Recurrent Neural Networks: Vanishing Gradients, Gated Variants

Ling 575j: Deep Learning for NLP

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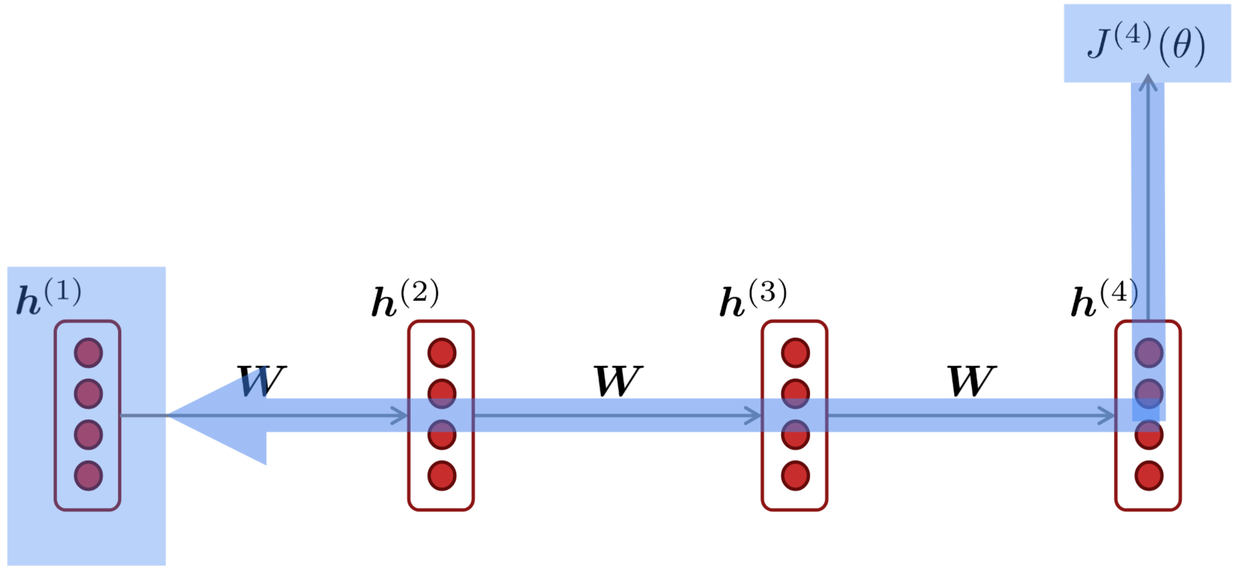
Spring 2023

Vanishing Gradients

Vanishing/Exploding Gradients Problem

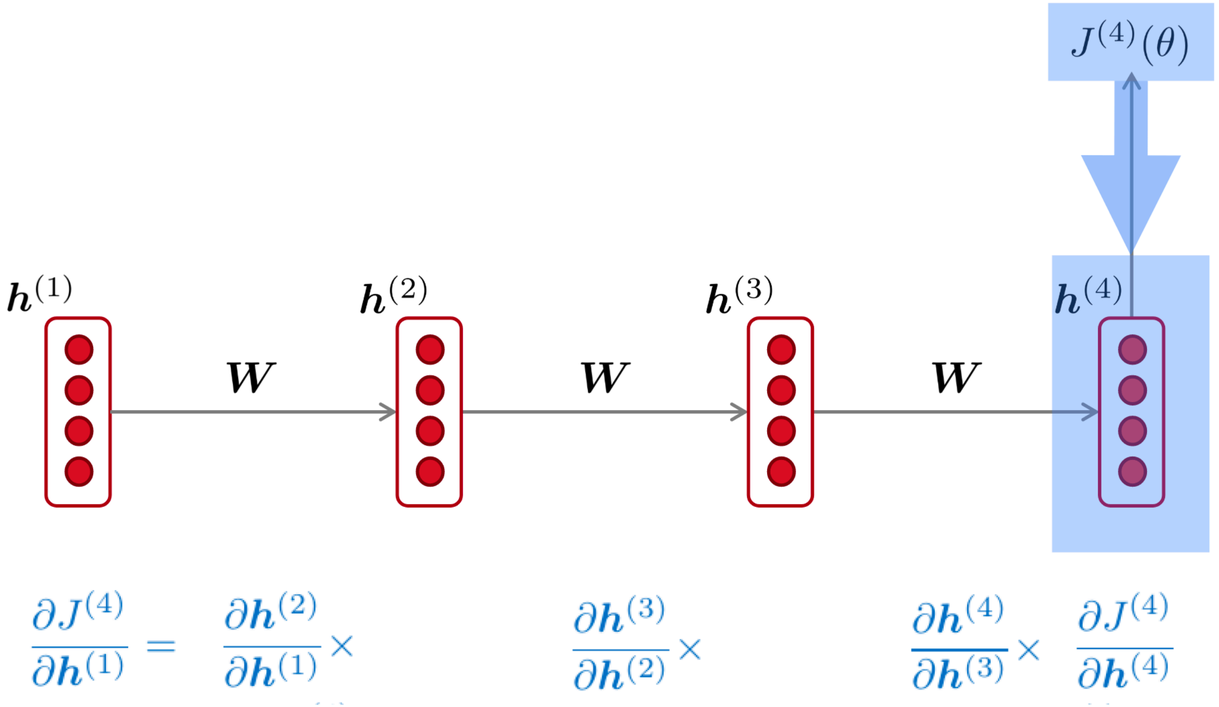
- BPTT with vanilla RNNs faces a major problem:
  - The gradients can *vanish* (approach 0) across time
  - This makes it hard/impossible to learn *long distance dependencies*, which are rampant in natural language

Vanishing Gradients



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \ ?$$

Vanishing Gradients



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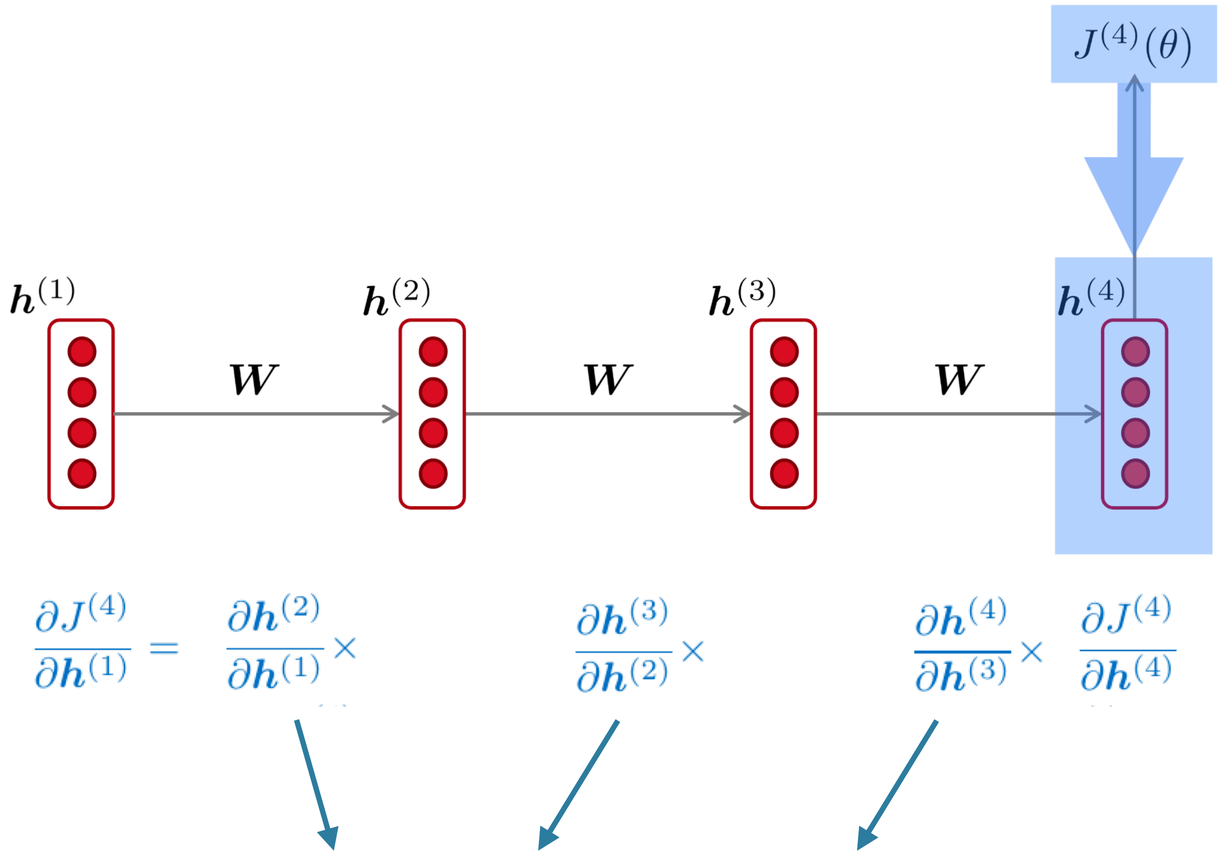
Vanishing Gradients



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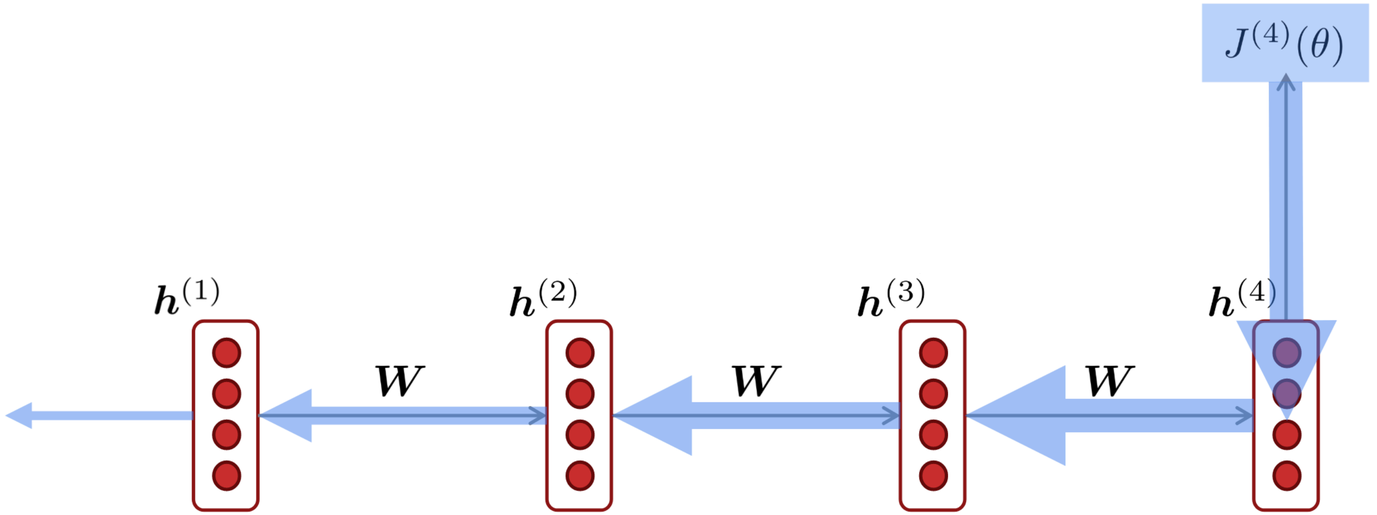
If these are small (depends on W), the effect from t=4 on t=1 will be very small

Vanishing Gradients



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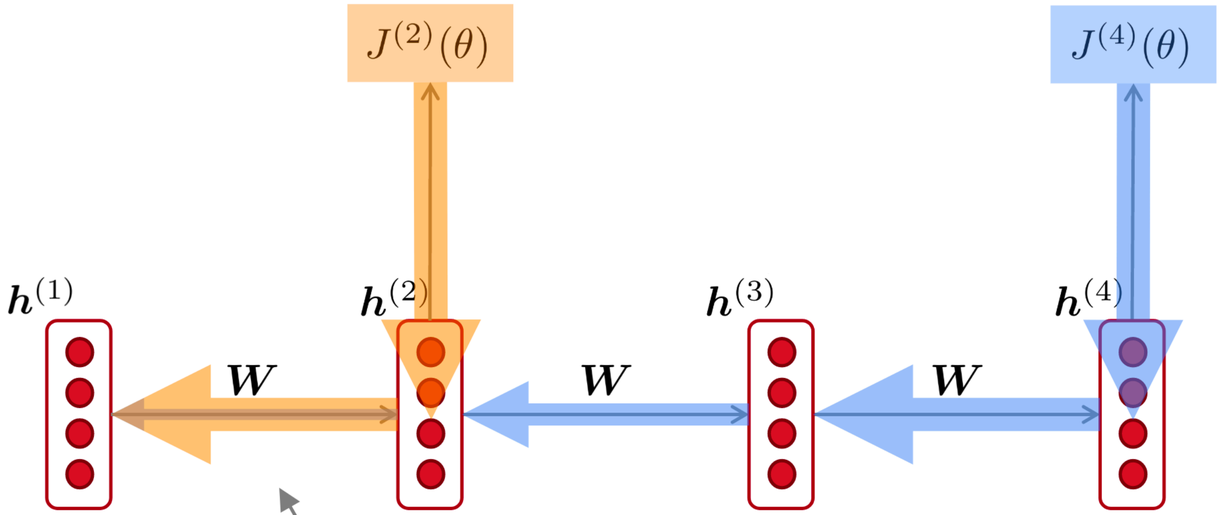
Vanishing Gradient Problem



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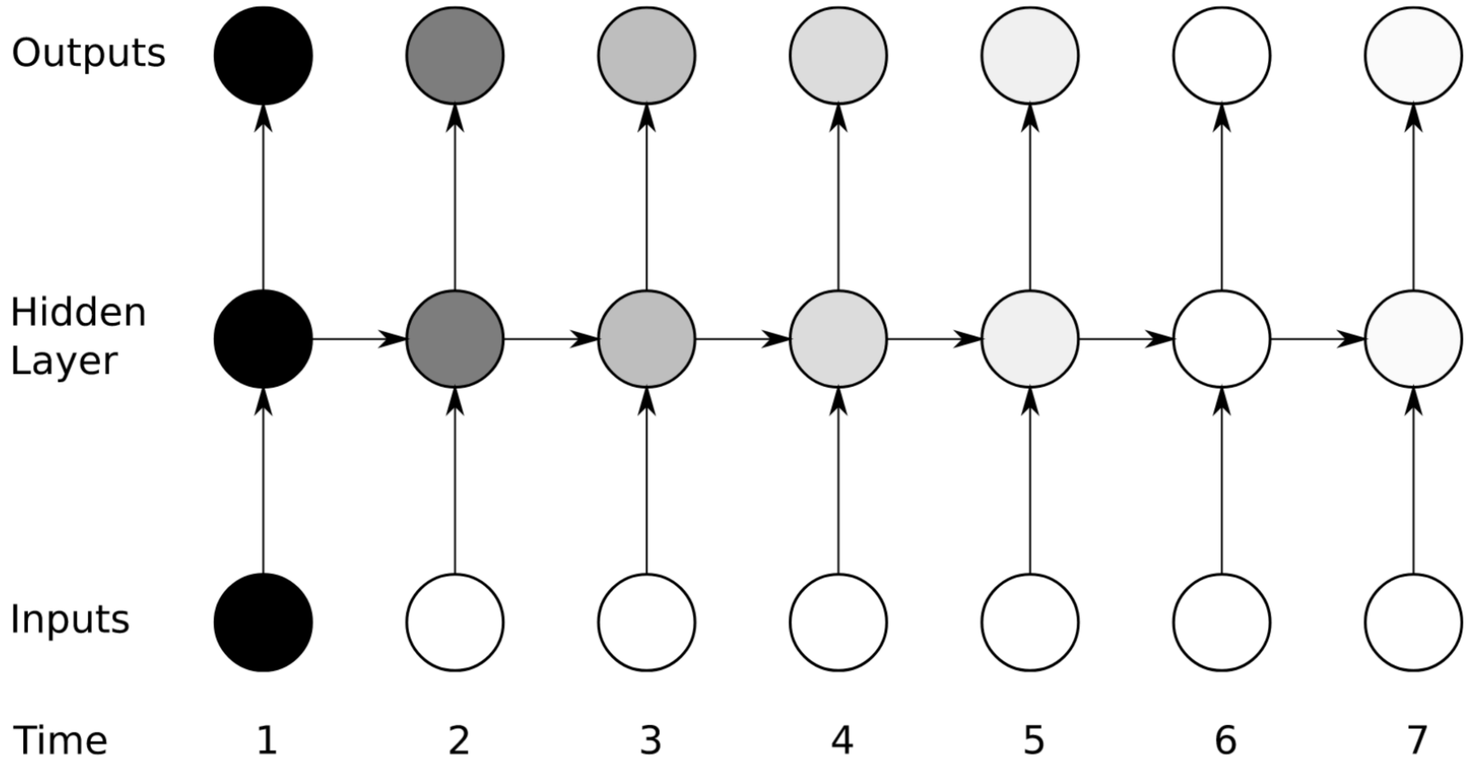
Gradient signal from faraway is lost because it's much smaller than gradient signal from close-by.

So model weights are updated only with respect to near effects, not long-term effects.

Vanishing Gradient Problem



Graves 2012

Vanishing Gradient Problem

- Gradient measures the effect of the past on the future
- If it vanishes between t and t+n, can't tell if:
  - There's no dependency in fact
  - The weights in our network just haven't yet captured the dependency

Examples of long-distance dependencies

- Number agreement
  - The keys _____
  - The keys on the table _____
  - The keys next to the book on top of the table _____
- Selectional Preferences
  - The **family** moved from the city because they wanted a larger **house**.
  - The **team** moved from the city because they wanted a larger **market**.

Gating Based RNNs: LSTM and GRU

LSTMs

- Long Short-Term Memory (Hochreiter and Schmidhuber 1997)
- The gold standard / default RNN
  - If someone says “RNN” now, they almost always mean “LSTM”
- Originally: to solve the vanishing/exploding gradient problem for RNNs
  - Vanilla: re-writes the entire hidden state at every time-step
  - LSTM: separate hidden state and memory
    - Read, write to/from memory; can preserve long-term information

LSTMs

$$f_t = \sigma\left(W^f \cdot h_{t-1}x_t + b^f\right)$$

$$i_t = \sigma\left(W^i \cdot h_{t-1}x_t + b^i\right)$$

$$\hat{c}_t = \tanh\left(W^c \cdot h_{t-1}x_t + b^c\right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \hat{c}_t$$

$$o_t = \sigma\left(W^o \cdot h_{t-1}x_t + b^o\right)$$

$$h_t = o_t \odot \tanh\left(c_t\right)$$

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LSTMs

- Key innovation:
  - $c_t, h_t = f(x_t, c_{t-1}, h_{t-1})$
  - c_t: a *memory cell*

- Reading/writing (smooth) controlled by *gates*
  - f_t: forget gate
  - i_t: input gate
  - o_t: output gate

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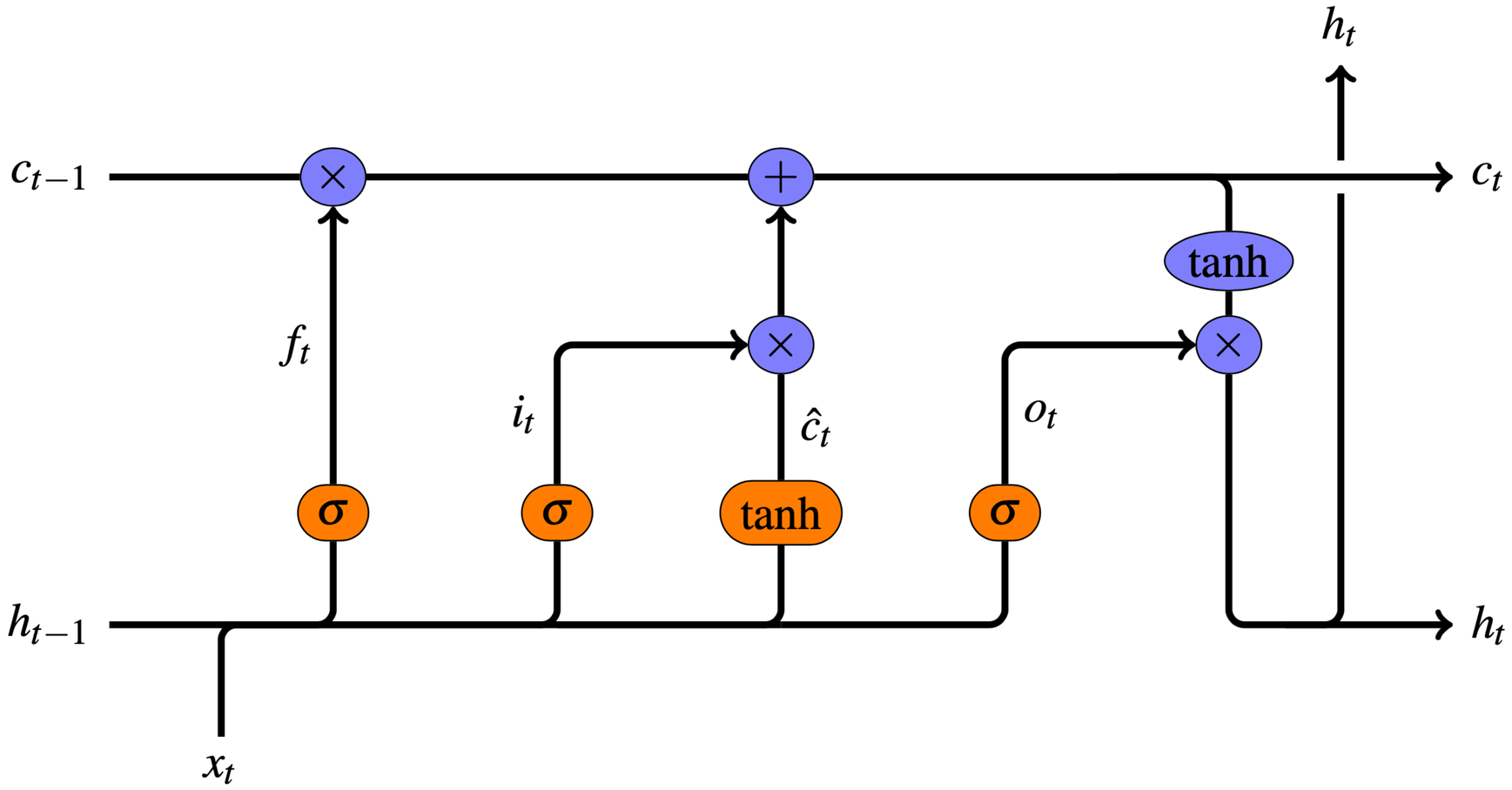

Steinert-Threlkeld and Szymanik 2019; Olah 2015

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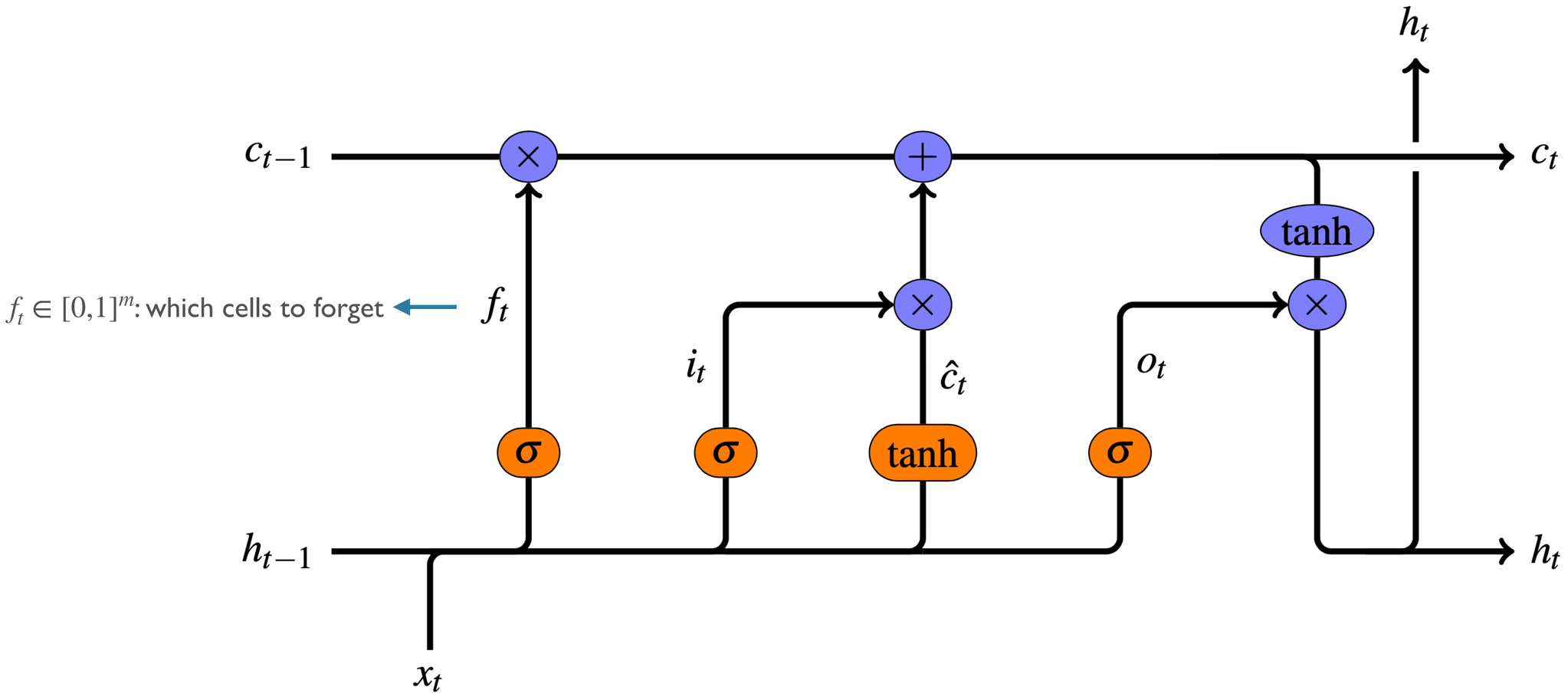

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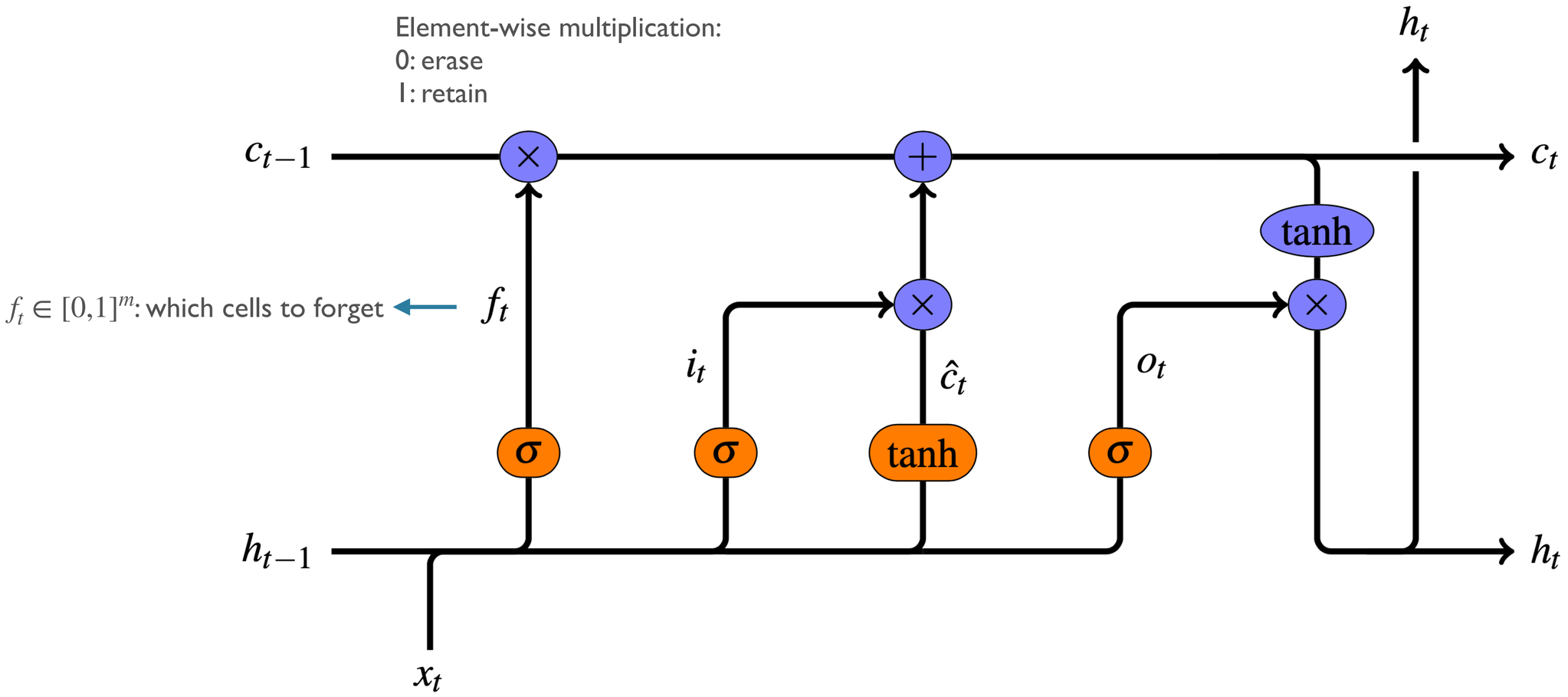

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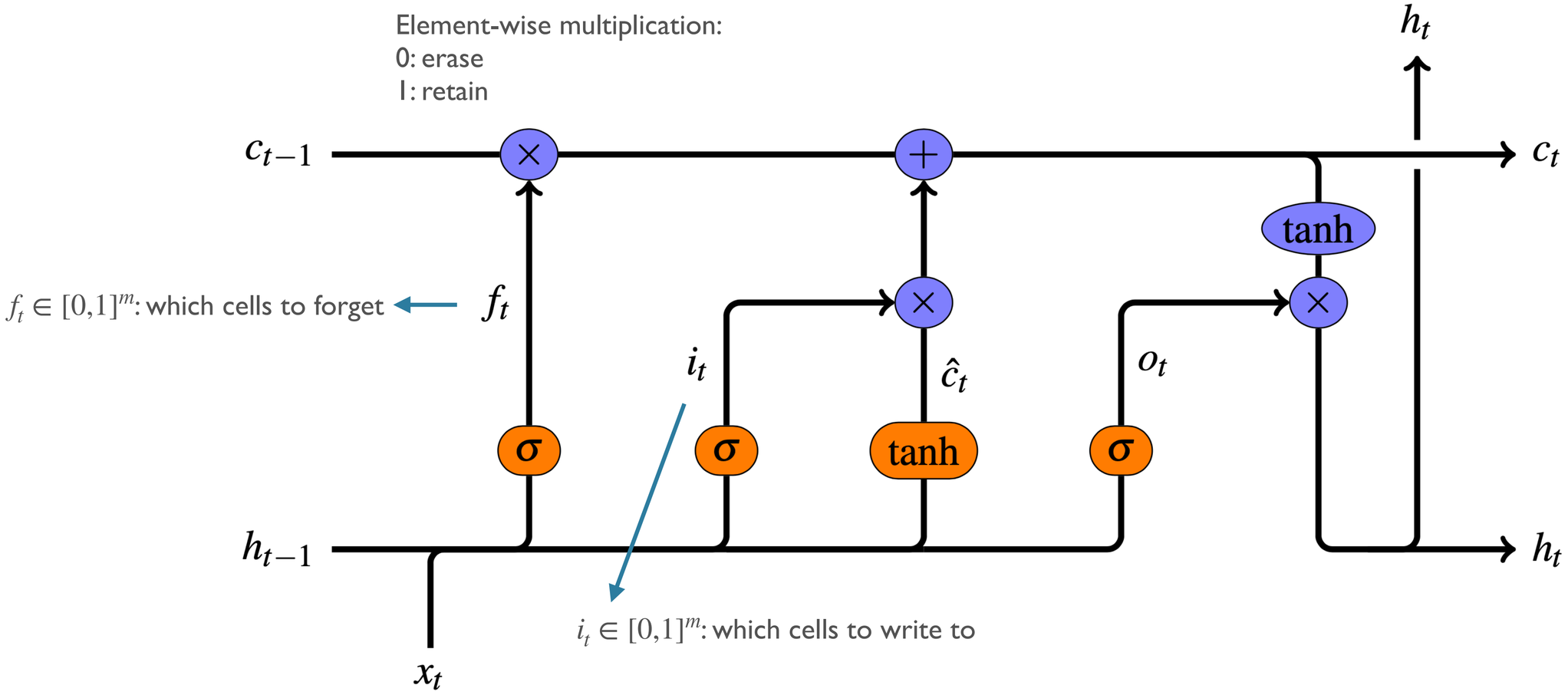

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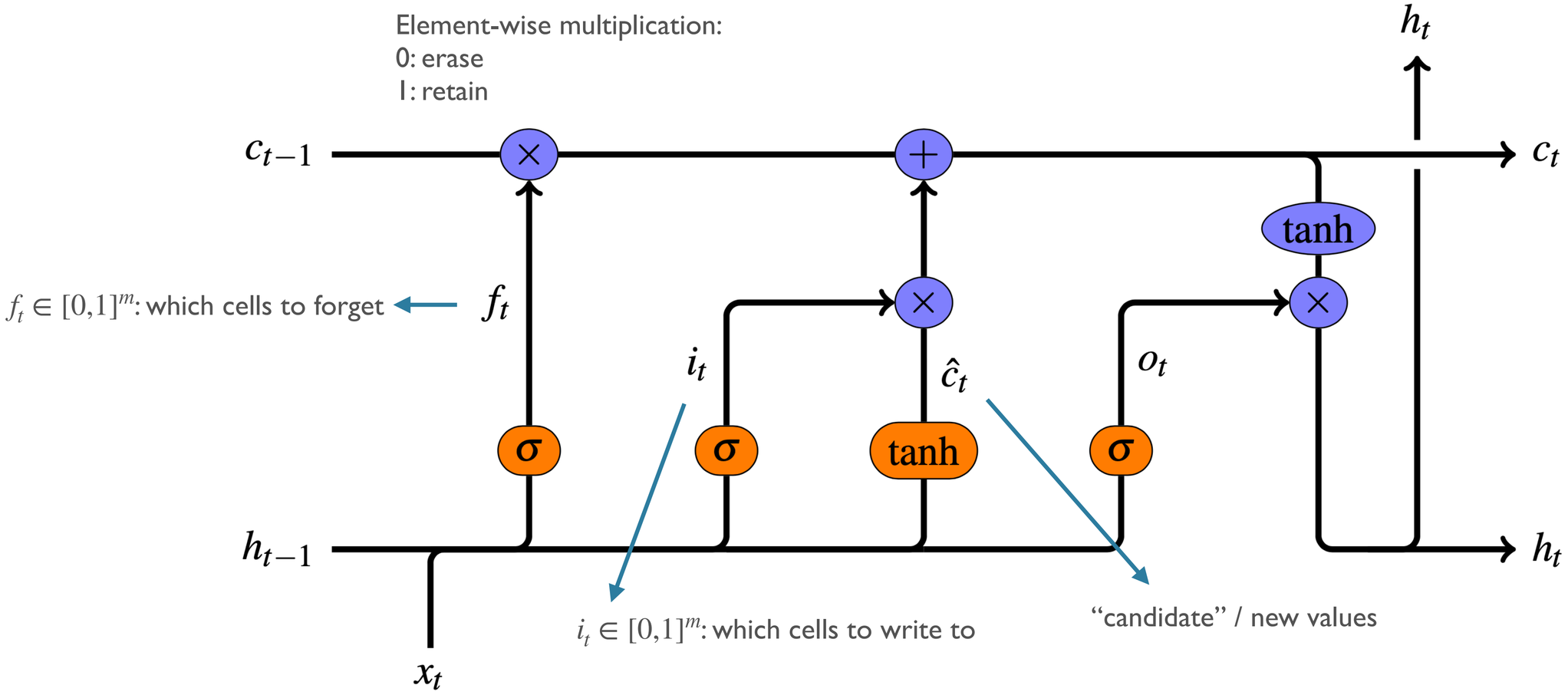

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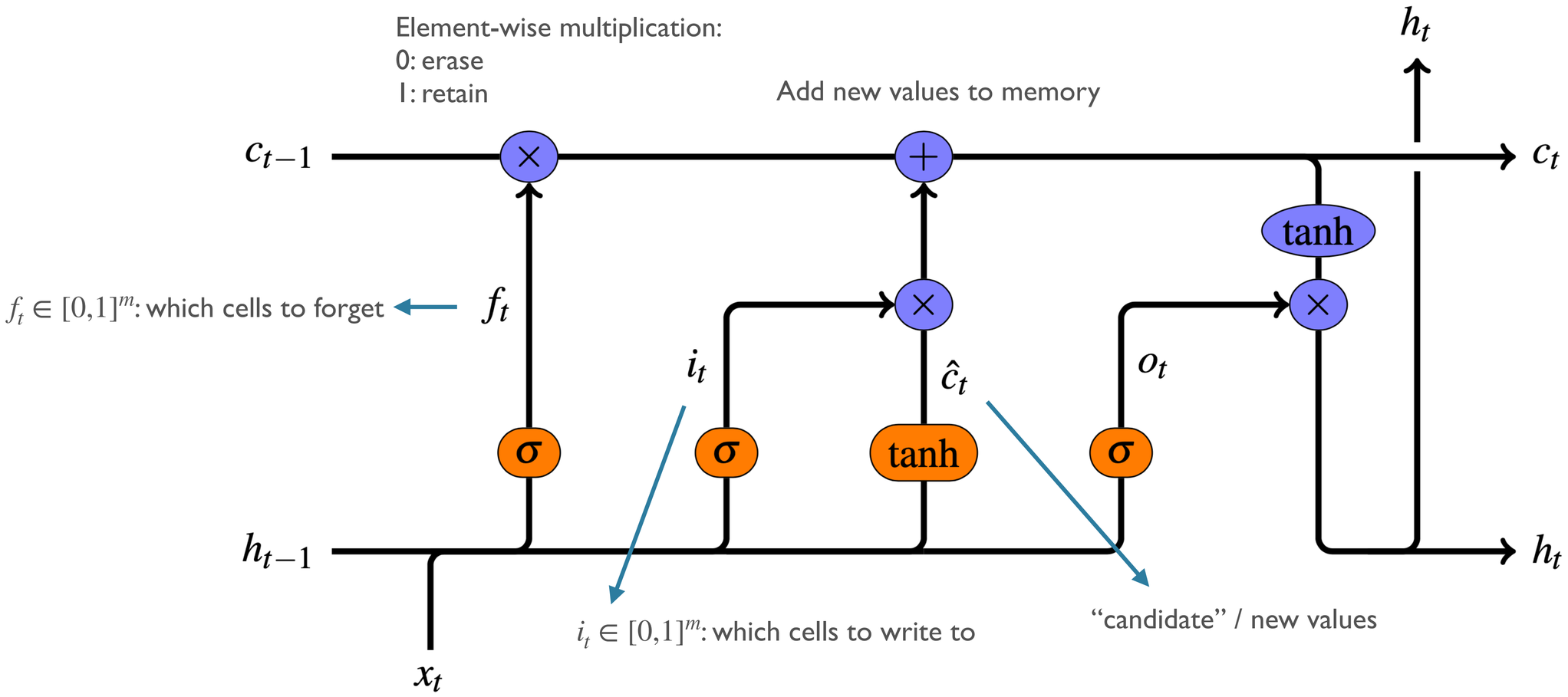

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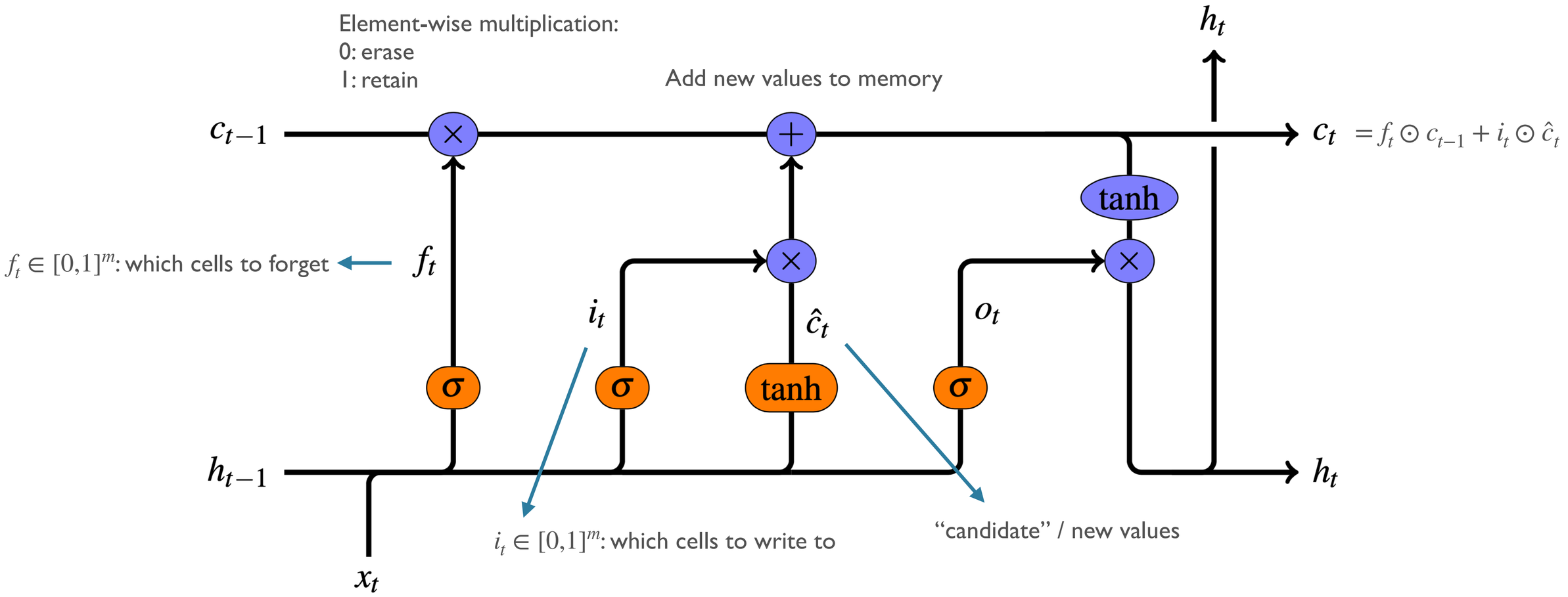

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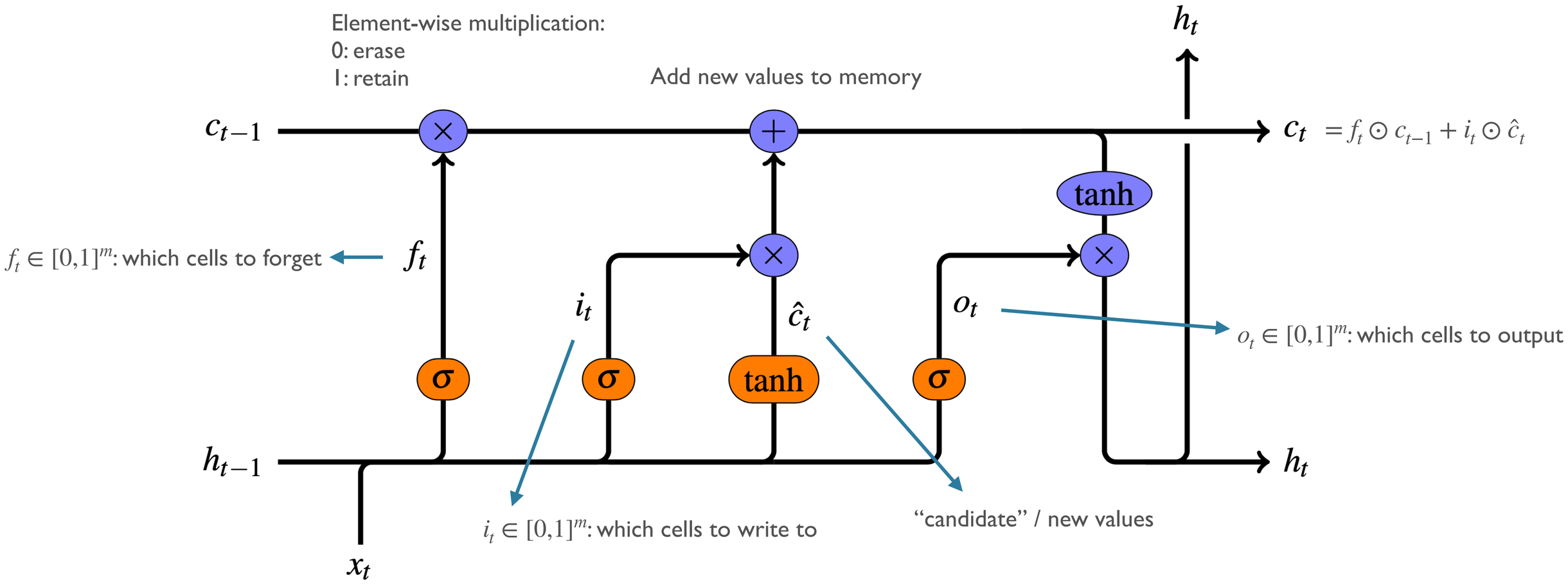

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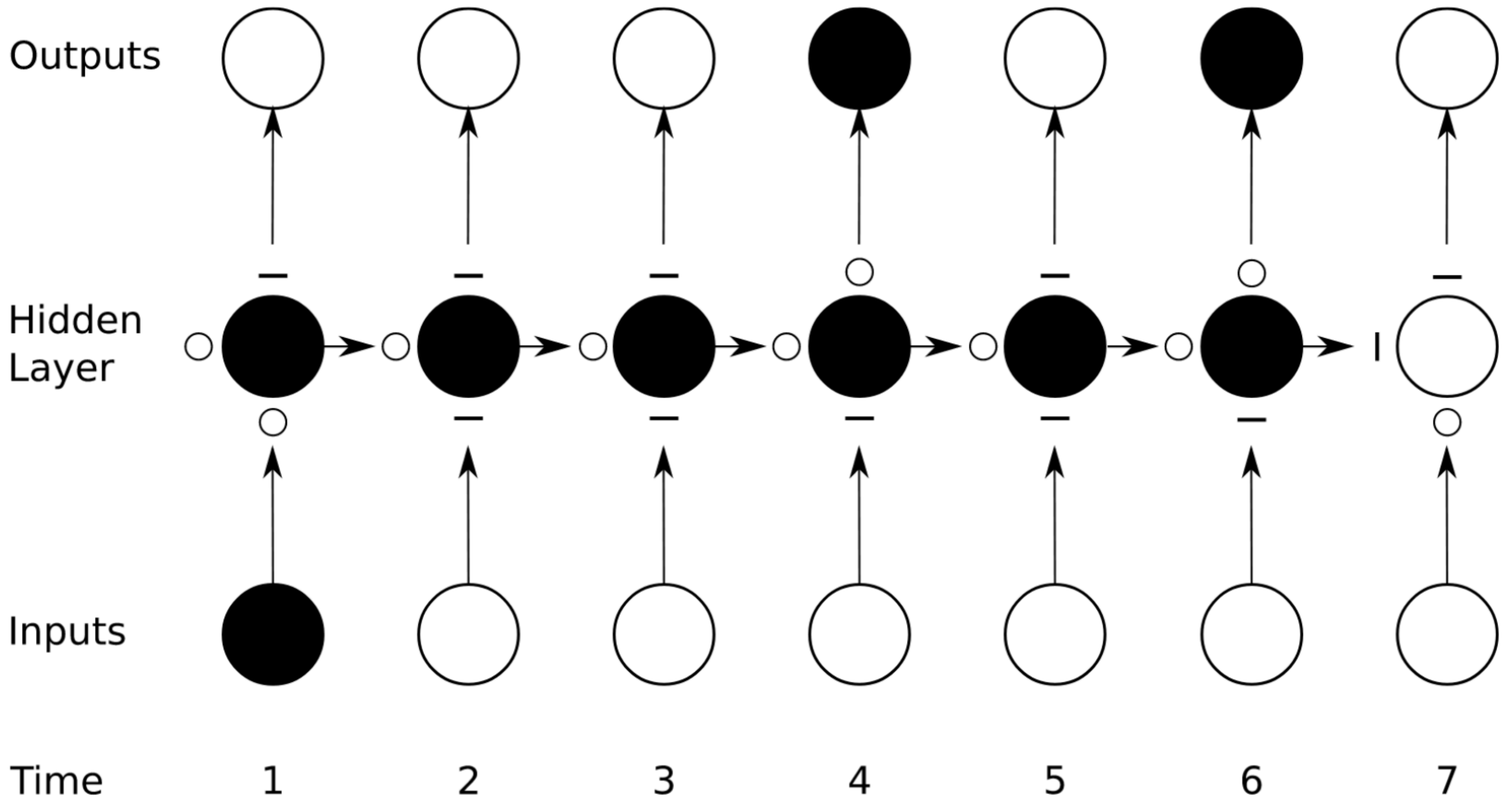
LSTMs solve vanishing gradients



Graves 2012

The Emergence of Number and Syntax Units in LSTM Language Models

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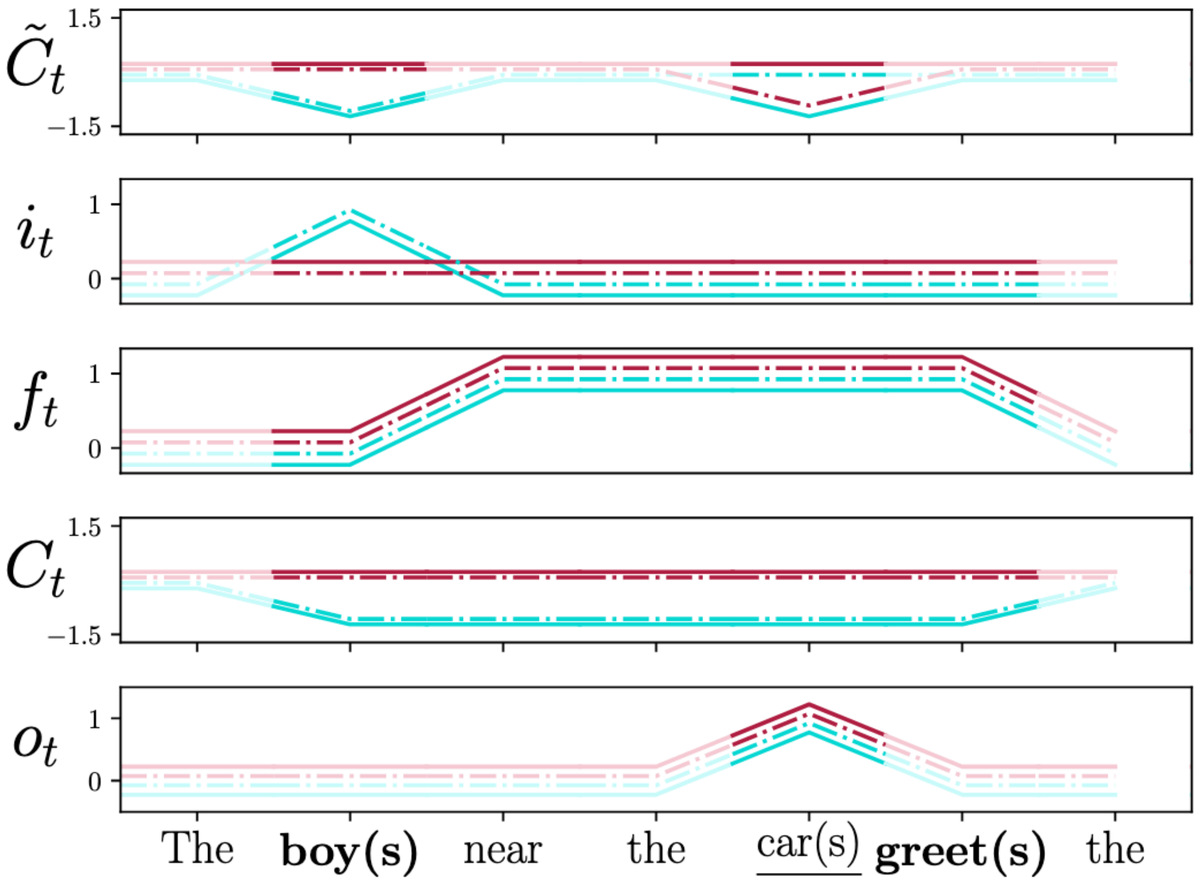
Cell dynamics for storing number info



“The BiLSTM Hegemony”

- Chris Manning, in 2017:

To a first approximation,
the de facto consensus in NLP in 2017 is
that no matter what the task,
you throw a BiLSTM at it, with
attention if you need information flow

source

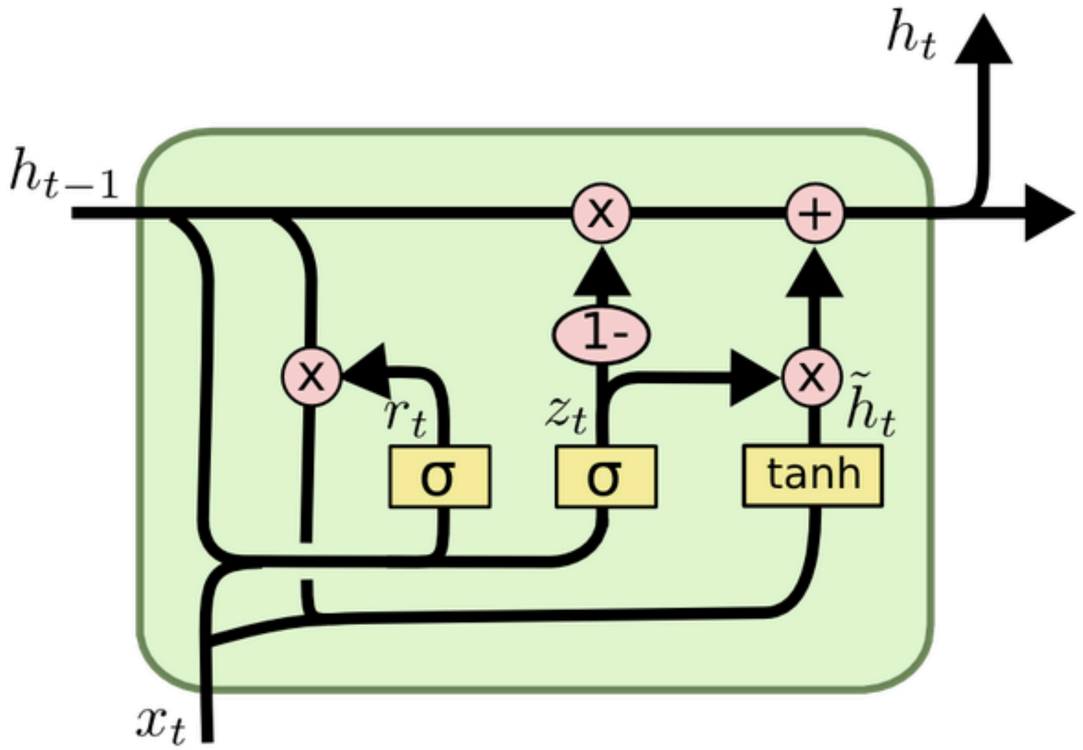
Gated Recurrent Unit (GRU)

- Cho et al 2014: gated like LSTM, but no separate memory cell
  - "Collapses" execution/control and memory
- Fewer gates = fewer parameters, higher speed
  - Update gate $u_t = \sigma(W_u h_{t-1} + U_u x_t + b_u)$
  - Reset gate $r_t = \sigma(W_r h_{t-1} + U_r x_t + b_r)$

$$\tilde{h}_t = \tanh(W_h(r_t \odot h_{t-1}) + U_h x_t + b_h)$$

$$h_t = (1 - u_t) \odot h_{t-1} + u_t \odot \tilde{h}_t$$

Gated Recurrent Unit



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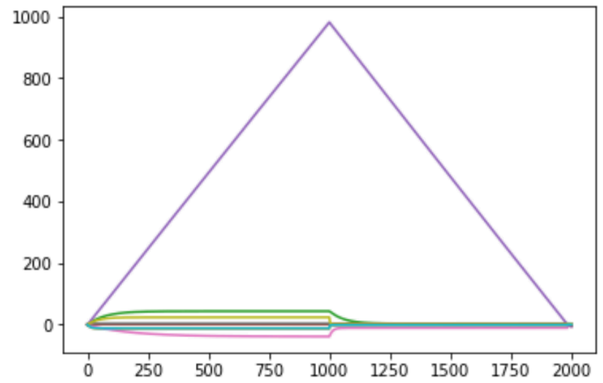
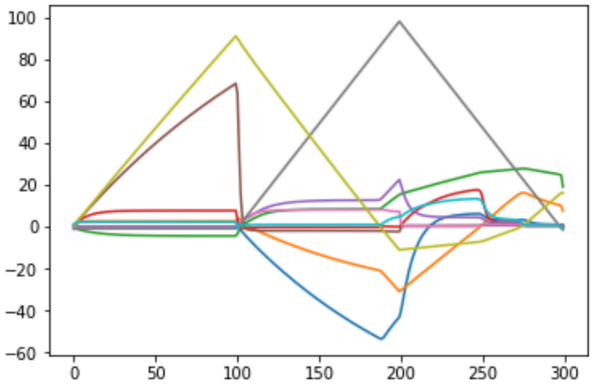
LSTM vs GRU

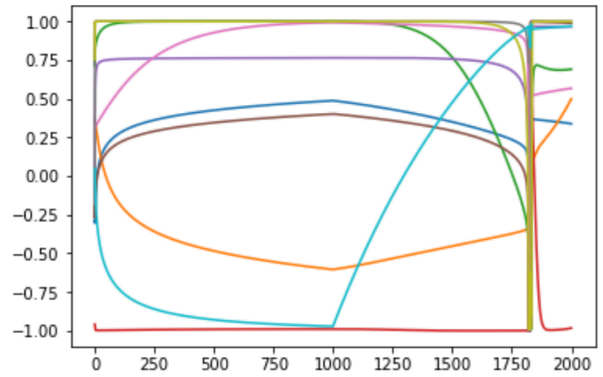
- Generally: LSTM a good default choice
  - GRU can be used if speed and fewer parameters are important
- Full differences between them not fully understood
- Performance often comparable, but: LSTMs can store unboundedly large values in memory, and seem to e.g. count better



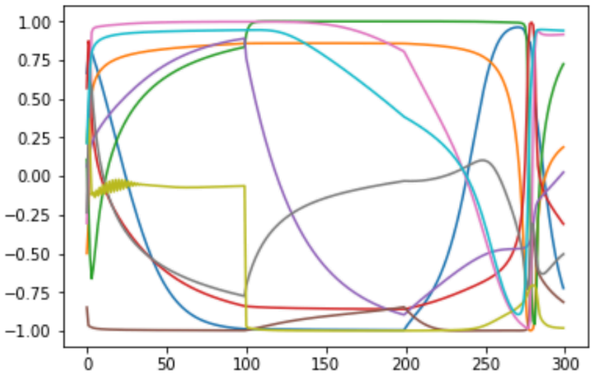
(a) $a^n b^n$-LSTM on $a^{1000} b^{1000}$



(b) $a^n b^n c^n$-LSTM on $a^{100} b^{100} c^{100}$



(c) $a^n b^n$-GRU on $a^{1000} b^{1000}$



(d) $a^n b^n c^n$-GRU on $a^{100} b^{100} c^{100}$

source

Odds and Ends

Fun with LSTM (character) LMs

- Generating text with an LM:
  - Feed initial token (e.g. BOS, or just a word/character)
  - Generate probability over next tokens
    - Sample next token from this distribution
  - Repeat until [EOS | max length | other criterion]

Fun with LSTM (character) LMs

```
and sexual power post. Many governments recognize the military housing of the
[[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]],
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http://karpathy.github.io/2015/05/21/rnn-effectiveness/

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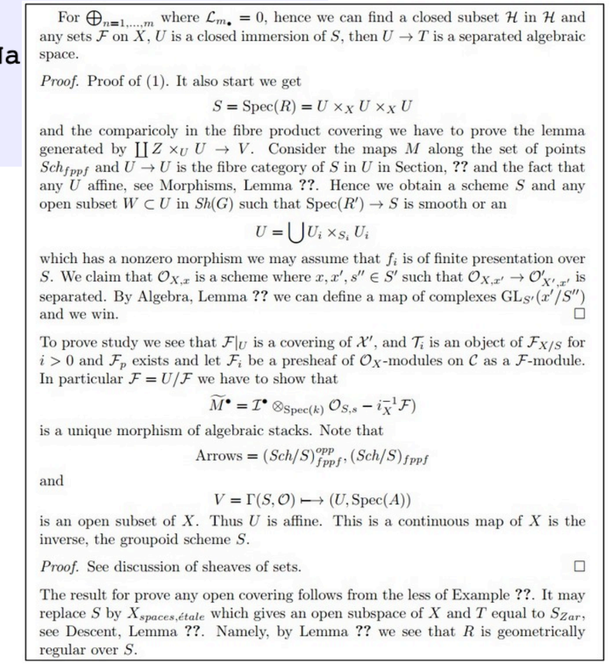

For $\bigoplus_{n=1,\ldots,m}$ where $\mathcal{L}_{m_\bullet} = 0$, hence we can find a closed subset $\mathcal{H}$ in $\mathcal{H}$ and any sets $\mathcal{F}$ on X, U is a closed immersion of S, then $U \to T$ is a separated algebraic space.

Proof. Proof of (1). It also start we get

$$S = \mathrm{Spec}(R) = U \times_X U \times_X U$$

and the compariconly in the fibre product covering we have to prove the lemma generated by $\coprod Z \times_U U \to V$. Consider the maps M along the set of points Sch_{fppf} and $U \to U$ is the fibre category of S in U in Section, ?? and the fact that any U affine, see Morphisms, Lemma ??. Hence we obtain a scheme S and any open subset $W \subset U$ in $Sh(G)$ such that $\mathrm{Spec}(R') \to S$ is smooth or an

$$U = \bigcup U_i \times_{S_i} U_i$$

which has a nonzero morphism we may assume that f_i is of finite presentation over S. We claim that $\mathcal{O}_{X,x}$ is a scheme where $x, x', s'' \in S'$ such that $\mathcal{O}_{X,x'} \to \mathcal{O}'_{X',x'}$ is separated. By Algebra, Lemma ?? we can define a map of complexes $\mathrm{GL}_{S'}(x'/S'')$ and we win. □

To prove study we see that $\mathcal{F}|_U$ is a covering of $\mathcal{X}'$, and $\mathcal{T}_i$ is an object of $\mathcal{F}_{X/S}$ for $i > 0$ and $\mathcal{F}_p$ exists and let $\mathcal{F}_i$ be a presheaf of $\mathcal{O}_X$-modules on $\mathcal{C}$ as a $\mathcal{F}$-module. In particular $\mathcal{F} = U/\mathcal{F}$ we have to show that

$$\widetilde{M}^\bullet = \mathcal{I}^\bullet \otimes_{\mathrm{Spec}(k)} \mathcal{O}_{S,s} - i_X^{-1}\mathcal{F})$$

is a unique morphism of algebraic stacks. Note that

$$\mathrm{Arrows} = (Sch/S)^{opp}_{fppf}, (Sch/S)_{fppf}$$

and

$$V = \Gamma(S, \mathcal{O}) \longmapsto (U, \mathrm{Spec}(A))$$

is an open subset of X. Thus U is affine. This is a continuous map of X is the inverse, the groupoid scheme S.

Proof. See discussion of sheaves of sets. □

The result for prove any open covering follows from the less of Example ??. It may replace S by $X_{spaces,étale}$ which gives an open subspace of X and T equal to S_{Zar}, see Descent, Lemma ??. Namely, by Lemma ?? we see that R is geometrically regular over S.

Lemma 0.1. *Assume (3) and (3) by the construction in the description.*

Suppose $X = \lim |X|$ *(by the formal open covering* X *and a single map* $\underline{Proj}_X(\mathcal{A}) = \mathrm{Spec}(B)$ *over* U *compatible with the complex*

$$Set(\mathcal{A}) = \Gamma(X, \mathcal{O}_{X,\mathcal{O}_X}).$$

When in this case of to show that $\mathcal{Q} \to \mathcal{C}_{Z/X}$ *is stable under the following result in the second conditions of (1), and (3). This finishes the proof. By Definition ?? (without element is when the closed subschemes are catenary. If* T *is surjective we may assume that* T *is connected with residue fields of* S*. Moreover there exists a closed subspace* $Z \subset X$ *of* X *where* U *in* X' *is proper (some defining as a closed subset of the uniqueness it suffices to check the fact that the following theorem*

(1) f *is locally of finite type. Since* $S = \mathrm{Spec}(R)$ *and* $Y = \mathrm{Spec}(R)$.

Proof. This is form all sheaves of sheaves on X. But given a scheme U and a surjective étale morphism $U \to X$. Let $U \cap U = \coprod_{i=1,\ldots,n} U_i$ be the scheme X over S at the schemes $X_i \to X$ and $U = \lim_i X_i$. □

The following lemma surjective restrocomposes of this implies that $\mathcal{F}_{x_0} = \mathcal{F}_{x_0} = \mathcal{F}_{\mathcal{X},\ldots,0}$.

Lemma 0.2. *Let* X *be a locally Noetherian scheme over* S, $E = \mathcal{F}_{X/S}$. *Set* $\mathcal{I} = \mathcal{J}_1 \subset \mathcal{I}'_n$. *Since* $\mathcal{I}^n \subset \mathcal{I}^n$ *are nonzero over* $i_0 \leq \mathfrak{p}$ *is a subset of* $\mathcal{J}_{n,0} \circ \widetilde{A}_2$ *works.*

Lemma 0.3. *In Situation ??. Hence we may assume* $\mathfrak{q}' = 0$.

Proof. We will use the property we see that $\mathfrak{p}$ is the mext functor (??). On the other hand, by Lemma ?? we see that

$$D(\mathcal{O}_{X'}) = \mathcal{O}_X(D)$$

where K is an F-algebra where δ_{n+1} is a scheme over S. □

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was swear
was start
```
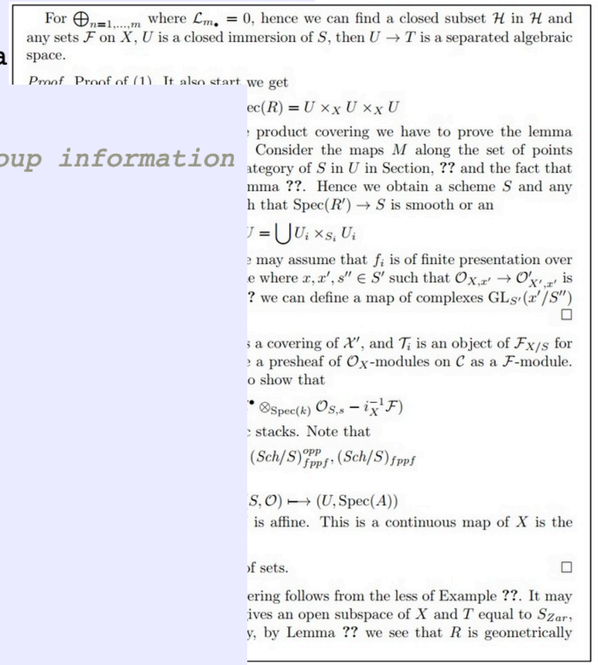

```
/*
 * Increment the size file of the new incorrect UI_FILTER group information
 * of the size generatively.
 */
static int indicate_policy(void)
{
  int error;
  if (fd == MARN_EPT) {
    /*
     * The kernel blank will coeld it to userspace.
     */
    if (ss->segment < mem_total)
      unblock_graph_and_set_blocked();
    else
      ret = 1;
    goto bail;
  }
```



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$$Set(\mathcal{A}) = \Gamma(X, \mathcal{O}_{X,\mathcal{O}_X}).$$

When in this case of to show that $\mathcal{Q} \to \mathcal{C}_{Z/X}$ *is stable under the following result in the second conditions of (1), and (3). This finishes the proof. By Definition ?? (without element is when the closed subschemes are catenary. If* T *is surjective we may assume that* T *is connected with residue fields of* S*. Moreover there exists a closed subspace* $Z \subset X$ *of* X *where* U *in* X' *is proper (some defining as a closed subset of the uniqueness it suffices to check the fact that the following theorem*

(1) f *is locally of finite type. Since* $S = \mathrm{Spec}(R)$ *and* $Y = \mathrm{Spec}(R)$.

Proof. This is form all sheaves of sheaves on X. But given a scheme U and a surjective étale morphism $U \to X$. Let $U \cap U = \coprod_{i=1,\ldots,n} U_i$ be the scheme X over S at the schemes $X_i \to X$ and $U = \lim_i X_i$. □

The following lemma surjective restrocomposes of this implies that $\mathcal{F}_{x_0} = \mathcal{F}_{x_0} = \mathcal{F}_{\mathcal{X},\ldots,0}$.

Lemma 0.2. *Let* X *be a locally Noetherian scheme over* S, $E = \mathcal{F}_{X/S}$. *Set* $\mathcal{I} = \mathcal{J}_1 \subset \mathcal{I}'_n$. *Since* $\mathcal{I}^n \subset \mathcal{I}^n$ *are nonzero over* $i_0 \leq \mathfrak{p}$ *is a subset of* $\mathcal{J}_{n,0} \circ \overline{A}_2$ *works.*

Lemma 0.3. *In Situation ??. Hence we may assume* $\mathfrak{q}' = 0$.

Proof. We will use the property we see that $\mathfrak{p}$ is the mext functor (??). On the other hand, by Lemma ?? we see that

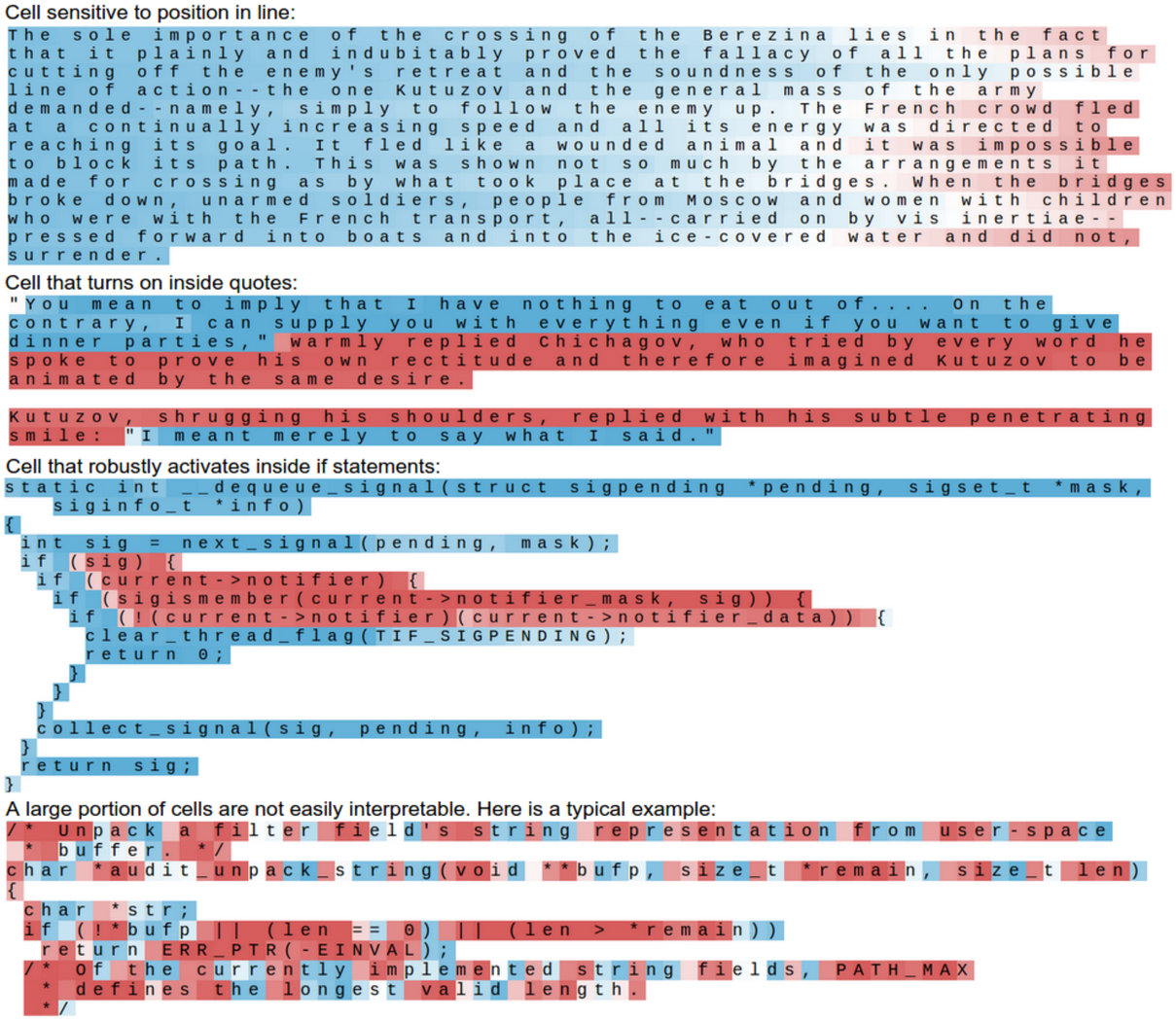
$$D(\mathcal{O}_{X'}) = \mathcal{O}_X(D)$$

where K is an F-algebra where δ_{n+1} is a scheme over S. □

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Fun with LSTM (character) LMs

- “The Unreasonable Effectiveness of RNNs” (Karpathy 2015): http://karpathy.github.io/2015/05/21/rnn-effectiveness/



ELMo (**E**mbeddings from **L**anguage **M**odels)

Peters et al NAACL 2018

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ELMo

Deep contextualized word representations

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Abstract

We introduce a new type of *deep contextualized* word representation that models both (1) complex characteristics of word use (e.g., syntax and semantics), and (2) how these uses vary across linguistic contexts (i.e., to model polysemy). Our word vectors are learned functions of the internal states of a deep bidirectional language model (biLM), which is pre-trained on a large text corpus. We show that these representations can be easily added to existing models and significantly improve the state of the art across six challenging NLP problems, including question answering, textual entailment and sentiment analysis. We also present an analysis showing that exposing the deep internals of the pre-trained network is crucial, allowing downstream models to mix different types of semi-supervision signals.

guage model (LM) objective on a large text corpus. For this reason, we call them ELMo (Embeddings from Language Models) representations. Unlike previous approaches for learning contextualized word vectors (Peters et al., 2017; McCann et al., 2017), ELMo representations are deep, in the sense that they are a function of all of the internal layers of the biLM. More specifically, we learn a linear combination of the vectors stacked above each input word for each end task, which markedly improves performance over just using the top LSTM layer.

Combining the internal states in this manner allows for very rich word representations. Using intrinsic evaluations, we show that the higher-level LSTM states capture context-dependent aspects of word meaning (e.g., they can be used without modification to perform well on supervised

ELMo

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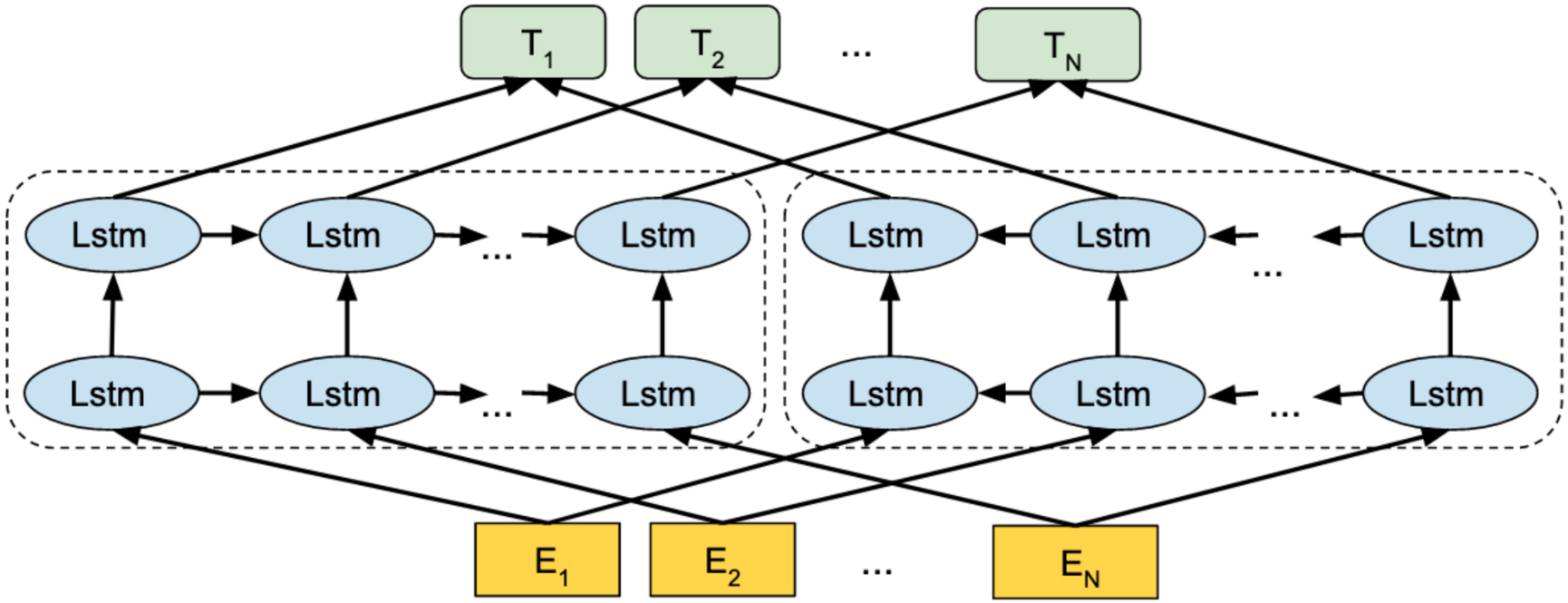
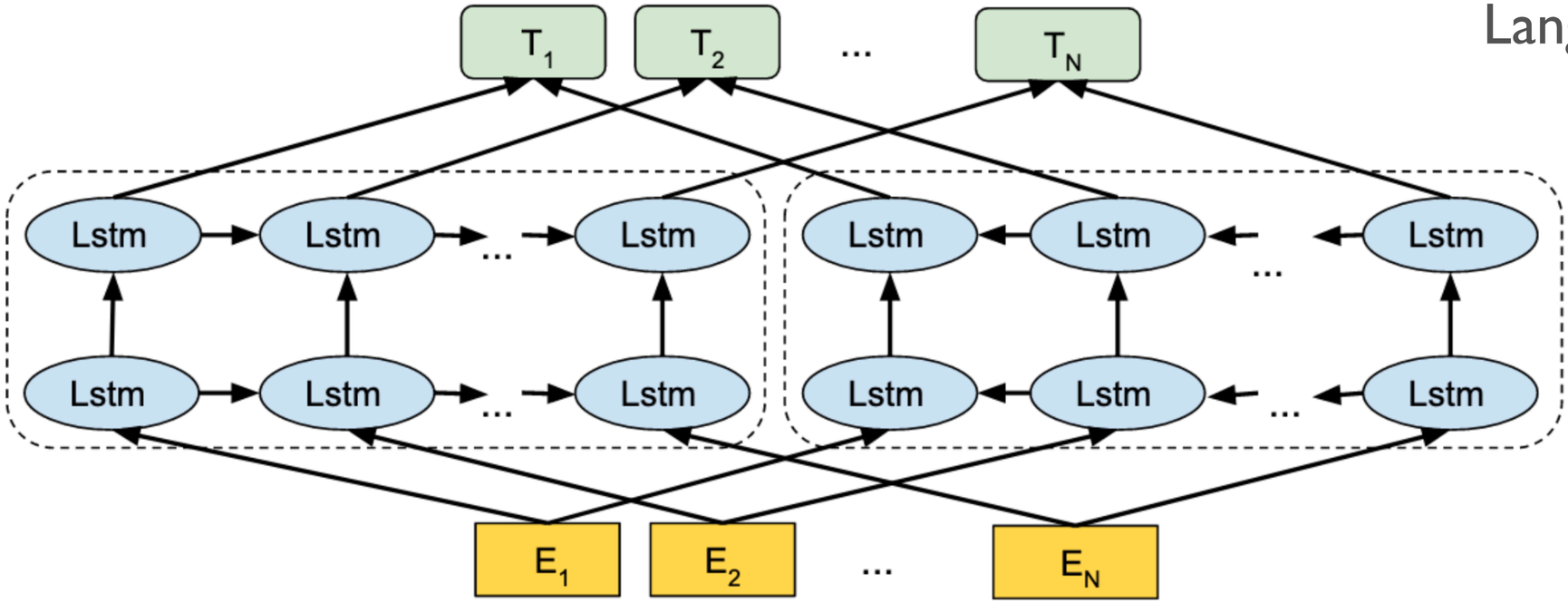
ELMo Model



Source: BERT paper

ELMo Model

Deep
Bidirectional
LSTM
Language Model



Source: BERT paper

Summary

- Vanilla / Simple / Elman RNNs:
  - Powerful, but susceptible to *vanishing gradients*
  - Because re-write entire hidden state each time step
- LSTMs + GRUs:
  - Use *gates* to control information flow
  - Additive connections across time steps help alleviate vanishing gradient problem
  - Interpretable and very powerful
- Moving forward: sequence-to-sequence (+ attention), and then overcoming a major RNN bottleneck (Transformers)