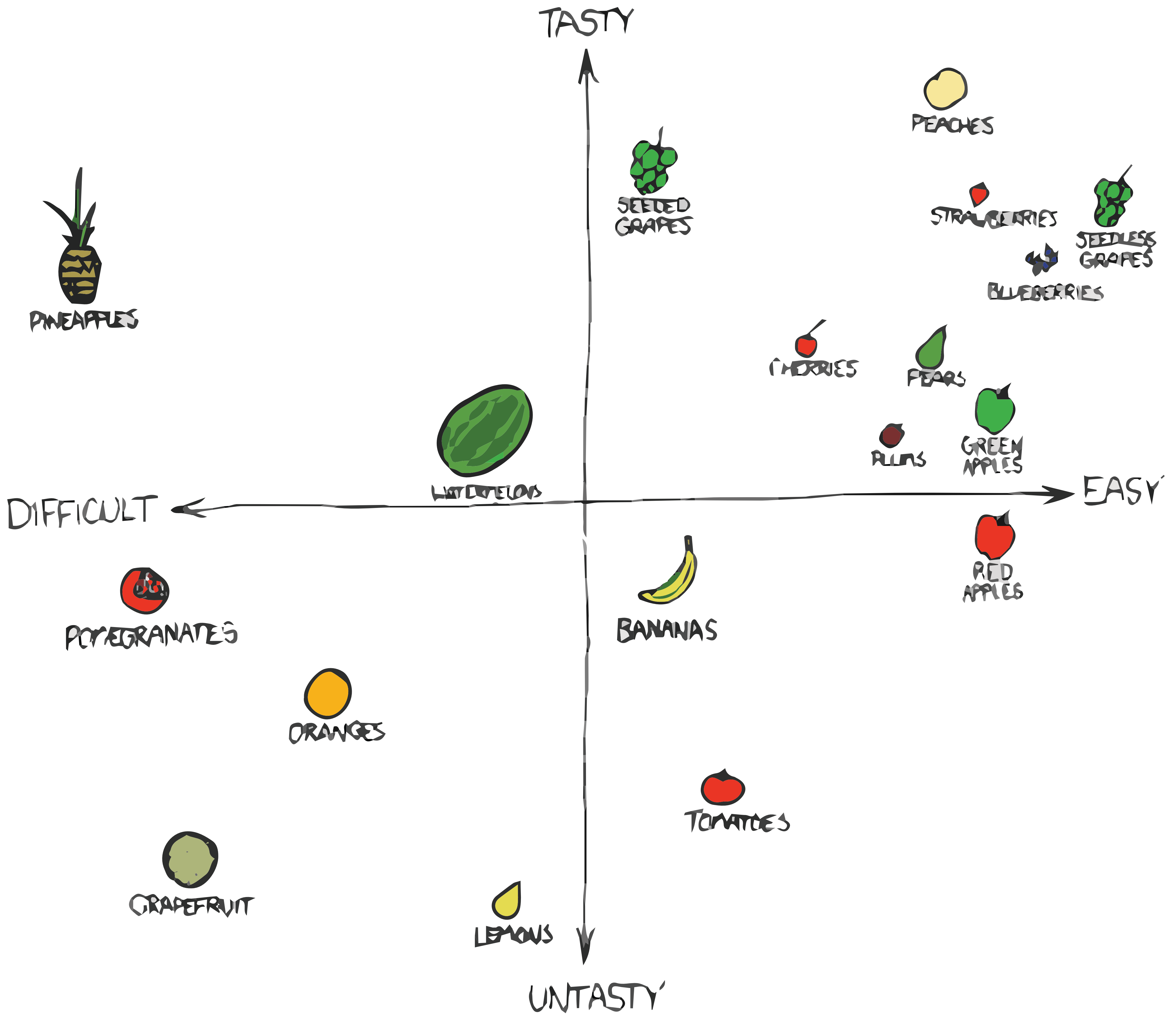
xkcd.com/388



Vector Length

- A vector's length is equal to the *square root of the dot product with itself*

$$\text{length}(x) = \|x\| = \sqrt{x \cdot x}$$

Vector Distances: Manhattan & Euclidean

- **Manhattan Distance**
  - (Distance as cumulative horizontal + vertical moves)
- **Euclidean Distance**
- Both are too sensitive to extreme values

$$d_{\text{manhattan}}(x, y) = \sum_i |x_i - y_i|$$

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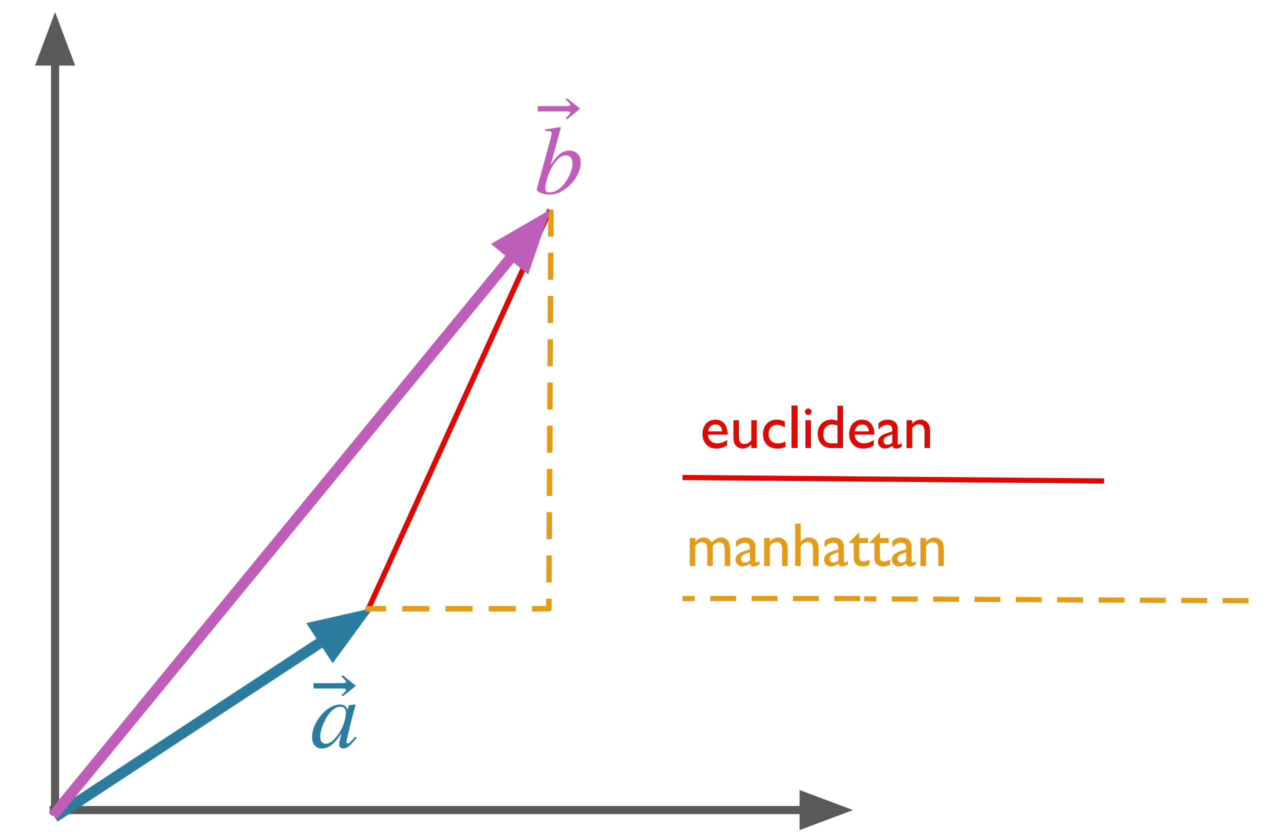

Vector Similarity: Dot Product

- Produces real number scalar from product of vectors' components
- Gives **higher similarity** to **longer** vectors

$$\text{sim}_{\text{dot}}(x, y) = x \cdot y = \sum_i x_i y_i$$

Vector Similarity: Cosine

- If you normalize the dot product for vector magnitude…
- …result is same as **cosine of angle** between the vectors

$$\text{sim}_{\text{cosine}}(x, y) = \frac{x \cdot y}{||x|| \, ||y||} = \frac{\sum_i x_i y_i}{\sqrt{\sum_i x_i^2} \sqrt{\sum_i y_i^2}}$$

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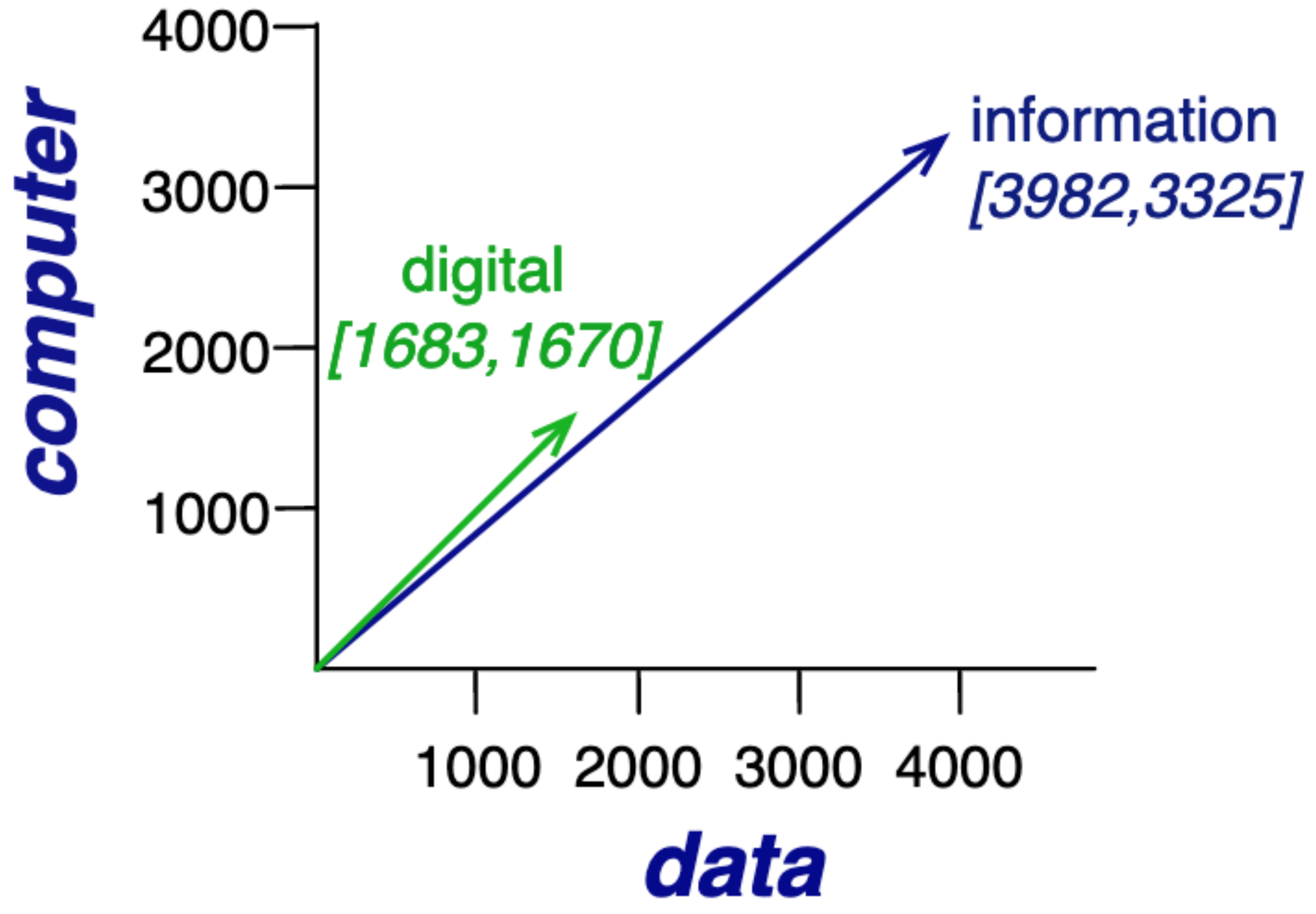
  - 'Company' = context
- Word represented by context feature vector
  - Many alternatives for vector
- Initial representation:
  - 'Bag of words' feature vector
  - Feature vector length *N*, where *N* is size of vocabulary
    - f_i+=1 if $word_i$ within window size *w* of *word*

Bag of Words Vectors

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...



Bag of Words Vectors

- Usually re-weighted, with e.g. tf-idf, ppmi
- Still sparse
- Very high-dimensional: |V|

	aardvark	...	computer	data	result	pie	sugar	...
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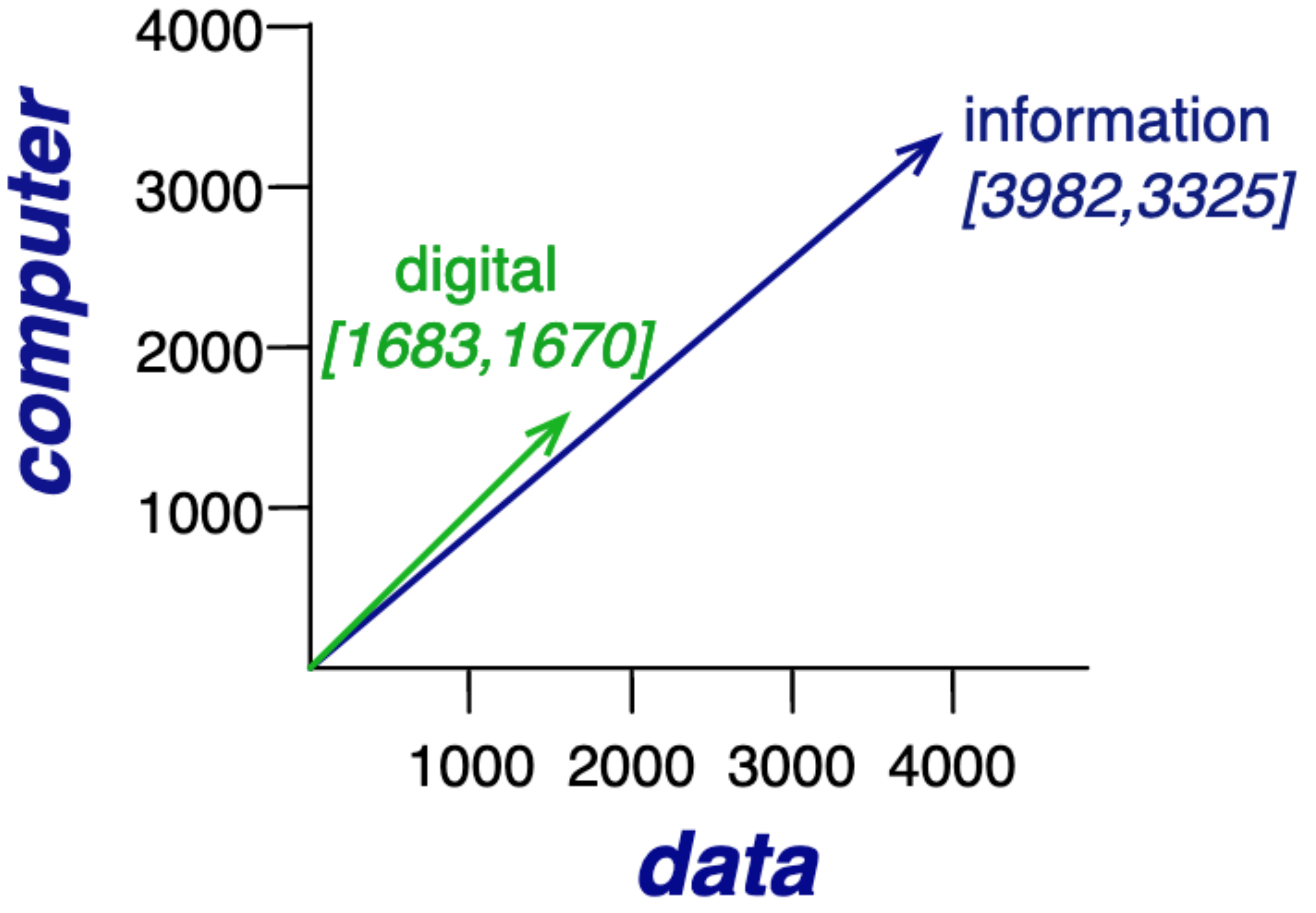

Gradient Descent

Supervised Learning

Supervised Learning

- Given: a dataset $D = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example

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- For example:
  - Sentiment analysis:
    - Input: bag of words representation of “This movie was great.”
    - Output: 4 [on a scale 1-5]
  - Language modeling:
    - Input: “This movie was”
    - Output: “great”

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  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example
- Goal: learn a function $f : X \rightarrow Y$ which:
  - "Does well" on the given data D
  - Generalizes well to unseen data

Parameterized Functions

- A learning algorithm searches for a function f amongst a space of possible functions
- Parameters define a family of functions
  - θ: general symbol for parameters
  - $\hat{y} = f(x; \theta)$: input x, parameters θ; model/function output $\hat{y}$
- Example: the family of linear functions $f(x) = mx + b$
  - $\theta = \{m, b\}$
- Later: neural network architecture defines the family of functions

Loss Minimization

- General form of optimization problem

$$\mathscr{L}(\hat{Y}, Y) = \frac{1}{|Y|} \sum_i \ell(\hat{y}(x_i), y_i)$$

- $\mathscr{L}(\hat{Y}, Y)$: loss function ("objective function");
  - How "close" are the model's outputs to the true outputs
  - $\ell(\hat{y}, y)$: local (per-instance) loss, averaged over training instances
  - More later: depends on the particular task, among other things
- View the loss *as a function of the model's parameters*

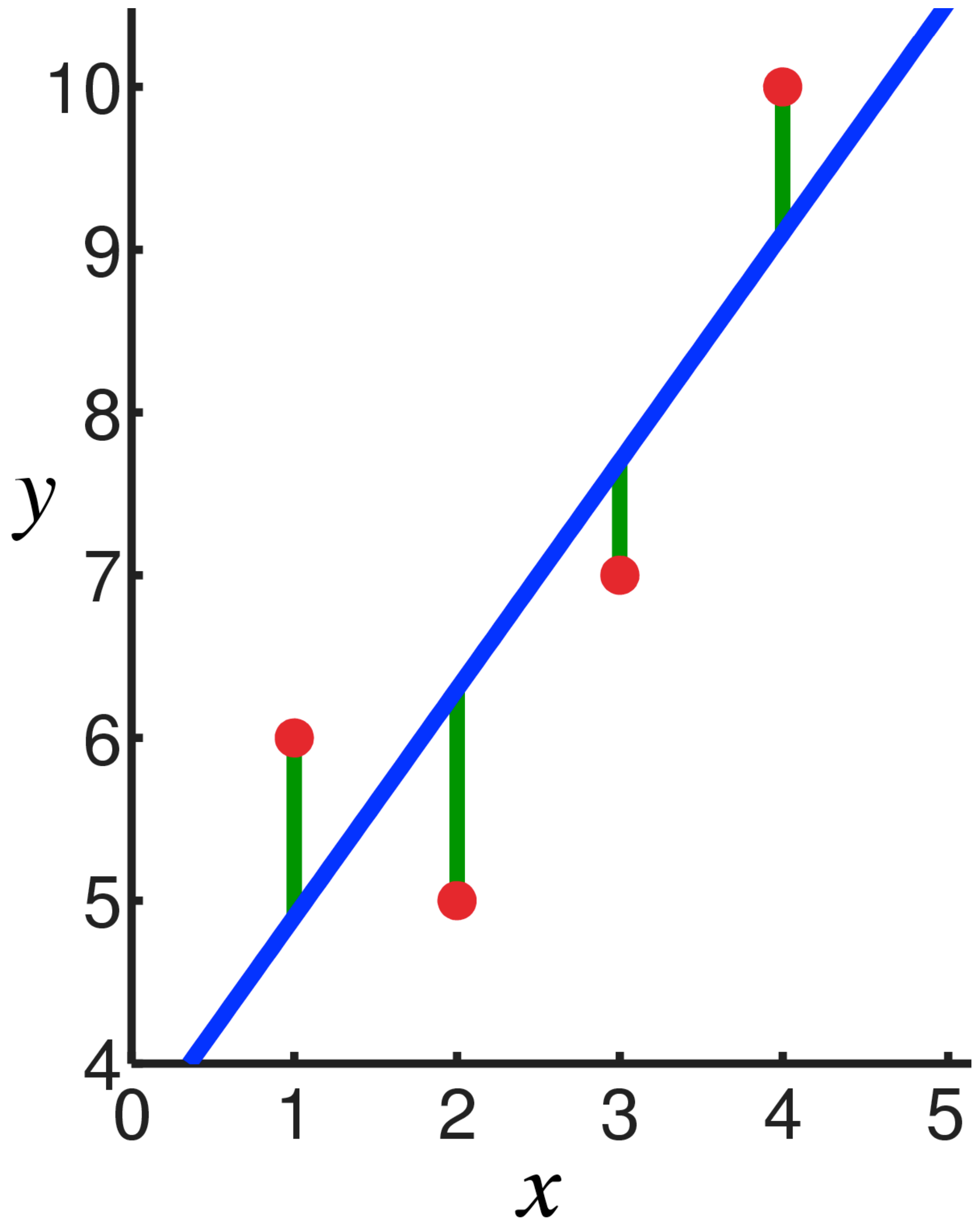
$$\mathscr{L}(\theta) := \mathscr{L}(\hat{Y}, Y) = \mathscr{L}(f(X;\theta), Y)$$

Loss Minimization

- The optimization problem:

$$\theta^* = \arg\min_{\theta} \mathcal{L}(\theta)$$

- Example: (least-squares) linear regression
  - $\ell(\hat{y}, y) = (\hat{y} - y)^2$



$$m^*, b^* = \arg\min_{m,b} \sum_i ((mx_i + b) - y_i)^2$$

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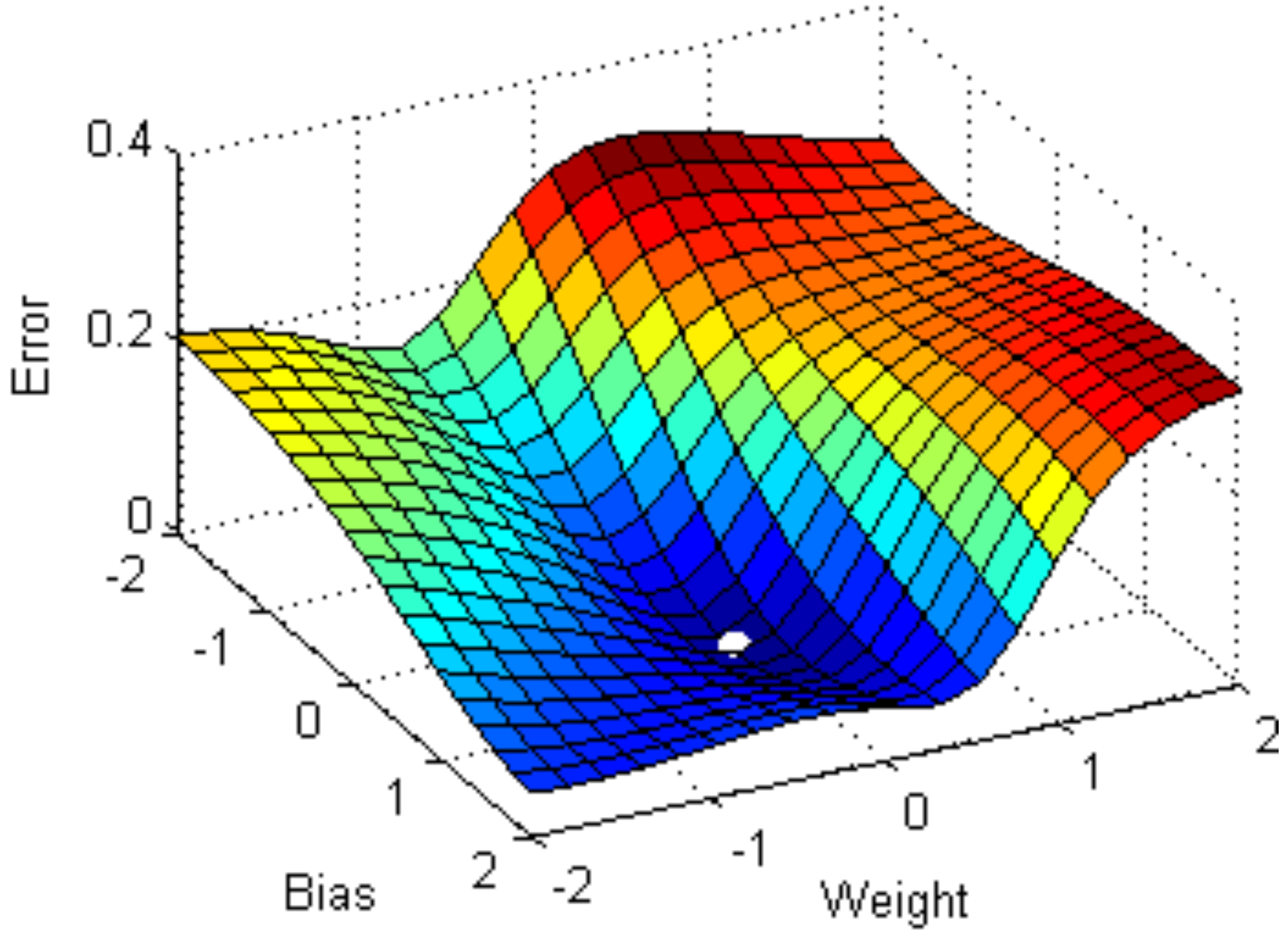
Learning: (Stochastic) Gradient Descent

Gradient Descent: Basic Idea



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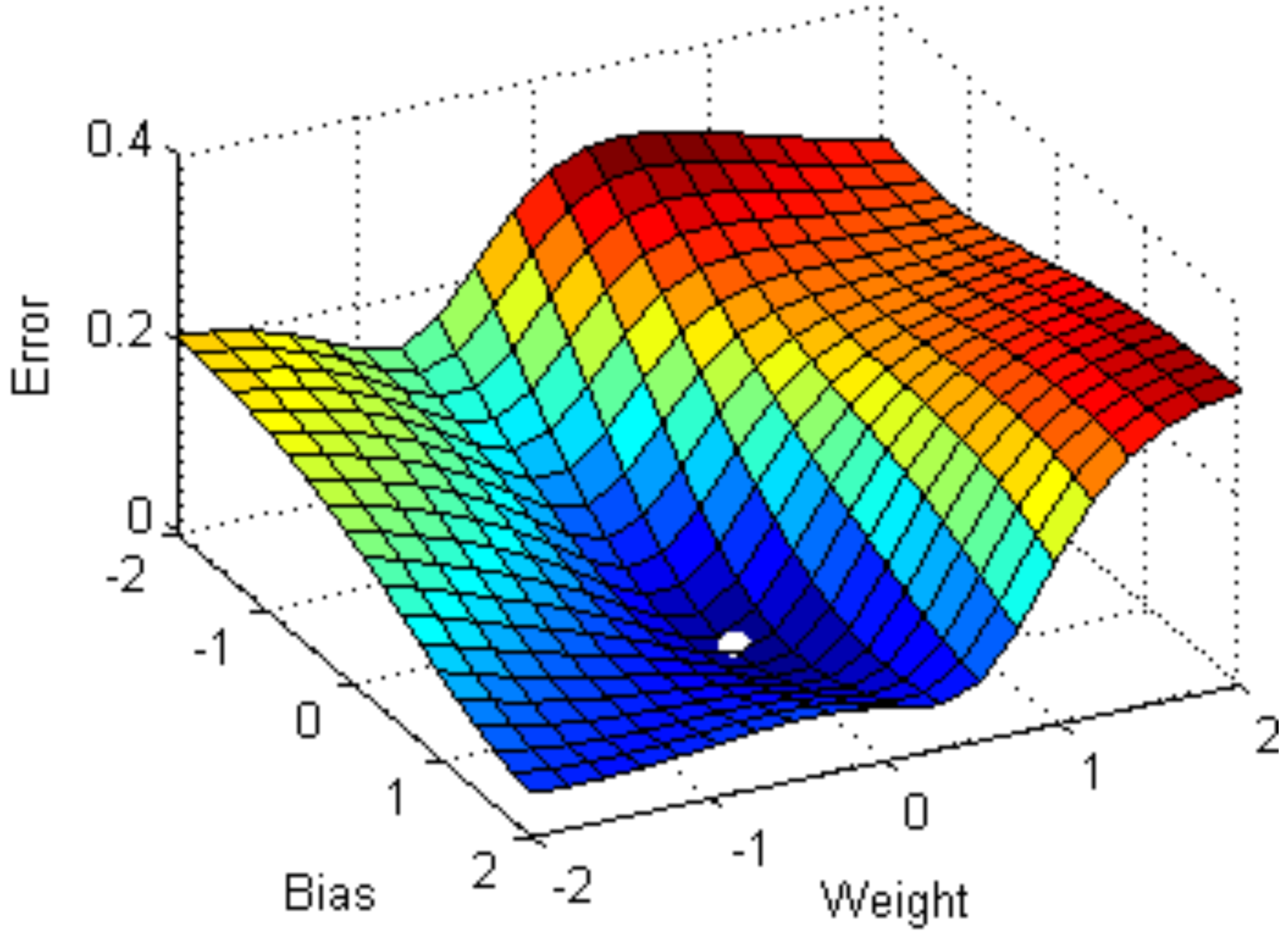

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Gradient Descent: Basic Idea

- The *gradient* of the loss w/r/t parameters tells which direction in parameter space to "walk" to make the loss smaller (i.e. to improve model outputs)
- Guaranteed to work in linear model case
  - Can get stuck in local minima for non-linear functions, like NNs
  - More precisely: if loss is a *convex* function of the parameters, gradient descent is guaranteed to find an optimal solution. For non-linear functions, the loss will generally *not* be convex.

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$$\frac{\partial f}{\partial y} = 20x^3y + 15xy^2 + 1$$

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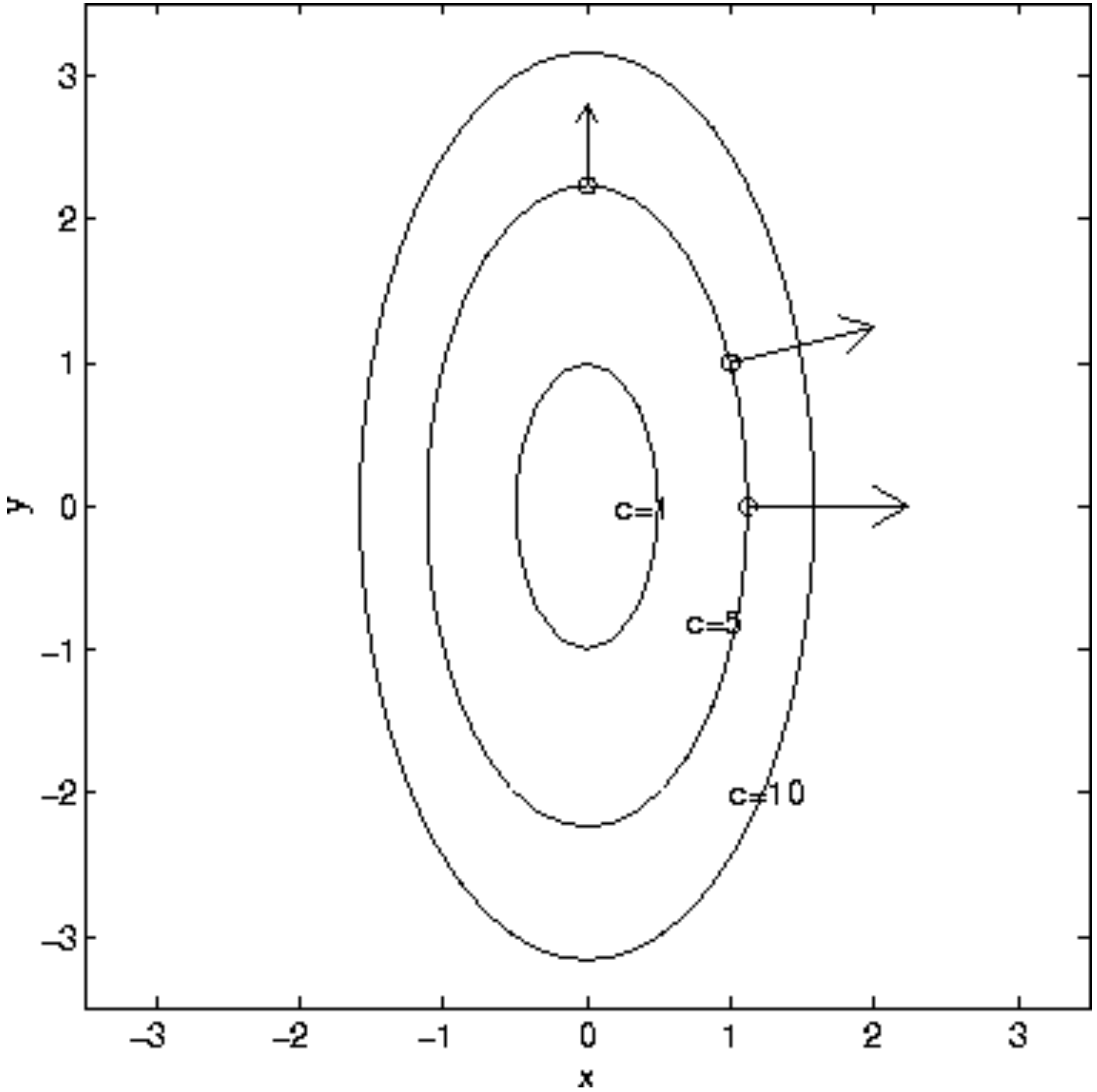
- The gradient is perpendicular to the *level curve* at a point
- The gradient points in the direction of greatest rate of increase of f

Gradient and Level Curves



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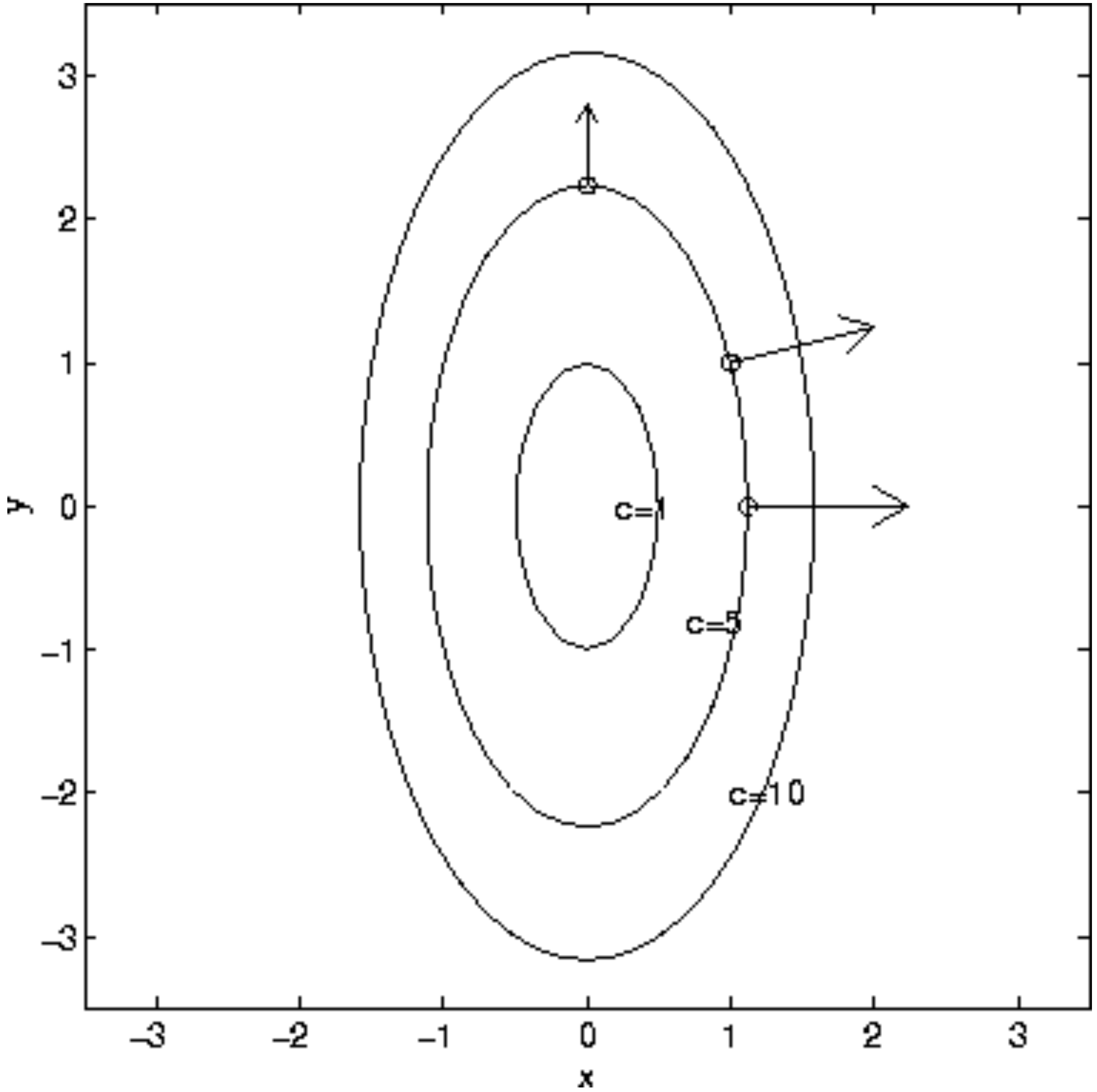

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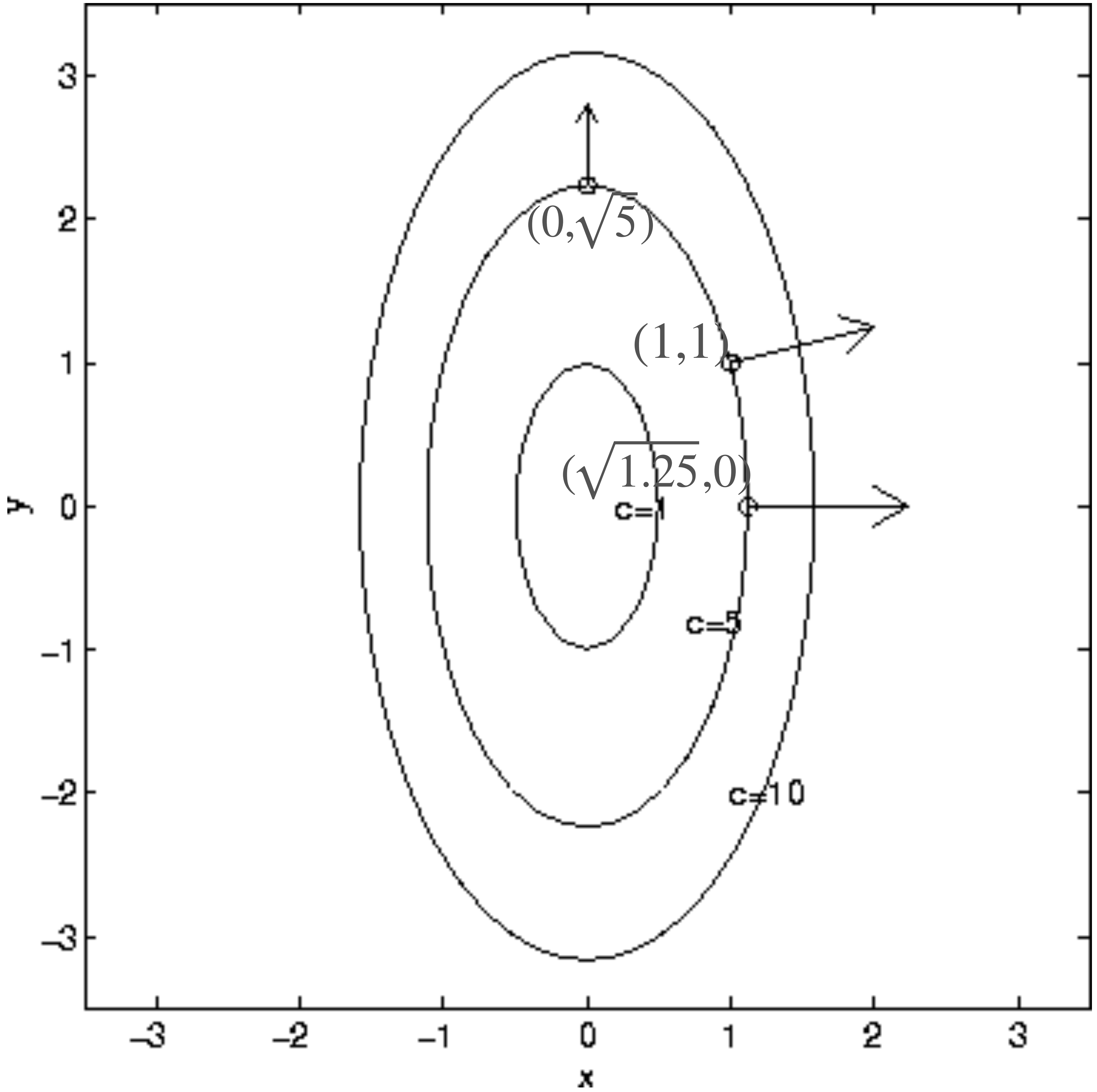

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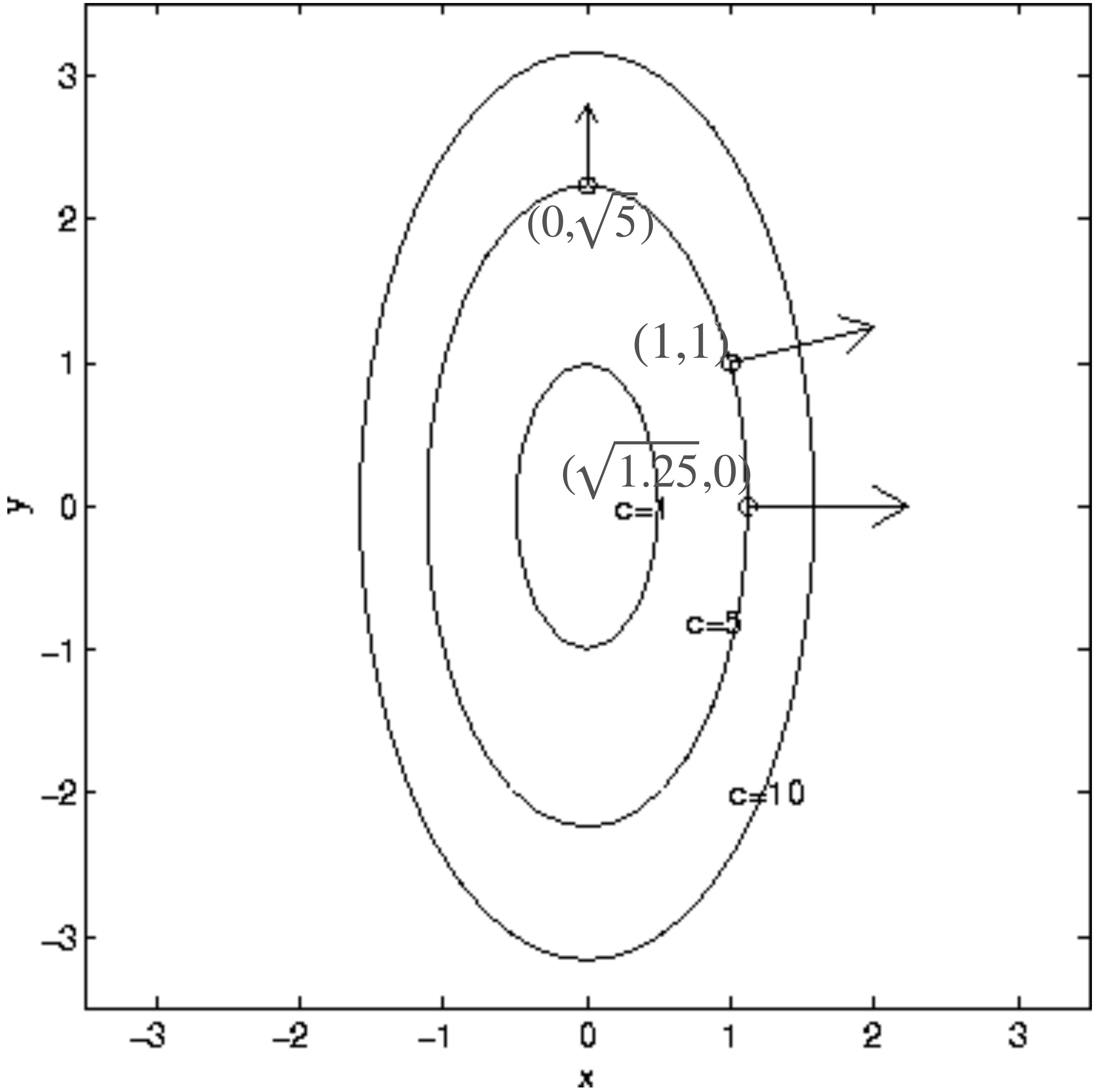

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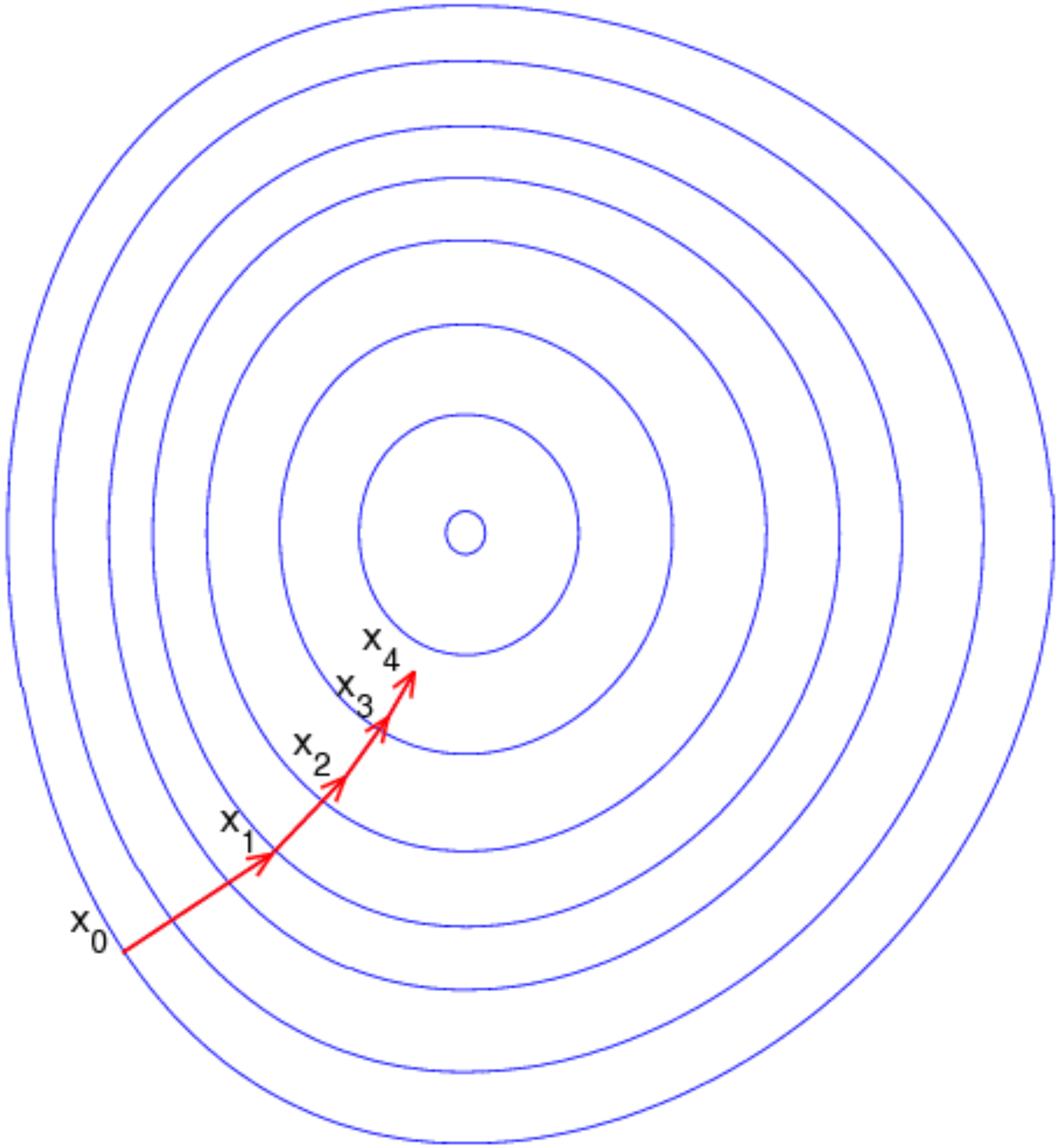
Q: what are the actual gradients at those points?

Gradient Descent and Level Curves



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Gradient Descent Algorithm

- Initialize θ_0
- Repeat until convergence:

$$\theta_{n+1} = \theta_n - \alpha \nabla \mathscr{L}(\hat{Y}(\theta_n), Y)$$

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Learning rate

Gradient Descent Algorithm

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- Repeat until convergence:

$$\theta_{n+1} = \theta_n - \underbrace{\alpha}_{\text{Learning rate}} \nabla \mathscr{L}(\hat{Y}(\theta_n), Y)$$

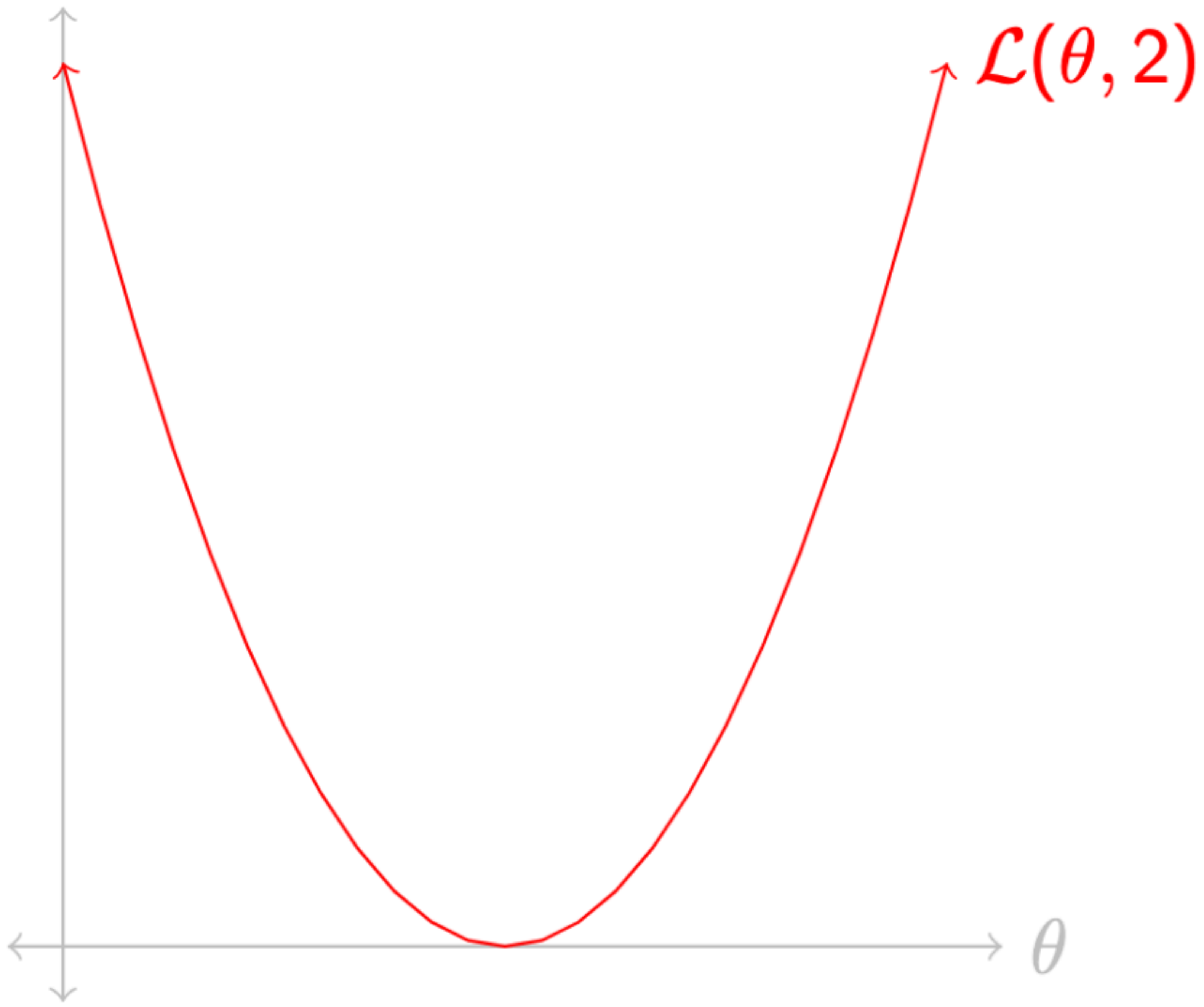
- High learning rate: big steps, may bounce and "overshoot" the target
- Low learning rate: small steps, smoother minimization of loss, but can be slow

Gradient Descent: Minimal Example

- Task: predict a target/true value $y = 2$
- "Model": $\hat{y}(\theta) = \theta$
  - A single parameter: the actual guess
- Loss: Euclidean distance

$$\mathcal{L}(\hat{y}(\theta), y) = (\hat{y} - y)^2 = (\theta - y)^2$$

Gradient Descent: Minimal Example



$$\frac{\partial}{\partial \theta}\mathcal{L}(\theta, y) = 2(\theta - y)$$

$$\theta_{t+1} = \theta_t - \alpha \cdot \frac{\partial}{\partial \theta}\mathcal{L}(\theta, y)$$

Stochastic Gradient Descent

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  - Break the data into “mini-batches”: small chunks of the data
  - Compute gradients and update parameters for each batch
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- Epoch: one pass through the whole training data

Stochastic Gradient Descent

```
initialize parameters / build model

for each epoch:

 data = shuffle(data)
 batches = make_batches(data)

 for each batch in batches:

  outputs = model(batch)
  loss = loss_fn(outputs, true_outputs)
  compute gradients
  update parameters
```

Next Time

- Skip-Gram with Negative Sampling
  - How optimization framework applies to this problem
- Introduction of two tasks that we will use throughout the class
  - Language modeling
  - Text classification [sentiment analysis]